Non-concurrent, Coplanar Force Systems

Non-concurrent Systems

• The FBD does not have to be a point.
• In addition to summing forces in two directions, we can sum moments about any point on the FBD to generate a third independent equation.
• On any planar FBD in equilibrium, we can have three unknowns and still solve for all.

Problem Solving Strategy

1. Identify ALL forces and moments acting on the body by making a "free-body diagram" (FBD).
2. Invoke Newton’s First Law. (Sum forces and set total equal to zero. Sum moments and set total equal to zero.)
3. Solve, validate.

ΣF_x = 0 ⇒ A_x + B_x = 0
ΣF_y = 0 ⇒ A_y + B_y = 0

CCW +

ΣM_P = 0 ⇒ M + A_y x_A + B_y x_B − A_x y_A − B_x y_B = 0
ΣM_A = 0 ⇒ M − B_x (y_B − y_A) + B_y (x_B − x_A) = 0
ΣM_B = 0 ⇒ M − A_x (y_A − y_B) − A_y (x_B − x_A) = 0

These are NOT five independent equations with five unknowns! Choose any three of the five equations!
Using concurrent force systems, find force in $BC$ as a function of the engine’s weight.

\[
\sum F_x = 0 \Rightarrow F_{AC} = F_{AB} \\
\sum F_y = 0 \Rightarrow (F_{AC} + F_{AB})\sin 55^\circ - mg \Rightarrow F_{AB} = \frac{mg}{2\sin 55^\circ} \quad (1)
\]

Using non-concurrent force systems, find force in $BC$ as a function of the engine’s weight. Assume that weight is centered, left-to-right.

FBD of all below bar:

\[
\sum F_x = 0 \Rightarrow F_{BC} + F_{AB}\cos 55^\circ = 0 \quad (2)
\]

From (1) and (2),

\[
F_{BC} = -\frac{mg}{2\tan 55^\circ}
\]

\[
\sum F_y = 0 \Rightarrow T_1 + T_2 = mg \\
(+CCW) \sum M_g = 0 \Rightarrow T_1\left(\frac{L}{2}\right) - T_2\left(\frac{L}{2}\right) = 0 \Rightarrow T_1 = \frac{mg}{2} \quad (1)
\]

(Could also establish result above by considering symmetry.)

\[
\sum M_A = 0 \Rightarrow T_1(d\cos 55^\circ) + F_{bc}(d\sin 55^\circ) = 0 \\
F_{bc} = -\frac{T_1}{\tan 55^\circ} \quad (2)
\]

From (1) and (2),

\[
F_{bc} = -\frac{mg}{2\tan 55^\circ}
\]
Surface at B is smooth. Find reactions at A and B.

\[ \sum F_x = 0 \Rightarrow A_x - F = 0 \Rightarrow A_x = F = 8 \text{ lb} \]

\[ \sum M_A = 0 \Rightarrow -B_y(2 \text{ ft}) + F(1.5 \text{ ft}) = 0 \Rightarrow B_y = \frac{3}{4} F \]

\[ \sum F_y = 0 \Rightarrow A_y + B_y = 0 \Rightarrow A_y = 6 \text{ lb} \]

No vertical support at A. Find maximum reaction magnitudes at A and B if

\[ 1.5 \text{ ft} \leq x \leq 7.5 \text{ ft} \]

Find the force in the strut BC.

\[ \sum M_A = 0 \Rightarrow \frac{3}{5} F_{BC}(7 \text{ m}) + \frac{3}{5} F_{BC}(0.4 \text{ m}) - (8 \text{ kN})(3 \text{ m}) = 0 \]

\[ F_{BC} = 5.31 \text{ kN (T)} \]

(+CCW) \[ \sum M_A = 0 \]

Either \[ -W_x + (F_B \sin \theta)(x) = 0 \]

or \[ -W_x + (F_B \cos \theta)(4 \text{ m}) = 0 \]

\[ x \sin \theta = \frac{x(4 \text{ m})}{\sqrt{(4 \text{ m})^2 + x^2}} \]

\[ (4 \text{ m}) \cos \theta = \frac{(4 \text{ m})x}{\sqrt{(4 \text{ m})^2 + x^2}} \]

\[ \sum M_B = 0 \Rightarrow A_x(4 \text{ m}) - W(x) = 0 \Rightarrow A_x = \frac{x}{4 \text{ m}} W \]
Do these results make sense?

\[ F_B = \frac{\sqrt{(4 \text{ m})^2 + x^2}}{4 \text{ m}} W \]

\[ A_x = \frac{x}{4 \text{ m}} W \]
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2  When the forces acting on a FBD do not intersect at a common point, the system of forces is said to be non-concurrent. Newton’s first law will then yield three independent equations, which will allow us to solve for as many as three unknowns.

3 For concurrent systems, the FBD is a point. But for a non-concurrent force system, the FBD is not a point. We can sum moments about any point and expect to find a useful equation.

4 Because we can sum moments around any point, the number of equations that we can generate from Newton’s first law is unlimited. But because it can be proven that the number of independent equations cannot exceed three, we cannot solve for more than three unknowns.

5, 6 In this system, the desired force can be found without using a non-concurrent force system. Two FBDs and three force summations are enough to get the answer.
7, 8 We can also solve the problem using non-concurrent force systems. Note that the amount of work is about the same, whether we use concurrent or non-concurrent force systems. But as we shall see in the next example, some force systems simply are not concurrent.

9 Because each of the three equations used here contains only one of the unknown reactions, we do not need to solve equations simultaneously. Another way to find $A_y$ would be to sum moments about $B$. And yet another, faster way to get $A_y$ would be to realize that because the structure is a three-force member, the force at $A$ must pass through the point where $F$ and $B_y$ intersect and the force’s components must have the ratio of 3 to 4.

10 As seen in previous notes, $BC$ is a two-force member, which tells us the direction of the force $F_{bc}$. Although the moment of that force could be calculated using a cross product, the scalar method is often easier for coplanar systems. That is because the moment arms of the force components are often easily computed in such systems.

11 As seen in previous notes, this FBD uses the fact that because the structure is a three-force member, the direction of $F_y$ is known.
12 We begin by summing moments about point \( A \). Both versions of the moment summation are correct. To get the first one, visualize \( F_b \) acting at the point at the lower right where \( A_x \) and \( W \) intersect. Then only its vertical component would have a moment about \( A \). To get the second moment summation, visualize \( F_b \) acting at \( B \). Then only its horizontal component would have a moment about \( A \). Either approach is correct because sliding a force along its line of action does not change its moment arm. The two resulting equations may appear to give different results, but trigonometry shows that the results are the same.

We could have obtained the result for \( A_x \) first. Then we could have obtained the result for \( F_b \) by either force summation or moment summation.

13 Do the results seem reasonable? They are dimensionally correct. Also, both reactions increase as the applied load increases, which makes sense. But the result for \( A_x \) indicates that it could be larger than \( W \) or smaller than \( W \), depending on the value of \( x \). In fact, the result indicates that \( A_x \) could be zero. This makes sense when we realize that \( A_x \) is the only force that counteracts the moment of \( W \) about \( B \). When their moment arms are equal, \( A_x \) equals \( W \). When \( W \) has the greater moment arm, \( A_x \) must be greater than \( W \). When \( W \) has the smaller moment arm, \( A_x \) must be smaller than \( W \). And when \( x = 0 \), \( W \) has no moment about \( B \), so \( A_x \) vanishes. In that case, \( F_b \) all of \( W \) is supported by \( F_b \).
14 By summing moments about $B$, we avoid the need to compute the moments of the separate components of the 1.4 kN force.