**Kinetics - Normal & Tangential Coordinates**

Newton’s 2\textsuperscript{nd} Law *still* governs: \[ \sum \vec{F} = m \vec{a} \]

In component form:
\begin{align*}
\sum F_t &= ma_t \\
\sum F_n &= ma_n
\end{align*}

Remember:
\[ \vec{a} = \dot{v} \hat{e}_t + \frac{v^2}{\rho} \hat{e}_n = a_t \hat{e}_t + a_n \hat{e}_n \]

Time rate of change of the *direction* of the velocity

Time rate of change of the *magnitude* of the velocity

Nothing else changes: 1) Still use 8 step process  
2) May still need to use *kinematics.*

**Note:** Occasionally we may need to add the "third" dimension and draw *FBD* and *KD* from *two different* perspectives (viewpoints).

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**Uniform Circular Motion**

Motion along a circular path with constant *speed* 
\[ v = \text{constant} \]

\[ \vec{v} = v \hat{e}_t \]

\[ \vec{a} = \dot{v} \hat{e}_t + \frac{v^2}{\rho} \hat{e}_n = a_t \hat{e}_t + a_n \hat{e}_n \]

\[ \vec{a} = \frac{v^2}{r} \hat{e}_n = a_n \hat{e}_n \]

\[ \sum F_t = ma_t = 0 \]

\[ \sum F_n = ma_n = \frac{mv^2}{\rho} \]

Note that the *acceleration* and the *force* are *inward.* They are a *centripetal* acceleration and force.
Eight Step Process
1) Decide what needs to be isolated. (may be the hardest part).
2) Draw the isolated Free Body & Kinetic Diagrams (complete with all external boundaries) and set them equal to each other.
3) Choose a Coordinate System (C.S.).
4) a) Add all EXTERNALLY APPLIED forces & moments acting ON the Free Body Diagram:
   a1) Given forces and moments including weight.
   a2) Support reactions (where the body is cut from the rest of the world).
   b) Add all mass*acceleration terms to the Kinetic Diagram.
5) Add all necessary dimensions.
6) Enforce Newton’s 2nd Law: \[ \sum F = ma \]
7) a) If necessary, set up any required Kinematic equations.
   b) Solve ALL equations for ALL unknowns.
8) Check work and answers for units, directions, proper notation, S.D., reasonableness, etc.

Example Problem 1

**Given:** Your 50 kg instructor is flying a jet that pulls up into a vertical curve as shown. At \( \theta = 30^\circ \) the speed is 750 km/h. The radius of curvature \( \rho \) is 1.5 km at this point. Assume 2 SF.

**Required:** The magnitude of the force exerted by the seat onto your instructor.

Step 1: What do we isolate? \( Me \), Why?
Problem 1 Cont.

Step 2: Draw FBD & KD

Step 3: C.S.

Step 4: a) External Forces & Moments ON FBD  
  b) Mass * Acceleration ON KD

Step 5: Dimensions

Step 6: Enforce Newton’s 2nd Law \( \sum \vec{F} = m \vec{a} \)

Step 7: a) Set up Kinematic equations

We need to do this - but we did this in Module 1 and on slides 1 and 2.

Problem 1 Cont.

What do we know?

\[ \rho = 1.50 \text{ km} \left( \frac{1000 \text{ m}}{\text{km}} \right) = 1500 \text{ m} \]

\[ v = 750 \frac{\text{km}}{\text{hr}} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{\text{hr}}{3600 \text{ sec}} \right) = 208 \frac{\text{m}}{\text{sec}} \]

Plug into the acceleration equation.

\[ \ddot{a} = \frac{v^2}{\rho} \hat{e}_n = \frac{\left(208 \frac{\text{m}}{\text{sec}}\right)^2}{1500\text{m}} \hat{e}_n = 28.9 \frac{\text{m}}{\text{sec}^2} \hat{e}_n \]
Problem 1 Cont.

\[ a_i = 0 \]
\[ a_n = 28.9 \frac{m}{s^2} \]

7b) Solve equations:

\[ \sum F_i = ma_i \Rightarrow +F_B - 490.5 \text{ N} \cos 60^\circ = +ma_i = 0 \]
\[ \Rightarrow F_B = +245.25 \text{ N} \]
\[ \sum F_n = ma_n \Rightarrow +F_S - 490.5 \text{ N} \sin 60^\circ = +ma_n = 50 \text{ kg} \left(28.9 \frac{m}{s^2}\right) \]
\[ \Rightarrow F_S = +1869.8 \text{ N} \]
\[ \therefore F = \sqrt{(F_B)^2 + (F_S)^2} = \sqrt{(245.25 \text{ N})^2 + (1869.8 \text{ N})^2} = 1886 \text{ N} \]

Step 8: Check Everything

\[ \therefore F = 1900 \text{ N} \]

Example Problem 2

Given: An airplane is flying at a constant speed of 400 mi/hr and making a horizontal turn with a 2.00 mi radius.

Required: Find the proper bank angle \( \theta \).

Note: The force exerted by the air is normal to the wing’s surface.

Step 1: What do we isolate? The plane, Why?
Step 2: Draw FBD & KD

Step 3: C.S.

Step 4:
   a) External Forces & Moments ON FBD
   b) Mass * Acceleration ON KD

Step 5: Dimensions

Step 6: Enforce Newton's 2nd Law

\[ \sum F = ma \]

\[ \sum F_i = ma_i \Rightarrow 0 = 0 \]

\[ \sum F_x = ma_x \Rightarrow +N \sin \theta = +ma_x \Rightarrow \left( \frac{mg}{\cos \theta} \right) \sin \theta = +ma_x = \frac{v^2}{\rho} \]

\[ \sum F_z = ma_z \Rightarrow +N \cos \theta - W = +ma_z \Rightarrow N = + \frac{W}{\cos \theta} = + \frac{mg}{\cos \theta} \]

Step 7: a) Set up Kinematic equations: \( a_s = \frac{v^2}{\rho} \)

b) Solve equations:

\[ \therefore g \tan \theta = \frac{v^2}{\rho} \]

\[ \Rightarrow \theta = \tan^{-1} \left( \frac{v^2}{g \rho} \right) \]

\[ \rho = 2 \text{mi} \left( \frac{5280 \text{ ft}}{\text{mi}} \right) = 10560 \text{ ft} \]

\[ v = 400 \text{ mi/hr} \left( \frac{5280 \text{ ft}}{\text{mi}} \right) \left( \frac{hr}{3600 \text{ s}} \right) = 587 \frac{\text{ft}}{\text{s}} \]

\[ \theta = \tan^{-1} \left( \frac{(587 \text{ ft/s})^2}{32.2 \text{ ft/s} (10560 \text{ ft})} \right) \]

\[ \sum F_i = ma_i \Rightarrow 0 = 0 \quad \text{Ans: 45.5°} \]

\[ \sum F_x = ma_x \Rightarrow +N \sin \theta = +ma_x \Rightarrow \left( \frac{mg}{\cos \theta} \right) \sin \theta = +ma_x = \frac{v^2}{\rho} \]

\[ \sum F_z = ma_z \Rightarrow +N \cos \theta - W = +ma_z \Rightarrow N = + \frac{W}{\cos \theta} = + \frac{mg}{\cos \theta} \]

Step 8: Check Everything