Module 2, Lecture 2  
**Pendulums**

**Simple Pendulum**
- Displacement along arc: \( x = l \theta \)
- If restoring force is proportional to \( \theta \) then motion will be simple harmonic motion.

![Pendulum diagram](image)

\[
ml \frac{d^2 \theta}{dt^2} + mg \theta = 0
\]

Simple harmonic motion

\[
\omega = \sqrt{\frac{g}{l}} \quad T = 2\pi \sqrt{\frac{l}{g}}
\]

Small angle approximation:

\[
\theta \approx \sin \theta
\]

**Physical Pendulum**
- Any object, if suspended and then displaced so the gravitational force does not run through the center of mass, will oscillate due to the torque.

\[
\omega = \sqrt{\frac{mgr}{I}}
\]

\[
m = \text{Total mass of object}
\]

\[
r = \text{Parallel distance between axis of interest and axis parallel to this through center of mass}
\]

Parallel Axis Theorem:

\[
I = I_{CM} + mr^2
\]

\[I = \text{Moment of inertia about any axis}\]

\[I_{CM} = \text{Moment of inertia about an axis through the center of mass}\]

\[m = \text{Total mass of object}\]

**Example: Pendulum**

What length of pendulum has a period of exactly 1 second?

![Pendulum example](image)

**Thin Rod Physical Pendulum**
- Determine the frequency of oscillation.

\[
I_{CM} = \frac{1}{12} mL^2
\]

Hey, I remember this from EF 151!
Mass Moment of Inertia of a Bowling Pin

A 38.1 cm tall, 1.5 kg bowling pin can be balanced at a point 14.7 cm above its base. When pivoted about the top, it oscillates with a period of 1.10 seconds. Find the moment of inertia about top and the moment of inertia about the center of mass.

Harmonic Motion: Damped Motion

Damping force proportional to velocity: \( F_{\text{damping}} = -b \cdot \nu \)

\[
\begin{align*}
&< \sqrt{4mk} \quad \text{underdamped} \\
&= \sqrt{4mk} \quad \text{critically damped} \\
&> \sqrt{4mk} \quad \text{overdamped}
\end{align*}
\]

A shock absorber is:
1. underdamped
2. critically damped
3. overdamped

Harmonic Motion: Underdamped Motion

\[
F_D = -bv = -b \frac{dx}{dt}
\]

\[
m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0
\]

\[
x = Ae^{-\alpha t} \sin(\omega t + \delta)
\]

\[
\alpha = \frac{b}{2m}, \quad \omega = \sqrt{\omega^2 - \frac{b^2}{4mk}}
\]

What decreases with time?
1. Frequency
2. Amplitude
3. Wavelength
4. Period

Harmonic Motion: Driven Motion

External harmonic force is applied at its own frequency, \( \omega \)

\[
F = F_0 \sin(\omega t)
\]

\[
\omega = \text{forcing frequency} \quad \omega_0 = \text{natural frequency}
\]

\[
A = \frac{F_0}{\sqrt{m^2 (\omega^2 - \omega_0^2)^2 + b^2 \omega^2}}
\]