EXAMPLE

Given: Charges, locations, and net force of q₁ and q₂ on q₃ as shown in figure.

Required:
   a. Distances b and s.
   b. Charge q₂.

Solution:

\[
\left| F_{13} \right| = k \frac{q_1 q_3}{r^2} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{4 \times 10^{-6} C(3 \times 10^{-6} C)}{(0.05 m)^2} = 43.2 N
\]

Sum forces in x and y direction:

\[
x: - F_{13} \cos(40^\circ) + F_{23} \cos(\alpha) = 0
\]

\[
y: - F_{13} \sin(40^\circ) - F_{23} \sin(\alpha) = -45 N
\]

Substitute \( F_{13} = 43.2 \) N into equations, and use Solver in Excel or Matlab to solve simultaneous nonlinear equations. Results are \( F_{23} = 37.3 N \) and \( \alpha = 27.5^\circ \).

Solve for \( \beta = 180^\circ - 40^\circ - 27.5^\circ = 112.5^\circ \). Use law of sines to find b and s.

\[
\frac{s}{\sin(40^\circ)} = \frac{0.05 m}{\sin(27.5^\circ)} = \frac{b}{\sin(112.5^\circ)}
\]

Solving, \( s = 0.0696 m \) and \( b = 0.1000 m \).

Use Coulomb’s law to solve for q₂. Since \( F_{23} \) must be in the fourth quadrant, q₂ must be a negative charge.

\[
\left| F_{23} \right| = k \frac{q_2 q_3}{r^2} \Rightarrow 37.3 N = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{4 \times 10^{-6} C(q_2)}{(0.0696 m)^2}
\]

Solving, \( |q_2| = 5.02 \mu C \). Therefore, \( q_2 = -5.02 \mu C \).