Module 1, Lecture 8
Bernoulli’s Principle

Measuring Fluid Flow: Venturi Tube

Bernoulli’s Principle

Continuity Equation

Algebra

\[ v_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho (1 - (A_1/A_2)^2)}} = \sqrt{\frac{2(p_1 - p_2)}{\rho \left(\frac{d_1}{d_2}\right)^2 - 1}} \]

How does \( P_2 \) compare to \( P_1 \)?

Measuring Fluid Flow: Pitot Tube

Air speed:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

Stream flow:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

Measuring Fluid Flow: Pitot Tube

Water

Air speed:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

Stream flow:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

Bernoulli’s Principle: Airplane Wings

Real airplane wing: flow quite complicated due to circulation, viscosity, and turbulence.

\[ L \approx \rho KA v^2 \]

\( L \) = lift
\( \rho \) = fluid (air) density
\( K \) = empirical constant (about 0.12 for an airplane wing)
\( A \) = wing area
\( v \) = airspeed

Boeing 747

W = 3.4 MN
\( A \) = 511 m\(^2\)
\( \rho \) = 1.20 kg/m\(^3\)
\( K \) = 0.12
\( v \) = 215 m/s = 480 mph

Bernoulli’s Principle: Curve Balls

Home Plate

Counter-clockwise rotation

Boeing 747

W = 3.4 MN
\( A \) = 511 m\(^2\)
\( \rho \) = 1.20 kg/m\(^3\)
\( K \) = 0.12
\( v \) = 215 m/s = 480 mph

Other applications:
Top spin of tennis ball: ball drops
Back spin of golf ball: ball lifts

Note final position: palm up
Ball should spin counter-clockwise
Draining a tank:
Bernoulli’s Principle
Continuity Equation
Velocity

Put it all together:
Differential Equation
Calculus

\[ h(t) = h_0 - \frac{A_2}{A_1} \sqrt{2gh_0 \frac{g}{2} + \frac{A_1^2}{A_2^2} t^2} \]

Time to drain
\[ t_f = \frac{A_1}{A_2} \sqrt{\frac{2h_0}{g}} \]

After a fun afternoon of playing ‘Dunk the Dean’, it is time to drain the tank. The tank is 5 ft in diameter, has a ¾ in. diameter drain hole, and there is 4 ft of water in the tank. How long does it take to drain the tank?

Half the tank?

Real Fluids - Viscosity

- Coefficient of viscosity ______
- Depends upon
  - ______________
  - ______________

Units:
- ______________
- Poise = ____________

Examples
- Water (20°C):_________
- SAE #30 Motor Oil:_________
- Heinz Ketchup:_________

Fluid flow through a real pipe

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 + \text{Loss} \]

For a horizontal pipe with laminar flow:

Poiseuille’s equation:
(horizonal pipe)

\[ Q = \frac{\pi r^4 (P_1 - P_2)}{8 \eta L} \]

Dr. Bennett enjoys a Cracker Barrel breakfast of eggs, bacon, sausage, hash browns, and biscuits and gravy. His cholesterol shoots up by 40 points, causing a 50% blockage (by area) of his arteries. By what factor must Dr. Bennett’s blood pressure increase to have the same blood-flow rate?

Real Fluids – Drag

_______________ gives \( F_{\text{drag}} \) of a sphere moving through a fluid

\[ F_{\text{drag}} = 6 \pi \eta rv \]

Determine the terminal velocity of a 2.5 cm diameter iron sphere falling through SAE #10 motor oil.

\( \rho_{\text{iron}} = 8.5 \times 10^3 \text{ kg/m}^3 \)
\( \rho_{\text{oil}} = 0.88 \times 10^3 \text{ kg/m}^3 \)
\( \eta_{\text{oil}} = 0.20 \text{ Pa·s} \)

Which is the correct FBD?