Introduction
Two instruments were designed and constructed to play Rocky Top. The first instrument was a single-string guitar. We used a guitar string attached to a 2x4 and a hanging weight to provide constant tension in the string. We named this instrument the Bucket Banjo. Our second instrument was a wind instrument made with two PVC pipes and water. We called this instrument the Water Trombone. As a bonus instrument, we constructed a percussion instrument using various wrenches hung in a frame and played as chimes. We called this instrument the Wrench Chimes.

Final Design: Instrument #1 (The Bucket Banjo)
The team decided one of the instruments should be a stringed instrument. Original ideas included an instrument with multiple strings on a fret board, similar to a guitar or banjo, or free strings in tension like a harp. Some preliminary testing indicated significant difficulty in maintaining the correct tension in multiple strings thus leading the team to create an instrument with only one string. Changing pitch could be accomplished by changing the tension in the string or altering the length of the string. The team decided on the latter.

A guitar string runs the length of a 2x4. One end of the string is mounted to the wood while the other end is looped and hooked to a 5 gallon bucket. Three 5-lb weights are placed in the bucket to give the string a constant tension of 15 lb. Sound is produced by plucking the string. The length and thus the pitch is changed by pressing down at various locations along the string.

Materials:
- 2x4
- guitar string
- 5-gallon bucket
- three 5-lb weights
- eyebolt
- small nail
- S-hook
- popsicle stick

Design Details:
The 2x4 is 50 inches long. A small nail is placed through the beaded end of the guitar string and nailed into the wood 4 \( \frac{1}{2} \) inches from one end. A 1” x \( \frac{3}{4} \) “ piece of popsicle stick is glued one inch down from the nail to create a bridge. A hole is drilled 5 \( \frac{5}{8} \) inches from the other end of the 2x4 and an eyebolt is inserted. The string is threaded through the eyebolt and looped around an s-hook. The other end of the hook holds the handle of the 5-gallon bucket. Three 5-lb weights are placed inside the bucket.
Analysis of the Bucket Banjo

The frequency of the string was obtained using the following equation:

\[ f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \]

where \( f_1 \) is the fundamental frequency, \( L \) is the length of the string, \( T \) is the tension in the string, and \( \mu \) is the mass per unit length.

The tension was determined based on a free body diagram of the bucket as shown in Figure 2.

![Free Body Diagram](image)

Weight of bucket: 15 lb
\[ \sum F_y = T - 15 \text{ lb} = 0 \]
\[ T = 15 \text{ lb} \]

Figure 3. Free Body Diagram

The unit weight of the string given by the manufacturer was 0.00011682 lb/in. Mass per unit length was calculated as:

\[ \mu = \frac{0.00011682 \text{ lb/in}}{322 \text{ ft/s}^2} = 4.354 \times 10^{-5} \text{ slug/ft} \]

We decided to play Rocky Top based on the lowest note of the song (D3) being the fundamental frequency of the string (146.8 Hz). We determined the other necessary frequencies of notes based on the corresponding music notes. We then calculated the lengths required to produce the corresponding frequencies. Rearranging the equation for frequency of a wave on a string yielded our equation for length:

\[ L = \frac{1}{2f} \sqrt{\frac{T}{\mu}} \]

Thus the total length of the string was calculated:
\[ L = \frac{1}{2(146.8 \text{ Hz})} \sqrt{\frac{15 \text{ lb}}{4.354 \times 10^{-5} \text{ slug/ft}}} = 27.5 \text{ in} \]

<table>
<thead>
<tr>
<th>Note</th>
<th>freq (Hz)</th>
<th>length (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D3</td>
<td>146.8</td>
<td>27.5</td>
</tr>
<tr>
<td>E3</td>
<td>164.8</td>
<td>24.4</td>
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<td>F#3</td>
<td>185.0</td>
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<td>A3</td>
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<td>B3</td>
<td>247.0</td>
<td>16.25</td>
</tr>
<tr>
<td>C4</td>
<td>261.6</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Table 1: Length calculations for corresponding frequencies (Excel)
Final Design: Instrument #2 (Water Trombone)

To complement the Bucket Banjo, the team decided to build a wind instrument. Our inspiration for this was a combination of a simple pan pipe and a slide whistle. The problem with a pan pipe, besides being declared too simple for this project, is the difficulty in changing notes quickly enough because of having to blow on different pipes. Our team began to brainstorm ways to change the length of the pipe while playing, as with a slide whistle or trombone. We initially considered having a pipe in a bucket of water, then blowing across the pipe. We ran into difficulty finding a bucket deep enough to create a tall column of water, thus leading to our decision to use a bigger PVC pipe to hold the water and a smaller PVC pipe to move up and down inside the bigger pipe while blowing across the smaller pipe.

![Diagram of water trombone]

The outer PVC pipe was 28 inches long and had a diameter of ¾”. The inner PVC pipe was 21 inches long and had a diameter of ½”. After much deliberation on how best to seal the end of the outer pipe to be able to hold water, a rubber bouncy ball was discovered and promptly wedged into one end of the pipe. Hot glue was used to seal any remaining gaps.

The instrument was played by blowing across one end of the smaller PVC pipe, while the other end was inserted into the larger pipe and thus into the water. Higher frequencies when the air column was shortened by allowing more water to fill the pipe.

Though the water trombone behaves as a closed wind pipe and the calculations should be straightforward, it would be difficult to see the markings on the pipe while playing to know where the notes should be.

Final Design: Bonus Instrument (Wrench Chimes)

Though a third instrument was not required, the team decided the band needed a percussion instrument. While looking through tools and dropping several on the concrete floor in Estabrook 13, it was discovered that different wrenches produce different pitches of sound when they come into contact with a hard surface. The team decided to hang the wrenches from fishing line and strike them with a metal rod.

Unfortunately, the wrenches resonated too much when hit such that it was difficult to distinguish the fundamental note from all the overtones. The fishing line was traded for nylon string in efforts to help deaden the vibrations thereby limiting the overtones produced. Further difficulty was the limitation on being able to modify the pitch. Slight modifications could be made in where along the wrench it was struck, but if significant pitch differences were required, a different wrench altogether must be used.

Frequency is a function of both stiffness and mass: $\omega = \sqrt{\frac{k}{m}}$. In general, as the formula suggests, increasing the mass of the wrench decreased the frequency. However, stiffness also changed from various wrenches further complicating any calculations on this instrument.
Results
The Bucket Banjo was easy to build and consistent in pitch. Having a constant tension provided by a known weight made the calculations and the performance relatively easy. The accuracy of the notes relied heavily on the player’s ability to move his fingers quickly to the correct positions. Also, it was found that plucking the string closer to the bucket-end provided more resonance. It took practice and repetition to be able to pluck the string in the optimal location while pressing the string for the right notes.

The Water Trombone was easy to construct and fun to play. It does, however, require some technique while blowing to get a good sound and also requires a good ear for note placement.

The Wrench Chime was a good idea in theory, but much more difficult to adjust to get the correct pitches. The team was relieved that calculations were not required for this instrument.

Conclusion
A simple design and minimal construction requirements allowed more time for minor adjustments and practice. Calculating the theoretical values prior to construction improved the efficiency of the overall process instead of relying solely on trial-and-error.

References
Note frequencies: http://www.phy.mtu.edu/~suits/notefreqs.html

Noyce guitars: http://www.noyceguitars.com/Technotes/Articles/T3.html


Guitar string manufacturer website: http://www.daddario.com/upload/tension_chart_13934.pdf