FE/EI/EIT REVIEW

ENGINEERING ECONOMICS

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CASH FLOW ANALYSIS

The basic reason for performing economic analysis is to make a choice between mutually exclusive projects that are competing for limited resources. The cost performance of each project will depend on the timing and levels of its expenditures. The techniques of computing cash flow equivalence permit us to bring competing project cash flows to a common basis for comparison. The common basis depends on the prevailing interest rate. Two cash flows that are equivalent at a given interest rate will not be equivalent at a different interest rate. The basic techniques for converting cash flows from one point in time to another are presented in the next section.

Cash Flow Conversion Factors

Cash flow conversion involves the transfer of project funds from one point in time to another. The following notation is used for the variables involved in the conversion process:

- \( i \) = interest rate per period
- \( n \) = number of interest periods
- \( P \) = a present sum of money
- \( F \) = a future sum of money
- \( A \) = a uniform end-of-period cash receipt or disbursement
- \( G \) = a uniform arithmetic gradient increase in period-by-period payments or disbursements.

In many cases, the interest rate used in performing economic analysis is set equal to the minimum attractive rate of return (MARR) of the decision maker. The MARR is also sometimes referred to as hurdle rate, required internal rate of return (IRR), return on investment (ROI), or discount rate. The value of MARR is chosen with the objective of maximizing the economic performance of a project.

COMPOUND AMOUNT FACTOR

The procedure for the single payment compound amount factor finds a future sum of money, \( F \), that is equivalent to a present sum of money, \( P \), at a specified interest rate, \( i \), after \( n \) periods. This is calculated as:

\[
F = P(1 + i)^n.
\]

A graphic representation of the relationship between \( P \) and \( F \) is shown in Figure 1.

![Figure 1. Single Payment Compound Amount Cash Flow](image-url)
Example
A sum of $5,000 is deposited in a project account and left there to earn interest for 15 years. If the interest rate per year is 12%, the compound amount after 15 years can be calculated as shown below:

\[
F = 5,000(1 + 0.12)^{15}
\]

\[
= 27,367.85.
\]

PRESENT WORTH FACTOR
The present worth factor computes \( P \) when \( F \) is given. The present worth factor is obtained by solving for \( P \) in the equation for the compound amount factor. That is,

\[
P = F(1 + i)^{-n}.
\]

Suppose it is estimated that $15,000 would be needed to complete the implementation of a project five years from now, how much should be deposited in a special project fund now so that the fund would accrue to the required $15,000 exactly five years from now? If the special project fund pays interest at 9.2% per year, the required deposit would be:

\[
P = 15,000(1 + 0.092)^{-5}
\]

\[
= 9,660.03.
\]

UNIFORM SERIES PRESENT WORTH FACTOR
The uniform series present worth factor is used to calculate the present worth equivalent, \( P \), of a series of equal end-of-period amounts, \( A \). Figure 2 shows the uniform series cash flow. The derivation of the formula uses the finite sum of the present worths of the individual amounts in the uniform series cash flow as shown below.

\[
P = \sum_{i=1}^{n} A (1 + i)^{-t} = A \left[ \frac{(1 + i)^n - 1}{i(1 + i)^n} \right].
\]
Figure 2. Uniform Series Cash Flow

Example
Suppose the sum of $12,000 must be withdrawn from an account to meet the annual operating expenses of a multi-year project. The project account pays interest at 7.5% per year compounded on an annual basis. If the project is expected to last ten years, how much must be deposited in the project account now so that the operating expenses of $12,000 can be withdrawn at the end of every year for ten years? The project fund is expected to be depleted to zero by the end of the last year of the project. The first withdrawal will be made one year after the project account is opened, and no additional deposits will be made in the account during the project life cycle. The required deposit is calculated to be:

\[
P = \frac{12,000[(1 + 0.075)^{10} - 1]}{0.075(1 + 0.075)^{10}}
\]

\[
= \$82,368.92.
\]

UNIFORM SERIES CAPITAL RECOVERY FACTOR

The capital recovery formula is used to calculate the uniform series of equal end-of-period payments, \( A \), that are equivalent to a given present amount, \( P \). This is the converse of the uniform series present amount factor. The equation for the uniform series capital recovery factor is obtained by solving for \( A \) in the uniform series present amount factor. That is,

\[
A = P \left[ \frac{i(1 + i)^n}{(1 + i)^n - 1} \right].
\]

Example
Suppose a piece of equipment needed to launch a project must be purchased at a cost of $50,000. The entire cost is to be financed at 13.5% per year and repaid on a monthly installment schedule over four years. It is desired to calculate what the monthly loan payments will be. It is assumed that the first loan payment will be made exactly one month after the equipment is financed. If the...
interest rate of 13.5% per year is compounded monthly, then the interest rate per month will be $13.5\%/12 = 1.125\%$ per month. The number of interest periods over which the loan will be repaid is $4(12) = 48$ months. Consequently, the monthly loan payments are calculated to be:

$$A = \frac{50,000\left[0.01125(1 + 0.01125)^{48}\right]}{(1 + 0.01125)^{48} - 1} = $1,353.82.$$

**UNIFORM SERIES COMPOUND AMOUNT FACTOR**

The series compound amount factor is used to calculate a single future amount that is equivalent to a uniform series of equal end-of-period payments. The cash flow is shown in Figure 3. Note that the future amount occurs at the same point in time as the last amount in the uniform series of payments. The factor is derived as shown below:

$$F = \sum_{i=1}^{n} A(1+i)^{n-i} = A\left[\frac{(1+i)^n - 1}{i}\right].$$

![Figure 3. Uniform Series Compound Amount Cash Flow](image)

**Example**

If equal end-of-year deposits of $5,000 are made to a project fund paying 8% per year for ten years. How much can be expected to be available for withdrawal from the account for capital expenditure immediately after the last deposit is made?

$$F = 5,000\left[\frac{(1+0.08)^{10} - 1}{0.08}\right] = $72,432.50.$$
UNIFORM SERIES SINKING FUND FACTOR

The sinking fund factor is used to calculate the uniform series of equal end-of-period amounts, \( A \), that are equivalent to a single future amount, \( F \). This is the reverse of the uniform series compound amount factor. The formula for the sinking fund is obtained by solving for \( A \) in the formula for the uniform series compound amount factor. That is,

\[
A = F \left[ \frac{i}{(1 + i)^n - 1} \right].
\]

Example
How large are the end-of-year equal amounts that must be deposited into a project account so that a balance of \$75,000\) will be available for withdrawal immediately after the twelfth annual deposit is made? The initial balance in the account is zero at the beginning of the first year. The account pays 10% interest per year. Using the formula for the sinking fund factor, the required annual deposits are:

\[
A = \frac{75,000 \left[ \frac{0.10}{(1 + 0.10)^{12} - 1} \right]}{10.00}
= \$3507.25.
\]

CAPITALIZED COST FORMULA

Capitalized cost refers to the present value of a single amount that is equivalent to a perpetual series of equal end-of-period payments. This is an extension of the series present worth factor with an infinitely large number of periods. This is shown graphically in Figure 4.

Using the limit theorem from calculus as \( n \) approaches infinity, the series present worth factor reduces to the following formula for the capitalized cost:

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\[ P = \lim_{n \to \infty} A \left[ \frac{(1 + i)^n - 1}{i(1 + i)^n} \right] \]

\[ = A \left( \frac{1}{i} \right) \]

Example
How much should be deposited in a general fund to service a recurring public service project to the tune of $6,500 per year forever if the fund yields an annual interest rate of 11%? Using the capitalized cost formula, the required one-time deposit to the general fund is:

\[ P = \frac{6,500}{0.11} = 59,090.91. \]

The formulas presented above represent the basic cash flow conversion factors. The factors are widely tabulated, for convenience, in engineering economy books. Several variations and extensions of the factors are available. Such extensions include the arithmetic gradient series factor and the geometric series factor. Variations in the cash flow profiles include situations where payments are made at the beginning of each period rather than at the end and situations where a series of payments contains unequal amounts. Conversion formulas can be derived mathematically for those special cases by using the basic factors presented above.

ARITHMETIC GRADIENT SERIES

The gradient series cash flow involves an increase of a fixed amount in the cash flow at the end of each period. Thus, the amount at a given point in time is greater than the amount at the preceding period by a constant amount. This constant amount is denoted by \( G \). Figure 5 shows the basic gradient series in which the base amount at the end of the first period is zero. The size of the cash flow in the gradient series at the end of period \( t \) is calculated as:

\[ A_t = (t-1)G, \quad t = 1, 2, \ldots, n. \]

The total present value of the gradient series is calculated by using the present amount factor to convert each individual amount from time \( t \) to time 0 at an interest rate of \( i\% \) per period and summing up the resulting present values. The finite summation reduces to a closed form as shown below:
Example
The cost of supplies for a 10-yr project increases by $1,500 every year starting at the end of year two. There is no supplies cost at the end of the first year. If interest rate is 8% per year, determine the present amount that must be set aside at time zero to take care of all the future supplies expenditures. We have $G = 1,500$, $i = 0.08$, and $n = 10$. Using the arithmetic gradient formula, we obtain:

$$P = \sum_{t=1}^{n} A_t (1+i)^{-t}$$

$$= \sum_{t=1}^{n} (t-1)G(1+i)^{-t}$$

$$= G \sum_{t=1}^{n} (t-1)(1+i)^{-t}$$

$$= G \left[ \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \right]$$

In many cases, an arithmetic gradient starts with some base amount at the end of the first period and then increases by a constant amount thereafter. The nonzero base amount is denoted as $A_1$. Figure 6 shows this type of cash flow.
The calculation of the present amount for such cash flows requires breaking the cash flow into a uniform series cash flow of amount $A_1$ and an arithmetic gradient cash flow with zero base amount. The uniform series present worth formula is used to calculate the present worth of the uniform series portion while the basic gradient series formula is used to calculate the gradient portion. The overall present worth is then calculated as:

$$P = P_{\text{uniform series}} + P_{\text{gradient series}}$$

$$= A_1 \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + G \left[ \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \right]$$

**INCREASING GEOMETRIC SERIES CASH FLOW**

In an increasing geometric series cash flow, the amounts in the cash flow increase by a constant percentage from period to period. There is a positive base amount, $A_1$, at the end of period one. Figure 7 shows an increasing geometric series. The amount at time $t$ is denoted as:

$$A_t = A_{t-1}(1+j), \quad t = 2, 3, ..., n$$

where $j$ is the percentage increase in the cash flow from period to period. By doing a series of back substitutions, we can represent $A_t$ in terms of $A_1$ instead of in terms of $A_{n+1}$ as shown below:

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The formula for calculating the present worth of the increasing geometric series cash flow is derived by summing the present values of the individual cash flow amounts. That is,

\[ P = \sum_{i=1}^{n} A_i (1+i)^{-t} \]

\[ = \sum_{i=1}^{n} [A_i (1+j)^{t-1}] (1+i)^{-t} \]

\[ = \frac{A_1}{(1+j)} \sum_{i=1}^{n} \left( \frac{1+j}{1+i} \right)^{t} \]

\[ = A_1 \left[ \frac{1 - (1+j)^n (1+i)^{-n}}{(i-j)} \right], \quad i \neq j \]

If \( i=j \), the formula above reduces to the limit as \( i \to j \), shown below:

- \( A_2 = A_1(1+j) \)
- \( A_3 = A_2(1+j) = A_1(1+j)(1+j) \)

\[ \ldots \]

\[ A_i = A_1(1+j)^{i-1}, \quad t = 1, 2, 3, \ldots, n \]
\[ P = \frac{nA_1}{(1+i)} \quad i = j \]

**Example**

Suppose funding for a five-year project is to increase by 6% every year with an initial funding of $20,000 at the end of the first year. Determine how much must be deposited into a budget account at time zero in order to cover the anticipated funding levels if the budget account pays 10% interest per year. We have \( j = 6\% \), \( i = 10\% \), \( n = 5 \), \( A_1 = $20,000 \). Therefore,

\[
P = 20,000 \left[ \frac{1}{1 + 0.06} \left( \frac{1 + 0.10}{1 + 0.06} \right)^5 \right]
\]

\[
= 20,000 \times 4.2267
\]

\[
= $84,533.60
\]

**DECREASING GEOMETRIC SERIES CASH FLOW**

In a decreasing geometric series cash flow, the amounts in the cash flow decrease by a constant percentage from period to period. The cash flow starts at some positive base amount, \( A_1 \), at the end of period one. Figure 8 shows a decreasing geometric series. The amount at time \( t \) is denoted as:

\[ A_t = A_{t-1}(1-j), \quad t = 2, 3, ..., n \]

where \( j \) is the percentage decrease in the cash flow from period to period. As in the case of the increasing geometric series, we can represent \( A_t \) in terms of \( A_1 \).

\[
A_2 = A_1(1-j)
\]

\[
A_3 = A_2(1-j) = A_1(1-j)(1-j)
\]

\[
\ldots
\]

\[
A_t = A_1(1-j)^{t-1}, \quad t = 1, 2, 3, ..., n
\]

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Figure 8. Decreasing Geometric Series Cash Flow

The formula for calculating the present worth of the decreasing geometric series cash flow is derived by finite summation as in the case of the increasing geometric series. The final formula is:

\[ P = A_n \left[ \frac{1 - (1 - j)^n (1 + i)^{-n}}{(1 + j)} \right] \]

Example

The contract amount for a three-year project is expected to decrease by 10% every year with an initial contract of $100,000 at the end of the first year. Determine how much must be available in a contract reservoir fund at time zero in order to cover the contract amounts. The fund pays 10% interest per year. Since \( j = 10\% \), \( i = 10\% \), \( n = 3 \), \( A_1 = $100,000 \), we should have:

\[ P = 100,000 \left[ \frac{1 - (1 - 0.10)^3 (1 + 0.10)^{-3}}{0.10 + 0.10} \right] \]

\[ = $100,000(2.2615) \]

\[ = $226,150 \]

INTERNAL RATE OF RETURN

The internal rate of return (IRR) for a cash flow is defined as the interest rate that equates the future worth at time \( n \) or present worth at time 0 of the cash flow to zero. If we let \( i' \) denote the internal rate of return, then we have:

\[ FW_{t=n} = \sum_{t=0}^{n} (\pm A_t)(1 + i')^{n-t} = 0 \]

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where "+" is used in the summation for positive cash flow amounts or receipts and "-" is used for negative cash flow amounts or disbursements. \( A_t \) denotes the cash flow amount at time \( t \), which may be a receipt (+) or a disbursement (-). The value of \( i^* \) is referred to as discounted cash flow rate of return, internal rate of return, or true rate of return. The procedure above essentially calculates the net future worth or the net present worth of the cash flow. That is:

\[
\text{Net Future Worth} = (\text{Future Worth of Receipts}) - (\text{Future Worth of Disbursements})
\]

\[
\text{Net Present Worth} = (\text{Present Worth of Receipts}) - (\text{Present Worth of Disbursements})
\]

Setting the NPW or NFW equal to zero and solving for the unknown variable \( i \), determines the internal rate of return of the cash flow.

**BENEFIT-COST RATIO**

The benefit cost ratio of a cash flow is the ratio of the present worth of benefits to the present worth of costs. This is defined as:

\[
\frac{\sum_{t=0}^{n} B_t (1+i)^{-t}}{\sum_{t=0}^{n} C_t (1+i)^{-t}} = \frac{PW_{\text{benefit}}}{PW_{\text{costs}}}
\]

where \( B_t \) is the benefit (receipt) at time \( t \) and \( C_t \) is the cost (disbursement) at time \( t \). If the benefit-cost ratio is greater than one, then the investment is acceptable. If the ratio is less than one, the investment is not acceptable. A ratio of one indicates breakeven situation for the project.

**SIMPLE PAYBACK PERIOD**

Payback period refers to the length of time it will take to recover an initial investment. The approach does not consider the impact of the time value of money. Consequently, it is not an accurate method of evaluating the worth of an investment. However, it is a simple technique that is used widely to perform a "quick-and-dirty" assessment of investment performance. Also, the technique considers only the initial cost. Other costs that may occur after time zero are not included in the calculation. The payback period is defined as the smallest value of \( n \) \((n_{\text{min}})\) that satisfies the following expression:

\[
PW_{i=0} = \sum_{i=0}^{n} (\pm A_i) (1+i)^{-i} = 0
\]

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where $R_t$ is the revenue at time $t$ and $C_0$ is the initial investment. The procedure calls for a simple addition of the revenues period by period until enough total has been accumulated to offset the initial investment.

**Example**

An organization is considering installing a new computer system that will generate significant savings in material and labor requirements for order processing. The system has an initial cost of $50,000. It is expected to save the organization $20,000 a year. The system has an anticipated useful life of five years with a salvage value of $5,000. Determine how long it would take for the system to pay for itself from the savings it is expected to generate. Since the annual savings are uniform, we can calculate the payback period by simply dividing the initial cost by the annual savings. That is:

\[
\frac{n_{\text{min}}}{\sum_{t=1}^{n_{\text{min}}} R_t} \geq C_0
\]

\[
\begin{align*}
\text{Annual Savings} & = 20,000 \\
\text{Initial Cost} & = 50,000 \\
\text{Payback Period} & = \frac{50,000}{20,000} = 2.5 \text{ years}
\end{align*}
\]

Note that the salvage value of $5,000 is not included in the above calculation since the amount is not realized until the end of the useful life of the asset (i.e., after five years). In some cases, it may be desired to consider the salvage value. In that case, the amount to be offset by the annual savings will be the net cost of the asset. In that case, we would have:

\[
\begin{align*}
\text{Net Cost} & = 50,000 - 5,000 \\
\text{Payback Period} & = \frac{50,000 - 5,000}{20,000} = 2.25 \text{ years}
\end{align*}
\]

If there are tax liabilities associated with the annual savings, those liabilities must be deducted from the savings before calculating the payback period.

**INVESTMENT LIFE FOR MULTIPLE RETURNS**

The time it takes an amount to reach a certain multiple of its initial level is often of interest in many investment scenarios. The "Rule of 72" is one simple approach to calculating how long it will take an investment to double in value at a given interest rate per period. The Rule of 72 gives the following formula for estimating the doubling period:

\[
n = \frac{72}{i}
\]

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where \( i \) is the interest rate expressed in percentage. Referring to the single payment compound amount factor, we can set the future amount equal to twice the present amount and then solve for \( n \), the number of periods. That is, \( F = 2P \). Thus,

\[
2P = P(1 + i)^n.
\]

Solving for \( n \) in the above equation yields an expression for calculating the exact number of periods required to double \( P \):

\[
n = \frac{\ln(2)}{\ln(1 + i)}
\]

where \( i \) is the interest rate expressed in decimals. In the general case, for exact computation, the length of time it would take to accumulate \( m \) multiple of \( P \) is expressed as:

\[
n = \frac{\ln(m)}{\ln(1 + i)}
\]

where \( m \) is the desired multiple. For example, at an interest rate of 5% per year, the time it would take an amount, \( P \), to double in value (\( m=2 \)) is 14.21 years. This, of course, assumes that the interest rate will remain constant throughout the planning horizon. Table 1 presents a tabulation of the values calculated from both approaches. Figure 9 shows a graphical comparison of the Rule of 72 to the exact calculation.

<table>
<thead>
<tr>
<th>( i % )</th>
<th>( n ) (Rule of 72)</th>
<th>( n ) (Exact Value)</th>
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<tr>
<td>0.25</td>
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</table>

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Effects of Inflation

Inflation is a major player in financial and economic analyses of projects. Multi-year projects are particularly subject to the effects of inflation. Inflation can be defined as the decline in purchasing power of money. Some of the most common causes of inflation are:

- Increase in amount of currency in circulation
- Shortage of consumer goods
- Escalation of the cost of production
- Arbitrary increase of prices by resellers

The general effects of inflation are felt in terms of increase in the prices of goods and decrease in the worth of currency. In cash flow analysis, return on investment (ROI) for a project will be affected by time value of money as well as inflation. The real interest rate \( d \) is defined as the desired rate of return in the absence of inflation. When we talk of "today's dollars" or "constant dollars," we are referring to the use of real interest rate. Combined interest rate \( i \) is the rate of return combining real interest rate and inflation rate. If we denote the inflation rate as \( j \), then the relationship between the different rates can be expressed as:
Thus, the combined interest rate can be expressed as:

$$i = d + j$$

Note that if \(j=0\) (i.e., no inflation), then \(i = d\). We can also define commodity escalation rate \((g)\) as the rate at which individual commodity prices escalate. This may be greater than or less than the overall inflation rate. In practice, several measures are used to convey inflationary effects. Some of these are Consumer Price Index, Producer Price Index, and Wholesale Price Index. A "market basket" rate is defined as the estimate of inflation based on a weighted average of the annual rates of change in the costs of a wide range of representative commodities. A "then-current" cash flow is a cash flow that explicitly incorporates the impact of inflation. A "constant worth" cash flow is a cash flow that does not incorporate the effect of inflation. The real interest rate, \(d\), is used for analyzing constant worth cash flows. Figure 10 shows constant worth and then-current cash flows.

![Figure 10: Cash Flows for Effects of Inflation](image)

The then-current cash flow in the figure is the equivalent cash flow considering the effect of inflation. \(C_k\) is what it would take to buy a certain "basket" of goods after \(k\) time periods if there was no inflation. \(T_k\) is what it would take to buy the same "basket" in \(k\) time period if inflation was taken into account. For the constant worth cash flow, we have:

$$C_k = T_0, \quad k = 1, 2, \ldots, n,$$

and for the then-current cash flow, we have:

$$T_k = T_0(1 + j)^k, \quad k = 1, 2, \ldots, n$$

where \(j\) is the inflation rate. If \(C_k = T_0 = $100\) under the constant worth cash flow, then we mean $100 worth of buying power. If we are using the commodity escalation rate, \(g\), then we will have:

$$T_k = T_0(1 + g)^k, \quad k = 1, 2, \ldots, n$$

Thus, a then-current cash flow may increase based on both a regular inflation rate \((j)\) and a commodity escalation rate \((g)\). We can convert a then-current cash flow to a constant worth cash flow by using the following relationship:

$$C_k = T_0(1 + j)^k, \quad k = 1, 2, \ldots, n$$

If we substitute \(T_k\) from the commodity escalation cash flow into the expression for \(C_k\) above, we get:

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\[ C_k = T_0(1 + j)^k \]
\[ = T_0[(1 + g)(1 + j)]^k, \quad k = 1, 2, \ldots, n \]

Note that if \( g = 0 \) and \( j = 0 \), the \( C_k = T_0 \). That is, no inflationary effect. We now define effective commodity escalation rate \( (v) \) as:

\[ v = \frac{(1 + g)(1 + j)}{(1 + j)} - 1 \]

and we can express the commodity escalation rate \( (g) \) as:

\[ g = v + j + vj. \]

Inflation can have a significant impact on the financial and economic aspects of a project. Inflation may be defined, in economic terms, as the increase in the amount of currency in circulation, resulting in a relatively high and sudden fall in its value. To a producer, inflation means a sudden increase in the cost of items that serve as inputs for the production process (equipment, labor, materials, etc). To the retailer, inflation implies an imposed higher cost of finished products. To an ordinary citizen, inflation portends an unbearable escalation of prices of consumer goods. All these views are interrelated in a project management environment.

The amount of money supply, as a measure of a country’s wealth, is controlled by the government. With no other choice, governments often feel impelled to create more money or credit to take care of old debts and pay for social programs. When money is generated at a faster rate than the growth of goods and services, it becomes a surplus commodity, and its value (purchasing power) will fall. This means that there will be too much money available to buy only a few goods and services. When the purchasing power of a currency falls, each individual in a product’s life cycle has to dispense more of the currency in order to obtain the product. Some of the classic concepts of inflation are discussed below:

1. Increases in producer’s costs are passed on to consumers. At each stage of the product’s journey from producer to consumer, prices are escalated disproportionately in order to make a good profit. The overall increase, in the product’s price is directly proportional to the number of intermediaries it encounters on its way to the consumer. This type of inflation is called cost-driven (or cost-push) inflation.

2. Excessive spending power of consumers forces an upward trend in prices. This high spending power is usually achieved at the expense of savings. The law of supply and demand dictates that the more the demand, the higher the price. This type of inflation is known as demand-driven (or demand-pull) inflation.

3. Impact of international economic forces can induce inflation in a local economy. Trade imbalances and fluctuations in currency values are notable examples of international inflationary factors.

4. Increasing base wages of workers generate more disposable income and, hence, higher demands for goods and services. The high demand, consequently, creates a pull on prices. Coupled with this, employers pass on the additional wage cost to consumers through higher
prices. This type of inflation is, perhaps, the most difficult to solve because wages set by union contracts and prices set by producers almost never fall; at least not permanently. This type of inflation may be referred to as wage-driven (or wage-push) inflation.

5. Easy availability of credit leads consumers to “buy now and pay later” and, thereby, create another loophole for inflation. This is a dangerous type of inflation because the credit not only pushes prices up, but it also leaves consumers with less money later on to pay for the credit. Eventually, many credits become uncollectible debts, which may then drive the economy into recession.

6. Deficit spending results in an increase in money supply and, thereby, creates less room for each dollar to get around. The popular saying which indicates that "a dollar does not go far anymore," simply refers to inflation in lay man’s terms. The different levels of inflation may be categorized as discussed below:

**MILD INFLATION**
When inflation is mild (2 to 4 percent) the economy actually prospers. Producers strive to produce at full capacity in order to take advantage of the high prices to the consumer. Private investments tend to be brisk and more jobs become available. However, the good fortune may only be temporary. Prompted by the prevailing success, employers are tempted to seek larger profits and workers begin to ask for higher wages. They cite their employer’s prosperous business as a reason to bargain for bigger shares of the business profit. So, we end up with a vicious cycle where the producer asks for higher prices, the unions ask for higher wages, and inflation starts an upward trend.

**MODERATE INFLATION**
Moderate inflation occurs when prices increase at 5 to 9 percent. Consumers start purchasing more as an edge against inflation. They would rather spend their money now than watch it decline further in purchasing power. The increased market activity serves to fuel further inflation.

**SEVERE INFLATION**
Severe inflation is indicated by price escalations of 10 percent or more. Double-digit inflation implies that prices rise much faster than wages do. Debtors tend to be the ones who benefit from this level of inflation because they repay debts with money that is less valuable than the one borrowed.

**HYPERINFLATION**
When each price increase signals the increase in wages and costs, which again sends prices further up, the economy has reached a stage of malignant galloping inflation or hyperinflation. Rapid and uncontrollable inflation destroys the economy. The currency becomes economically useless as the government prints it excessively to pay for obligations.

Inflation can affect any project in terms of raw materials procurement, salaries and wages, and/or cost tracking dilemma. Some effects are immediate and easily observable. Other effects are subtle and pervasive. Whatever form it takes, inflation must be taken into account in long term

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project planning and control. Large projects may be adversely affected by the effects of inflation in terms of cost overruns and poor resource utilization. The level of inflation will determine the severity of the impact on projects.

**BREAK EVEN ANALYSIS**

Break even analysis refers to the determination of the balanced performance level where project income is equal to project expenditure. The total cost of an operation is expressed as the sum of the fixed and variable costs with respect to output quantity. That is,

\[ TC(x) = FC + VC(x), \]

where \( x \) is the number of units produced, \( TC(x) \) is the total cost of producing \( x \) units, \( FC \) is the total fixed cost, and \( VC(x) \) is the total variable cost associated with producing \( x \) units. The total revenue resulting from the sale of \( x \) units is defined as:

\[ TR(x) = px, \]

where \( p \) is the price per unit. The profit due to the production and sale of \( x \) units of the product is calculated as:

\[ P(x) = TR(x) - TC(x). \]

The break even point of an operation is defined as the value of a given parameter that will result in neither profit nor loss. The parameter of interest may be the number of units produced, the number of hours of operation, the number of units of a resource type allocated, or any other measure of interest. At the break even point, we have the following relationship:

\[ TR(x) = TC(x) \]

or

\[ P(x) = 0. \]

In some cases, there may be a known mathematical relationship between cost and the parameter of interest. For example, there may be a linear cost relationship between the total cost of a project and the number of units produced. The cost expressions facilitate straightforward break even analysis. Figure 11 shows an example of a break even point for a single project. Figure 12 shows examples of multiple break even points that exist when multiple projects are compared. When two project alternatives are compared, the break even point refers to the point of indifference between the two alternatives. In Figure 12, \( x_1 \) represents the point where projects A and B are equally desirable, \( x_2 \) represents where A and C are equally desirable, and \( x_3 \) represents where B and C are equally desirable. The figure shows that if we are operating below a production level of \( x_2 \) units, then project C is the preferred project among the three. If we are operating at a level more than \( x_2 \) units, then project A is the best choice.

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Example

Three project alternatives are being considered for producing a new product. The required analysis involves determining which alternative should be selected on the basis of how many units of the product are produced per year. Based on past records, there is a known relationship between the

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number of units produced per year, \( x \), and the net annual profit, \( P(x) \), from each alternative. The level of production is expected to be between 0 and 250 units per year. The net annual profits (in thousands of dollars) are given below for each alternative:

Project A: \( P(x) = 3x - 200 \)
Project B: \( P(x) = x \)
Project C: \( P(x) = \frac{1}{50}x^2 - 300 \).

This problem can be solved mathematically by finding the intersection points of the profit functions and evaluating the respective profits over the given range of product units. It can also be solved by a graphical approach. Figure 13 shows a plot of the profit functions. Such a plot is called a break even chart. The plot shows that Project B should be selected if between 0 and 100 units are to be produced. Project A should be selected if between 100 and 178.1 units (178 physical units) are to be produced. Project C should be selected if more than 178 units are to be produced. It should be noted that if less than 66.7 units (66 physical units) are produced, Project A will generate a net loss rather than net profit. Similarly, Project C will generate losses if less than 122.5 units (122 physical units) are produced.

![Figure 13. Plot of Profit Functions](image-url)