Trusses - Definition
- Framework of members joined at ends with frictionless pins to form a stable structure
- Basic shape is a triangle

Joints
- Pinned Joint
- Riveted Joint
  Frictionless pin is good assumption if members concentric

Types of Trusses
- Named after inventor
  - Howe
  - Pratt
- Named after Railroad
  - Baltimore

Two-Force Members
- Tension
- Compression
  Line of action of forces is along the line connecting the points where the forces act.
Two-Force Members

Member hinged at both ends with no external force applied to member

\[ \sum M_x = 0 \Rightarrow (F_B)_y L = 0 \]
\[ \therefore (F_B)_y = 0 \]

\[ \sum F_y = 0 \Rightarrow (F_A)_y + (F_D)_y = 0 \]
\[ \therefore (F_D)_y = 0 \]

\[ \sum F_x = 0 \Rightarrow (F_A)_x + (F_B)_x = 0 \]
\[ \therefore (F_A)_x = -(F_B)_x \]

Methods of Truss Analysis

- Method of Joints
- Method of Sections

**Method of Joints:**
Draw FBD of pins. Show forces acting ON pins, not members

\[ \sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0 \]

*Does not give independent equation*

Example – Method of Joints

Given:

Required: Force in each member using method of joints. Assume 3 SF.

1. Solve for support reactions of whole structure
   - Follow 8-step process; FBD of whole structure
   - Occasionally not necessary, but it will never hurt
2. Start at a joint with only 1 or 2 unknown forces
   - Follow 8-step process; FBD of joint, or pin
   - Member forces obtained using 3rd law partners
   - Suggest using Tension for all UNKNOWN members
3. “Step” through structure by repeating Step 2 at all joints.
1. Solve for support reactions of whole structure

2. Draw FBD of Joint A

3. Draw FBD of Joint B

4. Draw FBD of Joint C

\[ \sum F_x = 0 \Rightarrow \]

\[ \sum F_y = 0 \Rightarrow \]

\[ + \sum F_y = 0 \Rightarrow \]
Draw FBD of Joint ??

\[ \sum F_x = 0 \Rightarrow \]

\[ + \sum F_y = 0 \Rightarrow \]

Zero-Force Members

- Three members at a joint
- No applied loads or reactions
- Two members are collinear

Third member is a zero-force member.

(And the forces in the other two are equal!)
Three-Dimensional Trusses
- Solve the same way; just more complex geometry
- Method of Joints: 3 equations at a joint

Redundancy
- Statically determinate truss (can be solved only using statics).
  - \( m + 3 = 2j \)
    - \( m \) = number of members
    - \( m + 3 \) = number of unknowns
    - \( j \) = number of joints
    - \( 2j \) = number of equations
- Necessary but not sufficient conditions. Members and reactions must be properly placed to provide stability.

Simple Equation

\[
3x + 9 = 0 \\
3x = -9 \\
(3^{-1})(3x) = (3^{-1})(-9) \\
x = (3^{-1})(-9) = -3
\]

In general
\[
ax + b = 0 \\
x = (a^{-1})(-b)
\]

In Matlab
\[
\text{>> } a = 3 \\
a = 3 \\
\text{>> } b = 9 \\
b = 9 \\
\text{>> } x = a \backslash -b \\
x = -3
\]

Simultaneous Equations

\[
3x + 4y - 20 = 0 \\
2x + 4y - 10 = 0
\]

Matrix Form

\[
\begin{bmatrix}
3 & 4 \\
2 & 4
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} +
\begin{bmatrix}
-20 \\
-10
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

In Matlab
\[
\text{>> a = } \begin{bmatrix}
3 & 4 \\
2 & 4
\end{bmatrix} \\
a = \begin{bmatrix}
3 & 4 \\
2 & 4
\end{bmatrix} \\
\text{>> b = } \begin{bmatrix}
-20 \\
-10
\end{bmatrix} \\
b = \begin{bmatrix}
-20 \\
-10
\end{bmatrix} \\
\text{>> } x = a \backslash -b \\
x = \begin{bmatrix}
10.0000 \\
-2.5000
\end{bmatrix}
\]
Matrix Operations * and \ 

- Matrix Multiplication *
\[
\begin{bmatrix}
4 & 3 \\
-2 & 1 \\
\end{bmatrix}
\begin{bmatrix}
8 & 9 \\
7 & 6 \\
\end{bmatrix}
= \begin{bmatrix}
4(8) + 3(7) & 4(9) + 3(-6) \\
-2(8) + 1(7) & -2(9) + 1(-6) \\
\end{bmatrix}
= \begin{bmatrix}
53 & 18 \\
-9 & -24 \\
\end{bmatrix}
\]

- Left divide – backslash \ 
\([G]^{-1}[H] = [G] \backslash [H]\)
\[A][U] + [B] = 0\]
\[[U] = [A]^{-1}[-B] = [A] \backslash [-B]\]

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Problem Statement

- **Given:** The three bar truss shown at right.
- **Required:** Calculate the force of each member and the support reactions.

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FBD of Whole Structure

Equilibrium Equations
\[
\sum F_x = 0 \Rightarrow A_x + C_x = 0
\]
\[
\sum F_y = 0 \Rightarrow A_y - 3920N = 0
\]
\[
\sum M_C = 0 \Rightarrow -A_x(5m) - 3920N(5m)(\sin 30^\circ)(\sin 60^\circ) = 0
\]

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Joint FBDs

\[
\sum F_x = +A_x \cos 0^\circ + A_y \cos 90^\circ + F_{AC} \cos 270^\circ + F_{AB} \cos 300^\circ = 0
\]
\[
\sum F_y = +A_x \sin 0^\circ + A_y \sin 90^\circ + F_{AC} \sin 270^\circ + F_{AB} \sin 300^\circ = 0
\]
\[
\sum F_x = F_{AB} \cos 120^\circ + F_{BC} \cos 210^\circ + 3920N \cos 270^\circ = 0
\]
\[
\sum F_y = F_{AB} \sin 120^\circ + F_{BC} \sin 210^\circ + 3920N \sin 270^\circ = 0
\]
\[
\sum F_x = C_x \cos 0^\circ + F_{AC} \cos 30^\circ + F_{BC} \cos 90^\circ = 0
\]
\[
\sum F_y = C_x \sin 0^\circ + F_{AC} \sin 30^\circ + F_{BC} \sin 90^\circ = 0
\]
Modified Problem Statement

**Given:** The three bar truss shown at right.

**Required:** Calculate the force of each member and the support reactions.

Once set up in Matlab, it is easy to vary load conditions and solve for a variety of configurations.

Matrix Form

\[
\begin{bmatrix}
\cos(300) & \cos(270) & 0 & \cos(90) & 0 \\
\sin(300) & \sin(270) & 0 & \sin(90) & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{sa} \\
F_{sc} \\
F_{sc} \\
F_{sa} \\
F_{sc}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Matlab Solution

\[
U = A \backslash (-B)
\]

\[
U = \begin{bmatrix}
3395 \\
980 \\
-1960 \\
-1698 \\
3920
\end{bmatrix}
\]

\[
\sum F_x = -1698N + 1698N = 0
\]

\[
\sum F_y = +3920N - 3920N = 0
\]

\[
\sum M_C = -(-1698N)(5m) - 3920N(5m)\sin(30^\circ)\sin(60^\circ) = 0
\]
Matlab Solution

\[ U = A \backslash (-B) \]

\[ U = \begin{bmatrix} 3395 \\ -752 \\ -1960 \\ -1698 \\ 2188 \\ 2698 \end{bmatrix} \]

\[ \sum F_x = -1698N + 2698N - 2000N\sin30^\circ = 0 \]
\[ \sum F_y = +2188N - 3920N + 2000N\cos30^\circ = 0 \]
\[ \sum M_C = -(-1698N)(5m) - 3920N(5m)(\sin30^\circ)(\sin60^\circ) = 0 \]

Method of Joints using Matlab

- Label all joints and all members.
- Draw FBD of whole structure.
- Assume positive values for all unknown support reactions.
- Write out equilibrium equations.
- Draw FBD of each joint.
- Assume tension for all members.
- Calculate angles for all forces relative to positive X axis.
- Write \( \sum F_x \) and \( \sum F_y \) equations for all joints.
- Put equations in matrix form.
- Use Matlab to solve.
- Check results with FBD and equilibrium equations of whole structure.

Method of Joints using Matlab

- The truss must be stable.
  - \( m + 3 = 2j \)
    - \( m \) = number of members
    - \( j \) = number of joints
- The truss must be must not be redundant.
  - \( m + 3 = 2j \)
    - \( m \) = number of members
    - \( j \) = number of joints
- You must have 3 and only 3 support reactions.
  - Usually 1 pin and 1 roller.

Matlab IS Useful

- Solve for the force in EVERY member of this truss using the method of joints for every possible load condition.
- 29 members, 16 joints, 16 FBD, 32 equations, 32 unknowns

Can we "automate" the process?
What about functions?