Module 2, Lecture 3
Waves

A wave is a disturbance that propagates in a certain direction with a certain speed.

- 1D
- 2D
- 3D

Physical medium
- Waves in water
- Waves in elastic bodies
- Sound

Empty space (a vacuum)
Electromagnetic waves
Waves: Basic definitions

You are in a boat out on the ocean watching the waves go by. To fully describe the waves, you need three things:

1. \( A \) - Amplitude
2. \( \lambda \) - wavelength
3. \( f \) - frequency / waves/time

- \( T \), period – Time for one wavelength to pass \( T = 1/f \)
- \( \omega \), angular frequency – \( \omega = 2\pi f \) or \( \omega = \frac{2\pi}{T} \)
- \( v \), wave velocity – How fast the wave is moving \( v = \lambda f \)
- \( k \), wave number – \( 2\pi/\lambda \) or \( 2\pi/(\text{length}) \)
Examples: Tsunami and Ultrasound

Satellite photos of the December 26, 2004 tsunami showed a spacing of wave crests of 800 km and a period between the waves of 1 hour. Find the speed of the waves in km/hr.

\[ \lambda = 800 \text{ km} \quad v = \lambda f \quad f = \frac{1}{T} \]

\[ T = 1 \text{ hour} \quad v = \frac{\lambda}{T} = \frac{800 \text{ km}}{1 \text{ hour}} = 800 \text{ km/hr} \approx 500 \text{ mph} \]

In a typical ultrasound, waves travel through body tissue with a speed of 1500 m/s. For a good detailed image, the wavelength should be no more than 1 mm. Find the frequency required for a good scan.

\[ v = \lambda f \]

\[ f = \frac{v}{\lambda} = \frac{1500 \text{ m/s}}{0.001 \text{ m}} = 1,500,000 \text{ Hz} \Rightarrow 1.5 \text{ MHz} \]
Some Types of Waves

Transverse:
\[ \omega = \sqrt{\frac{k}{m}} \]  
\[ v = \lambda f \]  
\[ \frac{m/s}{v} = \frac{T}{\sqrt{\mu}} \]

Longitudinal:
\[ \frac{m/s}{v} = \sqrt{\frac{E}{\rho}} \]

Surface Waves (circular):

\[ T = \text{tension} \quad \text{N or lbf} \]
\[ \mu = \text{mass per unit length} \]
\[ E = \text{modulus of elasticity} \]
\[ \rho = \text{mass density} \]
Example: Underground cables

To determine the location of a break in an underground cable, the cable can be hit with an instrumented hammer to generate a longitudinal wave. Find the location of a break in the cable if it takes 0.038 seconds for the wave to travel down and back.

\[ V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{29 \times 10^6 \text{ psi}}{12\text{ in}^2/1\text{ ft}^2} \left(\frac{1\text{ in}^2}{1\text{ ft}^2} \right)^2 \left(\frac{490 \text{ lb/ft}^3}{32.2 \text{ lb/ft}^3} \right)} \]

\[ V = 16,560 \frac{\text{ft}}{\sec} \]

\[ V = \frac{\text{d}}{t} \quad d = V \cdot t = (16,560 \frac{\text{ft}}{\sec})(0.038 \sec) = 629 \text{ ft} \]

\[ d \div 2 = \frac{315 \text{ ft}}{2} \]

\[ E = 29 \times 10^6 \text{ psi} \]

\[ \gamma = 490 \text{ lb/ft}^3 \]

Weight density \[ W = mg \]
The Wave Equation

\[ y(x, t) = A \cos(\omega t) \]

Periodic in both space and time

Speed of the wave

\[ v = \frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi f}{\lambda} = \frac{\lambda f}{c} \]

This is for wave traveling to right (forward). For wave moving to the left, the argument is \((kx + \omega t)\)

\[ y = A \cos(kx - \omega t) \]

Motion of a particle:

\[ v = \frac{\partial y}{\partial t} = A \sin(kx - \omega t)(-\omega) = A \omega \sin(kx - \omega t) \]

\[ a = \frac{\partial^2 y}{\partial t^2} = A \omega^2 \cos(kx - \omega t)(-\omega) = -A \omega^2 \cos(kx - \omega t) = -\omega^2 y \]

Particle follows **SIMPLE HARMONIC MOTION**
Animation courtesy of Dr. Dan Russell, Kettering University
Power and Energy in Waves

Waves transfer energy, without the transfer of mass.

\[ E = \frac{1}{2} \rho A v f^2 A^2 \]

Average power transmitted:

\[ P = \langle P \rangle = \frac{E}{t} = 2\pi^2 \mu v f^2 A^2 \]

Instantaneous power:

\[ P = 4\pi^2 \mu v f^2 A^2 \cos^2(ka - \omega t) \]

Wave Intensity

Average power transferred across unit area perpendicular to energy flow.

\[ I = \frac{\bar{P}}{\text{Surface Area}} = \frac{\bar{P}}{4\pi r^2} \]

\[ I \propto \frac{1}{r^2} \]

\[ \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \]

\[ I = \frac{2\pi^2 \mu v f^2 A^2}{\text{Surface Area}} = 2\pi^2 \rho v f^2 A^2 \]

Ocean Waves: 1.5 m amplitude, 8 sec period
Power generated is 70.6 kW/m
At 20% efficiency, 0.08 m crest length needed per house
Example: Tornado Warning Siren

A tornado warning siren can produce a sound of intensity $I_1 = 0.25 \text{ W/m}^2$ at 15 m away.

A) What is the intensity at 30 m away?

1.) $2 \times I_1$

2.) $4 \times I_1$

3.) $I_1/2$

4.) $I_1/4$

B) What power is required to run the siren?

$$I = \frac{P}{A} \quad P = I \cdot A = \left(2.5 \frac{W}{m^2}\right) \left(15 \text{ m}^2\right) = 37.5 \text{ W}$$

C) To hear the siren, it should have a greater intensity than normal conversation, which is $1 \times 10^{-6} \text{ W/m}^2$. How far away can you hear the siren?

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\left(\frac{1 \times 10^{-6} \text{ W/m}^2}{0.25 \text{ W/m}^2}\right) = \left(\frac{15 \text{ m}}{r_2}\right)^2$$

$$r_2 = 7500 \text{ m} = 7.5 \text{ km} \approx 4.6 \text{ miles}$$

EF 152 Fall, 2013 Lecture 2-3