Harmonic Motion

\[ x(t) = A \sin(\omega t + \delta) = a_1 \sin(\omega t) + a_2 \cos(\omega t) \]

\[ v(t) = A\omega \cos(\omega t + \delta) = a_1 \omega \cos(\omega t) - a_2 \omega \sin(\omega t) \]

\[ a(t) = -A\omega^2 \sin(\omega t + \delta) = -a_1 \omega^2 \sin(\omega t) - a_2 \omega^2 \cos(\omega t) \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ A = \sqrt{a_1^2 + a_2^2} \quad a_1 = \frac{v_0}{\omega} \]

\[ a_2 = x_0 \]

\[ \delta = \tan^{-1}\left( \frac{a_2}{a_1} \right) \]

\[ T = \frac{2\pi}{\omega} \quad f = \frac{1}{T} \quad \omega = 2\pi f \]

Parallel Axis Theorem

\[ I = I_{CM} + Mr^2 \]

Pendulums

simple: \[ \omega = \sqrt{\frac{g}{l}} \]

physical: \[ \omega = \sqrt{\frac{Mg}{l}} \]

Speed of Sound

\[ v = \sqrt{\frac{B}{\rho}} \quad B - \text{Bulk Modulus} \]

\[ \rho - \text{mass density} \]

Speed of Sound in Air:

\[ v = (331 + .6T) \text{m/s} \quad (T \text{ in } ^\circ \text{C}) \]

Natural frequencies

String:

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = \frac{v}{\lambda} = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]

Air Columns:

\[ \lambda = \frac{4L}{n} \quad \text{closed} \quad (n = 1, 3, 5, ...) \]

\[ \lambda = \frac{2L}{n} \quad \text{open} \quad (n = 1, 2, 3, ...) \]

Wave Speed:

Cables, Ropes, etc.

\[ T - \text{Tension} \]

\[ \mu - \text{mass per unit length} \]

\[ E - \text{Modulus of elasticity} \]

\[ \rho - \text{mass density} \]

Transverse: \[ v = \sqrt{\frac{E}{\mu}} \]

Longitudinal: \[ v = \sqrt{\frac{E}{\rho}} \]

Sound Level

\[ \beta \text{ (dB)} = 10 \log \frac{I}{I_0} \]

\[ I - \text{intensity} \]

\[ I_0 - \text{reference intensity, } 1 \times 10^{-12} \text{W/m}^2 \]

Doppler Shift

\[ f' - \text{shifted frequency} \]

\[ v - \text{velocity of sound in medium} \]

\[ v_L - \text{velocity of listener} \]

\[ v_S - \text{velocity of source} \]

\[ f' = \frac{f_0 (v + v_L)}{v + v_S} + \text{listener to source} \]

Beat Frequency:

\[ |f_1 - f_2| \]

Wave Energy, Power, Intensity

\[ E - \text{energy} \]

\[ I - \text{intensity} \]

\[ P - \text{power} \]

\[ \bar{P} - \text{average power} \]

\[ E = 2\pi^2 \mu v f^2 A^2 \]

\[ \bar{P} = \langle P \rangle = 2\pi^2 \mu v f^2 A^2 \]

\[ P = 4\pi^2 \mu v f^2 A^2 \cos^2(kx - \omega t) \]

\[ I = \frac{\bar{P}}{4\pi r^2} = \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \]

Light Waves

Law of Reflection:

\[ \theta_r = \theta_a \]

Index of refraction:

\[ n = \frac{c}{v} \]

Snell's Law:

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

Light wavelength:

\[ \lambda = \frac{\lambda_0}{n} \]

Total Internal Reflection:

\[ \sin \theta_{\mathrm{crit}} = \frac{n_b}{n_a} \]

speed of light in vacuum:

\[ c = 3 \times 10^8 \text{m/s} \]
1. (2 pts) A mass on a spring in simple harmonic motion has amplitude A and period T. What is the total distance traveled by the mass after a time interval 3.5T?
   a. 1.75A  b. 3.5A  c. 7A  d. 10.5A  e. 12.25A  f. 14A  g. 16A

2. (2 pts) Suppose you shake the end of a stretched string to produce a wave. Moving your hand up and down a greater distance as you generate the wave will:
   a. cause the wave speed to increase  b. cause the wave speed to slow down  c. not affect wave speed

3. (2 pts) The traces below show beats that occur when two different pairs of waves interfere. For which case is the difference in frequency of the original waves greater?
   a. pair 1  b. pair 2  c. same for both pairs  d. impossible to tell by just looking

4. (2 pts) In a vacuum, red light has a wavelength of 700 nm and violet light has a wavelength of 400 nm. The speed of red light in a vacuum is:
   a. faster than violet light  b. the same as violet light  c. slower than violet light

5. (4 pts) What is the frequency of a wave created by an earthquake that has a wavelength of 650 m and gets to another city 82 km away in 11 s?

6. (4 pts) The intensity of a jet engine is 42 W/m² at 25 m away. What is the intensity at 50 m away?
7. (14 pts) A system is in simple harmonic motion with a maximum displacement of 0.23 ft and a maximum acceleration of 41 ft/s². The stiffness of the system is 570 lb/ft. Determine the mass of the system.

8. (14 pts) A blue object (BO) has a mass moment of inertia about its center of mass of 0.045 kg·m², and a mass of 1.4 kg. The BO is hung from a point 0.16 m above its center of mass and swings back and forth. Determine the length of a simple pendulum that would have same period of oscillation as the BO.
9. (14 pts) The fluitar is a 0.4m long instrument that has both an open pipe and a string. The instrument is played by blowing air through the pipe and simultaneously plucking the string. Determine the tension in the string such that the fundamental frequency of the string is 3 times the fundamental frequency of the pipe. The mass per unit length of the string is 0.0016 kg/m. Assume the speed of sound in air is 343 m/s.

10. (14 pts) Roger, Chris, and Kevin are singing Rocky Top. Roger sings at a sound level of 64 dB. Chris sings at a sound level of 72 dB. The combined sound level of all three is 74 dB. Determine the sound level at which Kevin is singing.
11. (14 pts) Sheldon Cooper has developed the most convoluted way of measuring temperature. Howard drives down the road at 18 m/s while blowing a horn at 1200 Hz. Leonard drives toward Howard at 14 m/s and hears a frequency of 1320 Hz. Determine the temperature.

12. (14 pts) Determine the angle $\beta$ such that there is total internal reflection at point B.