Module 3, Lecture 4: 1st Law of Thermodynamics

\[ \Delta U = U_B - U_A = Q - W \]

\[ Q = W + \Delta U \]

**U:** INTERNAL ENERGY, just a function of state of system (or moles) of MASS and TEMP

- We will see that U is a function of MASS and TEMP

**Q:** heat flow INTO the system

**W:** work done BY the thermal system

cons. of energy

EF 152 Spring, 2014 Lecture 3-4
Work Done by a Thermal System

Piston of area $A$ pushed out by pressure $p$

$$dW = \overrightarrow{F} \cdot d\overrightarrow{x} = p A \, dx$$

$$dW = p \, dV$$

integrate $W = \int_{v_1}^{v_2} p \, dV$

Work is the area under each path of the p-V curve.

Work Done by a Cyclic Path (Engine)

$$Q_{cycle} = W_{cycle}$$

Net work is the area enclosed by the path of the cycle.
Heat Capacity

Recall heat capacity is proportional to \( \Delta T \)

\[
Q = C \Delta T = n c' \Delta T
\]

\( c' \) = molar heat capacity, cal/(mol-°C)

With gases, \( c' \) is not a constant, but a function of how the temperature change is made and the state variables, \( p, V, \) and \( T \).

Two special cases

Constant volume: \[
Q = n c'_v \Delta T
\]

Constant pressure: \[
Q = n c'_p \Delta T
\]

\[c'_p - c'_v = R\]

Ideal monatomic gas:
(helium, neon)
\[
\begin{align*}
c'_p &= \frac{5}{2} R \\
c'_v &= \frac{3}{2} R
\end{align*}
\]
\[
\approx 20.87 \text{ J/mol-K} \quad \approx 12.47 \text{ J/mol-K}
\]

Ideal diatomic gas:
(oxygen, \( O_2 \), nitrogen, \( N_2 \))
\[
\begin{align*}
c'_p &= \frac{7}{2} R \\
c'_v &= \frac{5}{2} R
\end{align*}
\]
\[
\approx 29.1 \text{ J/mol-K} \quad \approx 20.78 \text{ J/mol-K}
\]

EF 152 Spring, 2014 Lecture 3-4

R=8.314 J/mol-K
**Work Done by an Isobaric Process**

**Isobaric: Constant Pressure**

\[ W = P(V_B - V_A) \]

\[ Q = nc_p' \Delta T \]

What is the change in internal energy (\(\Delta U\))?
Work Done by an Isochoric Process

Isochoric: Constant Volume (isovolumetric)

\[ W = 0 \quad \text{(no area!)} \]

\[ Q = n c_v' \Delta T \]

What is the change in internal energy (\(\Delta U\))? 

\[ Q = \Delta U + W \]
Work Done by an Isothermal Process

Isothermal: Constant Temperature

\[ pV = nRT \]
\[ P = \frac{nRT}{V} \]

\[ W = \int_{V_1}^{V_2} p \, dV \]

Just a little Calculus...

\[ W = \int_{V_1}^{V_2} \frac{nRT}{V} \, dV = nRT \int_{V_1}^{V_2} \frac{1}{V} \, dV \]

\[ W = nRT \left( \ln V_2 - \ln V_1 \right) \]
\[ = nRT \ln \left( \frac{V_2}{V_1} \right) \]
\[ \frac{\partial E}{\partial V} = P \ln \left( \frac{V_2}{V_1} \right) \]

What is the change in internal energy (\( \Delta U \))? It is a function of mass and temperature. Since constant temp. it follows that:

\[ \Delta U = 0 \]
Work Done by an Adiabatic Process

Adiabatic: No **heat** flows into or out of system, \( Q = 0 \)

Adiabatic process:
- Well **insulated**
- Happens **quickly**

What is the change in internal energy (\( \Delta U \))?

\[
\Delta U = -\nabla V
\]

\[
pV^{\gamma} = \text{constant}
\]

\[
TV^{\gamma - 1} = \text{constant}
\]

\[
T_p \left( \frac{1 - \gamma}{\gamma} \right) = \text{constant}
\]

Ideal monatomic gas: \( \gamma = \frac{5}{3} = 1.66 \)

Ideal diatomic gas: \( \gamma = \frac{7}{5} = 1.4 \)

\[
W = p_A V_A^\gamma \int_{V_1}^{V_2} \frac{dV}{V^{\gamma}}
\]

\[
W = p_A V_A^\gamma \left( \frac{1}{1 - \gamma} \right) \left( V_B^{1 - \gamma} - V_A^{1 - \gamma} \right) = \frac{1}{\gamma - 1} \left( p_A V_A - p_B V_B \right)
\]

EF 152 Spring, 2014 Lecture 3-4
Example: Diesel Engine

The compression ratio of a diesel engine is 15:1. The initial pressure is 1.01x10^5 Pa (1 atm) and the initial temperature is 27°C (80.6°F). This can be considered an adiabatic process. Air is mostly a diatomic gas, for which \( \gamma = 1.4 \). Find the final pressure and temperature.

\[
P_V \gamma = \text{constant} \quad P_A V_A^\gamma = P_B V_B^\gamma
\]

\[
P_B = P_A \frac{V_A}{V_B} = P_A \left( \frac{V_A}{V_B} \right)^\gamma = 1.01 \times 10^5 \text{Pa} \times (15)^{1.4} = 44.76 \times 10^5 \text{Pa}
\]

\[
T_V \gamma^{-1} = \text{constant} \quad T_A V_A^{\gamma^{-1}} = T_B V_B^{\gamma^{-1}}
\]

\[
T_B = T_A \left( \frac{V_A}{V_B} \right)^{\gamma^{-1}} = 300 \text{K} \times (15)^{1.4^{-1}} = 886 \text{K} = 613°C
\]

The initial volume of the cylinder in the previous problem is 1.0 L. Find the work done by gas in compression cycle.

\[
W = \frac{1}{\gamma - 1} \left( P_A V_A - P_B V_B \right) = -494 \text{J}
\]

\[
= \frac{1}{1.4 - 1} \left( 1.01 \times 10^5 \text{Pa} \times (0.001 \text{m}^3) - 44.76 \times 10^5 \text{Pa} \times \frac{0.001 \text{m}^3}{15} \right)
\]

EF 152 Spring, 2014 Lecture 3-4