Equilibrium of Concurrent, Coplanar Force Systems

ME 202

Concurrent, Coplanar Force System

• Coplanar system
  • The \textit{lines of action} of all forces lie in a common \textit{plane}.
• Concurrent system
  • The \textit{lines of action} of all forces intersect at a common \textit{point}.

Independent Equations

• In a given problem, we can find only as many \textit{unknowns} as we have independent equations.
• For a system of coplanar, concurrent forces, Newton’s First Law yields only two independent equations for a given FBD.

Find magnitude and direction of $\vec{F}$.

1. Make FBD.
2. Newton’s First Law
3. Solve, validate.

1. Figure is already a FBD.

2. \[ \sum \vec{F} = 0 \]
   \[ F (\cos \theta \hat{i} + \sin \theta \hat{j}) + (-4.5 \text{ kN})\hat{i} + \cdots \]
   \[ \cdots 7.5 \text{ kN}(-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}) + \cdots \]
   \[ \cdots 2.25 \text{ kN}(\cos 60^\circ \hat{i} - \sin 60^\circ \hat{j}) = 0 \]
3. Collect and equate components.
For the \( \hat{i} \) components:
\[ F \cos \theta - 4.5 \text{ kN} - 7.5 \text{ kN} \sin 30^\circ + 2.25 \text{ kN} \cos 60^\circ = 0 \] (1)
For the \( \hat{j} \) components:
\[ F \sin \theta - 7.5 \text{ kN} \cos 30^\circ - 2.25 \text{ kN} \sin 60^\circ = 0 \] (2)
Now, what?

**COUNT EQUATIONS AND UNKNOWNS!!!!!!**

Two equations, (1) and (2), and two unknowns, \( F \) and \( \theta \). Time to solve ….

From (1) and (2),
\[ F \cos \theta = 7.125 \text{ kN} \] (3)
\[ F \sin \theta = 8.444 \text{ kN} \] (4)
Now, what?
For the angle, divide (4) by (3) to get
\[ \tan \theta = 1.185 \Rightarrow \theta = 49.8^\circ \]
For the magnitude,
\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{(F \cos \theta)^2 + (F \sin \theta)^2} \]
\[ F = 11.0 \text{ kN} \]

Find the forces in cables \( AB \) and \( AC \).

\[ \sum F_x = 0 \Rightarrow F_{AB} \cos 12^\circ - F_{AC} \left( \frac{24}{25} \right) = 0 \] (1)
\[ \sum F_y = 0 \Rightarrow F_{AB} \sin 12^\circ + F_{AC} \left( \frac{7}{25} \right) - mg = 0 \] (2)

**COUNT EQUATIONS AND UNKNOWNS!!!!!!**

Solve (1) for \( F_{AB} \) in terms of \( F_{AC} \).
\[ F_{AB} = \frac{24}{25} F_{AC} \cos 12^\circ \] (3)
Use (3) to substitute for \( F_{AB} \) in (2).
\[ \left( \frac{24}{25} \sin 12^\circ + \frac{7}{25} \right) F_{AC} = mg \] (4)
Solve (4) for \( F_{AC} \).

\[
F_{AC} = \frac{25(12 \text{ kg})(9.81 \text{ m/s}^2)}{24 \tan 12^\circ + 7}
\]

\[
F_{AC} = 243 \text{ N}
\]  

(5)  

(6)

Use (5) for in (3) to get

\[ F_{AB} = 239 \text{ N} \]

Validation requires comparing results with someone else’s.

Find the forces in all five cables.

\[ \sum F_y = 0 \Rightarrow F_{AG} = mg = 294 \text{ N} \]

\[ \sum F_x = 0 \Rightarrow F_{BD} = \frac{4}{5} F_{BD} + F_{AB} \cos 60^\circ - F_{BC} = 0 \]

\[ F_{BC} = \frac{4}{5} F_{BD} + F_{AB} \cos 60^\circ = 562 \text{ N} \]
The free length of spring $AB$ is 2m. What is the mass of the suspended block?

\[ \sum F_y = 0 \Rightarrow F_{AD} = mg \quad (1) \]

\[ F_{AB} = k_{AB} \delta_{AB} = k_{AB} (l - l_0) = (30 \text{ N/m})(5 \text{ m} - 2 \text{ m}) = 90 \text{ N} \quad (2) \]

\[ \sum F_x = 0 \Rightarrow \frac{4}{5} F_{AB} - \frac{1}{\sqrt{2}} F_{AC} = 0 \Rightarrow F_{AC} = \frac{\sqrt{2}}{5} F_{AB} \quad (3) \]

\[ \sum F_y = 0 \Rightarrow \frac{3}{5} F_{AB} + \frac{1}{\sqrt{2}} F_{AC} - mg = 0 \quad (4) \]

From (2), (3) and (4),

\[ \frac{7}{5} F_{AB} = mg \]

\[ m = \frac{7}{5} \frac{F_{AB}}{g} = \frac{7}{5} \frac{90 \text{ N}}{9.81 \text{ m/s}^2} \]

\[ m = 12.8 \text{ kg} \]

For equilibrium, what is $F$ as a function of the angle $\theta$?

\[ \sum F_y = 0 \Rightarrow 2F \cos \theta - W = 0 \]

\[ F = \frac{W}{2 \cos \theta} \]

N. B. $\theta \to 90^\circ \Rightarrow F \to \infty$

(Latin “nota bene,” which means “note well.”)
Concurrent, Coplanar Force Systems

2 Concurrent, coplanar systems of forces are the easiest systems to analyze, because the FBD is a point. Since the size of a point is zero, moment arms do not exist, all moments are zero and moment summation is irrelevant.

3 The only independent equations available from the application of Newton’s first law to the FBD of a point are force summations in two perpendicular directions. With two independent equations, we can handle no more than two unknowns.

4 The entire solution plan can be reduced to the three steps inside the box on this slide. In the second step, we apply the generic equation, which is Newton’s first law, to the FBD to obtain specific equations for the problem. Since the problem is coplanar, and therefore two-dimensional, we have no more than two vector components to consider.

5 Once specific equations have been written, we can easily check that we have no more than two unknowns.
In this problem, the two equations are not linear, algebraic equations. The unknown angle is buried inside trigonometric functions, and so cannot be found by linear operations, such as addition, subtraction and multiplication. Instead, we must use what we know about trigonometry to find a way to isolate one variable. Inspection of the equations shows that in dividing one by the other, the unknown force cancels, leaving the angle as the only unknown in the resulting equation. Given the angle, we could use it in either Eq. (1) or Eq. (2) to find the force. But the method shown here also works, and would work even if we had not found the angle.

Validation of these results would require comparing them with someone else’s.

In the FBD for this problem, we use what we know about the directions of cable forces. Thus we find that we have only two unknowns and can expect to solve the problem using only Newton’s first law. If we did not use what we know about cable forces and so drew unknown horizontal and vertical components for each cable, we would have four unknowns and not enough equations.

The two specific equations generated by applying Newton’s first law can be solved with linear algebraic operations. One approach is shown on these two slides.
10 A FBD of the suspended mass gives one equation with one unknown, so we can immediately find the force in the cable connecting the mass to point $A$. To find the other cable forces, we will need to see them in other FBDs.

11 The FBD of point $A$ has two more unknown cable forces, but also yields two more equations. Inspection of the diagram shows that both unknown forces will appear in a summation of horizontal forces, but only one unknown force will appear in a summation of vertical forces. Doing the vertical summation first allows us to find one of the unknowns immediately. That result can than be used in the horizontal summation to find the other unknown force. Finding situations where we have one equation and one unknown, what we might call “knocking them off one at a time,” can often save time. It avoids the need to solve equations simultaneously.

12 The strategy used on the previous slide is used again here to find the remaining two required cable forces.

13 This is similar to the traffic light problem on slide 7, but now we account for stretching of the springs. Eq. (1) has two unknowns, the desired mass and one of the spring forces.
14, 15  Because we know both the current length, the free length and the stiffness of spring $AB$, we can compute the force in that spring. That reduces the number of unknowns in the FBD of point $A$, which allows us to solve for the other two forces. These two slides show how Eq. (4) then leads to the required mass.

16  Using what we know about tension in a cable wrapped around a pulley, we can find the desired relationship. The result shows that no amount of force could make the angle 90°. This is consistent with the fact that if the cable forces were horizontal, they could not support the weight.