Conservation of Mechanical Energy

We have seen:

\[ W_{1 \rightarrow 2} = U_1 - U_2 \]

But,

\[ K_1 + W_{1 \rightarrow 2} = K_2 \]

Implies,

\[ K_1 + U_1 - U_2 = K_2 \]

Rearrange,

\[ K_1 + U_1 = K_2 + U_2 \implies K_1 - K_2 + U_1 - U_2 = 0 \]

\[ \therefore \Delta K + \Delta U = 0 \quad \Delta K = -\Delta U \]

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Law of Conservation of Energy

Conservation of Energy: The SUM of ALL energies remains constant.

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and from one body to another, but the total amount remains constant.

\[ \therefore \Delta K + \Delta U + \Delta[\text{all other forms of energy}] = 0 \]

- Includes thermodynamic energy, strain energy, chemical energy, etc.
Given: A 75.0 kg parachutist jumps off a training tower that is 85.0 m high. He lands with a vertical speed of 5.00 m/s. Assume the drag force is constant.

Required: Calculate the drag force and compare it to the man’s weight.

Solution:

Where did the “lost work” go?

Potential Energy

- The amount of work that a conservative force would do if the force were to move from its current position to a given (defined) reference position.
- The reference position is called the datum.
- The symbol of Potential Energy is $U$.
- Types of forces that have Potential Energy:
  - Weight (i.e., the force due to gravity).
  - Spring.
  - ANY Constant Conservative Force.
- Friction and ALL other Non-Conservative forces DO NOT have a Potential Energy.
Newton’s Law of Universal Gravitation

- **Scalar form**  
  \[ F = G \frac{m_1 m_2}{r_{21}^2} \]

- **Vector form**  
  \[ \vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21} \]

  - Where
    - \( G \) = universal gravitational constant = \( 6.67 \times 10^{-11} \) N·m²/kg
    - \( m_1 \) = mass of body 1
    - \( m_2 \) = mass of body 2
    - \( \vec{r}_{21} \) = position vector from body 2 to body 1
    - \( r_{21} \) = magnitude of the position vector from body 2 to body 1
    - \( \hat{r}_{21} \) = position vector from body 2 to body 1

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Gravitational Potential

Locate the datum at \( \infty \)

\[ U(r) = \int_{r}^{\infty} -Fdr = -\int_{r}^{\infty} G \frac{mm}{r^2} d\vec{r} \]

\[ U(r) = -GmmE \int_{r}^{\infty} \frac{1}{r^2} d\vec{r} \]

\[ U(r) = -GmmE \left[ \frac{1}{r} \right]_{r}^{\infty} = -GmmE \left[ \frac{1}{r} - \frac{1}{\infty} \right] \]

\[ U(r) = -G \frac{mmE}{r} \]
Escape Speed

At what radial speed will an object “escape” the pull of Earth’s gravitation?

\[ K_1 + U_1 = K_2 + U_2 \]
\[ \frac{1}{2} M v_{esc}^2 - \frac{G M M_E}{r_E} = 0 + 0 \]
\[ v_{esc} = \sqrt{\frac{2G M_E}{r_E}} = \sqrt{\frac{2\left(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.97 \times 10^{24} \text{ kg}\right)}{6.38 \times 10^6 \text{ m}}} \]
\[ v_{esc} = 11,200 \text{ m/s} \]

Example

Astronomers discover a meteorite at a distance of 80,000 km from the center of Earth. The meteorite is moving directly toward Earth with a velocity of 2000 m/s. What will be the velocity of the meteorite when it hits Earth’s surface? Ignore all drag effects.

\[ K_1 + U_1 = K_2 + U_2 \]
\[ \frac{1}{2} M v_1^2 - \frac{G M M_E}{r} = \frac{1}{2} M v_2^2 - \frac{G M M_E}{r_E} \]
\[ v = \sqrt{\frac{v_1^2 + G M E (\frac{1}{r_E} - \frac{1}{r})}{\frac{1}{r} - \frac{1}{r_E}}} \]
\[ v = \sqrt{(2000 \text{ m/s})^2 + 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2}(5.97 \times 10^{24} \text{ kg})(\frac{1}{6.38 \times 10^6 \text{ m}} - \frac{1}{6.38 \times 10^6 \text{ m}}) \]
\[ v = \boxed{10,400 \text{ m/s}} \]