Cables, Springs and Pulleys

ME 202

Ideal Cable

- Neglect weight (massless)
- Neglect bending stiffness
- Force parallel to cable
- Force only tensile (cable taut)
- Neglect stretching (inextensible)

Linear Spring - 1

- Massless, like an ideal cable, but extensible and resists either stretching or shortening.
- Force is either tensile or compressive.
- When the force is zero, the spring is undeformed and its length is called its free length or natural length.
- When the force is not zero, it is proportional to the spring’s deformation (stretch or contraction).

Sketch a FBD of the traffic light.
Linear Spring - 2

- When the force is not zero, it is proportional to the spring’s deformation (stretch or contraction).
- The constant of proportionality between the force in a spring and its deformation is called the spring’s stiffness or its spring rate.
- Stiffness is usually represented by the letter k.

Equation for Spring

\[
F = k(l - l_0)
\]
(stretch)  \( l_0 = \) "free" length \((F = 0)\)

\[
F = k(l_0 - l)
\]
(contraction)

- Direction assumed for spring load cannot be wrong!
- Equation used for spring load can be wrong!

Pulleys

- Mass of pulley is negligible (zero).
- Friction at pin is negligible.
- Cable does not slip on pulley.
- Cable tension is constant. (On FBD of pulley, sum moments about pin at center of pulley.)
Sketch FBD for the pulley and the weight suspended from the pulley.

FBD for the pulley and the weight suspended from the pulley.
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2 Although none of these assumptions is exactly correct, each is approximately correct for many real systems. Neglecting bending stiffness means assuming that the cable can change direction at a sharp corner, that is, a corner with zero radius. This is more nearly correct for a piece of twine or string than it is for a wire cable. But even the wire cable has much greater resistance to being stretched than is does to being bent. It follows that neglecting bending stiffness is a good first approximation. Given that assumption, it can be shown that the force in the cable must be parallel to the cable and that it can only be tension. (As the saying goes, “You can’t push a rope.”) The final assumption on this slide expresses that fact that wire cables stretch relatively little unless they are “overloaded,” which is being subjected to tension larger than that for which they are designed. In this class, we assume that cables are not overloaded.

3 To sketch the FBD, we must cut the traffic light away from everything to which it is attached. That means cutting the two cables. At every place we cut, we must consider the possibility of reactions, which may be forces or moments.

With a FBD we should always include a coordinate system, because we will almost always need to resolve forces into Cartesian components.

Because cables have no bending stiffness, they cannot exert moments on the light. And because the forces on the cables are parallel to the cables and must be tensile, the FBD must be as shown. Note that the weight of
the light is written as the product of its mass and the local gravitational constant. Had we been given the light’s weight, rather than its mass, it would be appropriate to write the weight as a single symbol or number.

4 Unlike an ideal cable, a linear spring is able to resist forces that are tensile, which tend to stretch the spring, or forces that are compressive, which tend to shorten the spring. When it carries no force, a spring is neither stretched nor compressed.

5 A linear spring is called linear because the relationship between the force carried by the spring and the spring’s deformation is linear; each is proportional to the other.

6 It does not matter whether we assume that the force in a spring is tensile or compressive. (If we assume incorrectly, we will know soon because we will find a negative value for the force.) But once we have chosen between tension and compression, only one of the equations shown is correct. It is good practice always to assume tension or always to assume compression. That way, you need to remember only one equation and there is little chance that you will write the wrong one. Most introductory texts (ours included) suggest that tension is the better choice, simply because it appears more often than compression in such texts.
7 For a FBD of the block, we must cut the apparatus from which it is suspended. If we make the cut through the spring, then we know that the force is vertical, since it must be parallel to the spring. The only other force is the block's weight. If we assume tension in the spring, Newton's third law tells us that the spring is pulling up on the block.

To isolate point $A$, we must cut through all three springs. Each force is parallel to one of the springs. Then assuming tension in all springs leads to the FBD shown.

8 As we did with the ideal cable and the ideal spring, we assume several things for the ideal pulley. A FBD of a pulley will have three forces, one supporting the pulley at the central pin and two exerted by the two sides of the cable that wraps around part of the pulley. The force at the pin has no moment about the pin because its moment arm is zero. The moment arm of each of the other two forces is the radius of the pulley. One of these two moments is clockwise and the other counterclockwise. Since they must cancel each other when added, so that the sum of the moments is zero, the two forces must be the same.

9 It is not necessary to repeat the analysis described on the previous slide. We can immediately use the result of that analysis to show that the tension is the same on both sides of the cable.

When we cut the cable above the block, we use Newton's third law to see that the tension pulling up on the cable part below the cut is equal and opposite to the tension pulling down on the cable part above the cut.