Normal and Tangential Coordinates

Motion along a curved 2-D path

- The velocity is tangent to the path and in the direction of motion.
- Set up a C.S. centered on the particle. This C.S. will move with the particle.
- One axis is tangent to the path and in the direction of motion.
  Define \( \hat{e}_t \) as a unit vector tangent to the path and in the direction of motion.
- The other axis must be perpendicular to \( \hat{e}_t \).
  Define \( \hat{e}_n \) as a unit vector perpendicular to \( \hat{e}_t \) and in the direction of curvature.
  \[
  \mathbf{v} = v \hat{e}_t
  \]
  where \( v \) is the speed \( = |\mathbf{v}| \)

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Motion along a curved 2-D path

\[
\vec{v} = v \hat{e}_t
\]
\[
\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left( v \hat{e}_t \right) = \frac{dv}{dt} \left( \hat{e}_t \right) + v \left( \frac{d\hat{e}_t}{dt} \right)
\]
\[
\frac{dv}{dt} = \dot{v} = \text{time rate of change of the speed}
\]
\[
\frac{d\hat{e}_t}{dt} \neq 0 \quad \text{we need to calculate this!}
\]
Normal and Tangential Coordinates

What can we observe?

\( \Delta \hat{e}_t \approx \Delta \theta |\hat{e}_n| \) by the arc length

\[ \Delta \hat{e}_t = \Delta \theta \hat{e}_n \]

\( \Delta s = \rho \Delta \theta \) also by the arc length

\[ \Delta \theta = \frac{\Delta s}{\rho} \Rightarrow \Delta \hat{e}_t = \frac{\Delta s}{\rho} \hat{e}_n \]

Divide by \( \Delta t \)

\[ \frac{\Delta \hat{e}_t}{\Delta t} = \frac{1}{\rho} \frac{\Delta s}{\Delta t} \hat{e}_n \]

\[ \frac{d\hat{e}_t}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \hat{e}_t}{\Delta t} = \frac{1}{\rho} \frac{ds}{dt} \hat{e}_n \]

but

\[ \frac{ds}{dt} = v \Rightarrow \frac{d\hat{e}_t}{dt} = \frac{v}{\rho} \hat{e}_n \]

Normal and Tangential Coordinates

Motion along a curved 2-D path

\[ \vec{v} = v \hat{e}_t \]

\[ \vec{a} = \frac{dv}{dt} (\hat{e}_t) + v \frac{d\hat{e}_t}{dt} = \dot{v} \hat{e}_t + v \left( \frac{v}{\rho} \hat{e}_n \right) \]

\[ \vec{a} = \frac{v^2}{\rho} \hat{e}_n + a_t \hat{e}_t + a_n \hat{e}_n \]

Time rate of change of the direction of the velocity

Time rate of change of the magnitude of the velocity
**Uniform Circular Motion**

Motion along a circular path with constant speed

\[ v = \text{constant} \]

\[ \vec{v} = v \hat{e}_t \]

\[ \vec{a} = \vec{e}_r + \frac{v^2}{\rho} \hat{e}_n = a_r \hat{e}_r + a_n \hat{e}_n \]

\[ \vec{a} = \frac{v^2}{r} \hat{e}_n = a_n \hat{e}_n \]

**Note that the acceleration is inward.** It is a centripetal acceleration.

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**Example Problem 1**

**Given:** A jet pulls up into a vertical curve as shown. At \( \theta = 30.0^\circ \) its speed is 750.0 km/h. The radius of curvature is 1.50 km at this point.

**Required:** Find:

a) The magnitude of the total acceleration \( a \).

b) The horizontal and vertical components of the acceleration.
Problem 1 Cont.

What do we know?

\[ \rho = 1.50 \text{ km} \left( \frac{1000 \text{ m}}{\text{km}} \right) = 1500 \text{ m} \]

\[ v = 750 \text{ km/hr} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ sec}} \right) = 208 \text{ m/sec} \]

Plug into the acceleration equation.

\[ \ddot{\mathbf{a}} = \frac{v^2}{\rho} \mathbf{\hat{e}}_n = \frac{(208 \text{ m/sec})^2}{1500 \text{ m}} \mathbf{\hat{e}}_n = 28.9 \frac{\text{m}}{\text{sec}^2} \mathbf{\hat{e}}_n \]

Problem 1 Cont.

\[ \ddot{\mathbf{a}} = \frac{v^2}{\rho} \mathbf{\hat{e}}_n = +28.9 \frac{\text{m}}{\text{sec}^2} \mathbf{\hat{e}}_n \]

Now what do we do? Yes, add another C.S.

Draw the vector components.

\[ a_x = 28.9 \frac{\text{m}}{\text{sec}^2} \cos 120^\circ = -14.5 \frac{\text{m}}{\text{sec}^2} \]

\[ a_y = 28.9 \frac{\text{m}}{\text{sec}^2} \sin 120^\circ = 25.0 \frac{\text{m}}{\text{sec}^2} \]
Example Problem 2

Because the Earth rotates once per day, the effective acceleration of gravity at the equator is slightly less than it would be if the Earth did not rotate. Estimate the magnitude of this effect. What fraction of a g is this?

\[ a = \frac{v^2}{r} \]

\[ r = \frac{C}{2\pi} \]

\[ r = 21.06 \times 10^6 \text{ ft} \]

\[ v = \frac{C}{1 \text{ day}} = \frac{23,000 \text{ mi}}{1 \text{ day}} \left( \frac{1 \text{ day}}{24 \text{ hr}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \]

\[ v = 1529 \text{ ft/s} \]

\[ a = \frac{0.11}{32.2} = 0.0034 \text{ g} \]