Harmonic Motion

\[ x(t) = A \sin(\omega t + \delta) = a_1 \sin(\omega t) + a_2 \cos(\omega t) \]
\[ v(t) = A\omega \cos(\omega t + \delta) = a_1\omega \cos(\omega t) - a_2\omega \sin(\omega t) \]
\[ a(t) = -A\omega^2 \sin(\omega t + \delta) = -a_1\omega^2 \sin(\omega t) - a_2\omega^2 \cos(\omega t) \]
\[ \omega = \sqrt{\frac{k}{m}} \]
\[ A = \sqrt{a_1^2 + a_2^2} \]
\[ a_1 = \frac{v_0}{\omega} \quad a_2 = x_0 \]
\[ \delta = \tan^{-1}\left(\frac{a_2}{a_1}\right) \]
\[ T = \frac{2\pi}{\omega} \quad f = \frac{1}{T} \quad \omega = 2\pi f \]

Parallel Axis Theorem

\[ I = I_{CM} + mr^2 \]

Pendulums

simple: \[ \omega = \sqrt{\frac{g}{l}} \]

physical: \[ \omega = \sqrt{\frac{mg}{I}} \]

Speed of Sound

\[ v = \sqrt{\frac{B}{\rho}} \]

Speed of Sound in Air:

\[ v \approx (331 + .6T) \text{m/s} \quad (T \text{ in °C}) \]

Natural frequencies

String:

\[ \lambda_n = \frac{2l}{n} \]
\[ f_n = \frac{v}{\lambda} = \frac{n}{2l} \sqrt{\frac{T}{\mu}} \]

Air Columns:

\[ \lambda = \frac{4l}{n} \quad \text{closed (n = 1, 3, 5, ...)} \]
\[ \lambda = \frac{2l}{n} \quad \text{open (n = 1, 2, 3, ...)} \]

Wave Speed:

Cables, Ropes, etc.

\[ v = \frac{\sqrt{T}}{\sqrt{\mu}} \]

Transverse: \[ v = \frac{\sqrt{E}}{\sqrt{\rho}} \]

Longitudinal: \[ v = \frac{\sqrt{E}}{\sqrt{\rho}} \]

Sound Level

\[ \beta \text{ (dB)} = 10\log \left(\frac{I}{I_0}\right) \]

\[ I \quad \text{Intensity} \]
\[ I_0 \quad \text{reference intensity, } 1 \times 10^{-12} \text{W/m}^2 \]

Doppler Shift

\[ f' = f \left(\frac{v + v_s}{v + v_l}\right) + \text{listener to source} \]

Beat Frequency:

\[ |f_1 - f_2| \]

Wave Energy, Power, Intensity

\[ E \quad \text{energy} \]
\[ I \quad \text{intensity} \]
\[ P \quad \text{power} \]
\[ \bar{P} \quad \text{average power} \]
\[ E = 2\pi^2\mu vf^2A^2 \]
\[ \bar{P} = \langle P \rangle = 2\pi^2\mu vf^2A^2 \]
\[ P = 4\pi^2\mu vf^2A^2 \cos^2(kx - \omega t) \]
\[ I = \frac{mP}{4\pi^2r^2} \quad \frac{I_2}{I_1} = \frac{r_2^2}{r_1^2} \]

Light Waves

Law of Reflection:

\[ \theta_r = \theta_a \]

Index of refraction:

\[ n = \frac{c}{v} \]

Snell's Law:

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

Light wavelength:

\[ \lambda = \frac{n_0}{n} \]

Total Internal Reflection:

\[ \sin \theta_{crit} = \frac{n_b}{n_a} \]

speed of light in vacuum: \[ c = 3 \times 10^8 \text{m/s} \]
1. (2 pt) The 6th harmonic of an organ pipe is 1992 Hz. Determine the frequency of the 3rd harmonic.
   a. 332 Hz   b. 664 Hz   c. 996 Hz   d. 1328 Hz

2. (2 pt) Two transverse waves traveling the same speed in opposite directions have the same amplitude and a period of 2 sec. Which of the following represents the superposition (resultant) of the two waves after 1 sec?
   a.   b.   c.

3. (2 pts) On the graph shown, the quantity labeled Z represents which of the following:
   a. period   c. wavelength
   b. frequency   d. amplitude

4. (10 pts) A 30 inch long electric bass string has a weight per unit length of $8.0 \times 10^{-4}$ lb/in. Determine the tension that must be placed on the string to produce a fundamental frequency of 110 Hz.
5. (14 pts) A 13 lb bowling ball is connected to a vertical spring (k=8.5 lb/ft). The ball is given an initial displacement and released from rest. At t=2.2 seconds, the ball’s instantaneous velocity is 4.1 ft/sec. Assuming the ball oscillates in simple harmonic motion, determine its maximum acceleration.

6. (14 pts) A circular disk rotates about a point 0.12 m from the top 3 cycles every 5 seconds. Determine the radius of the disk. $I_{cm} = \frac{1}{2} mR^2$ where R is the radius of the disk.
7. (14 pts) Betty and Bobby both play the bassoon which behaves as an open pipe that is 2.54 m long. Betty’s bassoon is room temperature (20°C) while Bobby’s bassoon is 10°C after leaving it in his car overnight. Determine the beat frequency heard when Betty and Bobby play their fundamental frequency together. Assume the instruments do not change temperature.

8. (14 pts) A large speaker emits sound equally in all directions. At 2.0 m away, the sound level is 115 dB. Determine the sound level at 50 m away.
9. (14 pts) Kevin is driving at a constant speed and hears a frequency of 880 Hz while approaching a stationary siren. While driving away from the siren, he hears a frequency of 770 Hz. How fast is Kevin going? Assume speed of sound = 767 mph.

10. (14 pts) Based on the scenario shown, determine the angle $\theta$ at which light is reflected back into the glass.