Harmonic Motion

\[ x(t) = A \sin(\omega t + \delta) = a_1 \sin(\omega t) + a_2 \cos(\omega t) \]

\[ v(t) = A\omega \cos(\omega t + \delta) = -a_1 \omega \cos(\omega t) - a_2 \omega \sin(\omega t) \]

\[ a(t) = -A\omega^2 \sin(\omega t + \delta) = -a_1 \omega^2 \sin(\omega t) - a_2 \omega^2 \cos(\omega t) \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ A = \sqrt{a_1^2 + a_2^2} \]

\[ a_1 = \frac{v_0}{\omega} \quad a_2 = x_0 \]

\[ \delta = \tan^{-1}\left(\frac{a_2}{a_1}\right) \]

\[ T = \frac{2\pi}{\omega} \quad f = \frac{1}{T} \quad \omega = 2\pi f \]

Wave Equation

\[ v \quad \text{wave velocity} \]
\[ A \quad \text{amplitude} \]
\[ k \quad \text{wave number} \]
\[ \omega \quad \text{angular frequency} \]
\[ \lambda \quad \text{wavelength} \]
\[ f \quad \text{frequency} \]

\[ y(x, t) = A \cos(kx - \omega t) \]

\[ v \] is positive if sign of \( \omega \) is negative.

\[ v = \lambda f \]

\[ k = \frac{2\pi}{\lambda} \]

Parallel Axis Theorem

\[ l = I_{CM} + mr^2 \]

Pendulums

Simple:

\[ \omega = \sqrt{\frac{g}{l}} \]

Physical:

\[ \omega = \sqrt{\frac{mg\sin\theta}{l}} \]

Speed of Sound

\[ v = \sqrt{\frac{B}{\rho}} \]

\( B \) - Bulk Modulus
\( \rho \) - mass density

Speed of Sound in Air:

\[ v \approx (331 + .67T) \text{m/s} \quad (T \text{ in } ^\circ\text{C}) \]

Natural frequencies

String:

\[ \lambda_n = \frac{2\lambda}{n} \]

\[ f_n = \frac{v}{\lambda} = \frac{n}{2\pi} \sqrt{\frac{T}{\mu}} \]

Air Columns:

\[ \lambda = \frac{4\lambda}{n} \quad \text{closed} \quad (n = 1, 3, 5, \ldots) \]

\[ \lambda = \frac{2\lambda}{n} \quad \text{open} \quad (n = 1, 2, 3, \ldots) \]

Wave Speed:

Cables, Ropes, etc.

\[ T \quad \text{Tension} \]
\[ \mu \quad \text{mass per unit length} \]
\[ E \quad \text{Modulus of elasticity} \]
\[ \rho \quad \text{mass density} \]

Transverse:

\[ v = \sqrt{\frac{T}{\mu}} \]

Longitudinal:

\[ v = \sqrt{\frac{E}{\rho}} \]

Sound Level

\[ \beta \text{ (dB) } = 10 \log \frac{l}{l_0} \]

\( l \) - Intensity
\( l_0 \) - reference intensity, \( 1 \times 10^{-12} \text{ W/m}^2 \)

Doppler Shift

\[ f' = f \left( \frac{v + v_s}{v + v_l} \right) + \text{ listener to source} \]

Wave Energy, Power, Intensity

\[ E \quad \text{energy} \]
\[ I \quad \text{intensity} \]
\[ P \quad \text{power} \]
\[ \bar{P} \quad \text{average power} \]

\[ E = 2\pi^2 \mu vf^2A^2 \]

\[ \bar{P} = \langle P \rangle = 2\pi^2 \mu \bar{v}f^2A^2 \]

\[ P = 4\pi^2 \mu \bar{v}f^2A^2 \cos^2(kx - \omega t) \]

\[ l = \frac{\bar{P}}{4\pi^2r^2} = \frac{l_2}{l_1} = \frac{r_1^2}{r_2^2} \]

Light Waves

Law of Reflection:

\[ \theta_r = \theta_a \]

Index of refraction:

\[ n = \frac{c}{v} \]

Snell's Law:

\[ n_a \sin\theta_a = n_b \sin\theta_b \]

Light wavelength:

\[ \lambda = \frac{\lambda_0}{n} \]

Total Internal Reflection:

\[ \sin\theta_{\text{crit}} = \frac{n_b}{n_a} \]

Speed of light in vacuum:

\[ c = 3 \times 10^8 \text{ m/s} \]
1. (1 pt) Determine the number of periods shown in the graph to the right.
   a. 1.25
   b. 1.5
   c. 2
   d. 2.25
   e. 2.5
   f. 3

2. (1 pt) A tuning fork produces a steady 520 Hz tone. When struck and held near a vibrating guitar string, thirty beats are counted in 5 seconds. What are the possible frequencies produced by the guitar string?
   a. 490 Hz or 550 Hz
   b. 514 Hz or 526 Hz
   c. 515 Hz or 525 Hz
   d. 520 Hz

3. (14 pts) A wave has an amplitude of 0.82 m and is moving at 0.6 m/s. The distance between three wavecrests is 12 m. Write the wave equation for this wave.
4. (14 pts) An object is in simple harmonic motion and has a maximum displacement of 0.12 meters and a maximum acceleration of 95 m/s². The initial displacement is 0.08 m. Determine the magnitude of the initial velocity.

5. (14 pts) A 2.6 kg object is hung as shown. When the object is swung as a physical pendulum about the pivot point, it has a period of 0.9 seconds. Determine the mass moment of inertia of the object about point A.
6. (14 pts) A steel wire has a mass of 0.0048 kg. When pulled with a force of 580 N, the fundamental frequency of the wire is 440 Hz. Determine the length of the wire.

7. (14 pts) Three people are playing bassoons at a sound level of 68 dB each. Two people are playing oboes at a sound level of 66 dB each. Four people are playing clarinets at a sound level of 70 dB each. Determine the total sound level the nine people produce.
8. (14 pts) A car moving at 12 m/s sends out a signal at 1200 Hz. The signal bounces off a truck that is moving towards the car. The return frequency is 1320 Hz. Determine the speed of the truck. Assume the speed of sound in air is 343 m/s.

9. (14 pts) Determine the index of refraction of the prism shown.