Module 2, Lecture 2  Pendulums
Simple Pendulum

Displacement along arc: \( x = l \theta \)

If restoring force is proportional to \( \theta \) then motion will be simple harmonic motion.

\[
FBD = KD
\]

\[
\begin{align*}
\dot{x} &= -W \sin \Theta = m a_x = m \ddot{x} = m \frac{d^2 x}{dt^2} \\
\Theta &= m a \theta = m \ddot{\theta} = m \frac{d^2 \theta}{dt^2} \\
mg \Theta &= ml \ddot{x} \\
\end{align*}
\]

Small angle approximation:
\( \sin \Theta \approx \Theta \) in radians

Simple harmonic motion
\[
\omega = \sqrt{\frac{g}{l}} \quad T = 2\pi \sqrt{\frac{l}{g}}
\]

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Example: Pendulum

What length of pendulum has a period of exactly 1 second?

\[ \omega = \frac{2\pi}{T} \]

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

\[ L = \left( \frac{T}{2\pi} \right)^2 g \]

\[ = \left( \frac{1 \text{ sec}}{2\pi} \right)^2 \frac{32.2 \text{ ft/s}^2}{\text{s}^2} \left( \frac{12 \text{ in}}{\text{ft}} \right) \]

\[ = 9.8 \text{ inch} \]
Physical Pendulum

Any object, if suspended and then displaced so the gravitational force does not run through the center of mass, will oscillate due to the torque.

$$\omega = \sqrt{\frac{mgr}{I}}$$

- $m =$ mass
- $r =$ distance from pivot point to center of mass
- $I =$ mass moment of inertia about pivot point

Parallel Axis Theorem: $I = I_{CM} + mr^2$

- $I =$ Moment of inertia about any axis
- $I_{CM} =$ Moment of inertia about an axis through the center of mass
- $m =$ Total mass of object
- $r =$ Perpendicular distance between axis of interest and axis parallel to this through center of mass
Thin Rod Physical Pendulum

Determine the frequency of oscillation.

\[ \omega = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{mgL}{\frac{1}{12}mL^2 + mr^2}} \]

\[ I = I_{CM} + mr^2 \]

\[ \omega = \sqrt{\frac{g (0.4L)}{\frac{1}{12}L^2 + (0.4L)^2}} = \sqrt{\frac{1.644}{L}} \]

Thin rod

\[ I_{CM} = \frac{1}{12} mL^2 \]
Mass Moment of Inertia of a Bowling Pin

A 38.1 cm tall, 1.5 kg bowling pin can be balanced at a point 14.7 cm above its base. When pivoted about the top, it oscillates with a period of 1.10 seconds. Find the moment of inertia about top and the moment of inertia about the center of mass.

\[ \omega = \sqrt{\frac{mgR}{I}} \]

\[ \frac{2\pi}{1.1 \text{ sec}} = \sqrt{\frac{1.5 \text{ kg} \left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.234 \text{ m})}{I}} \]

\[ I = 0.1055 \text{ kg} \cdot \text{m}^2 \]

\[ I_{cm} = I - mr^2 = 0.1055 - 1.5(0.234) = 0.0234 \text{ kg} \cdot \text{m}^2 \]
Harmonic Motion: Damped Motion

Damping force proportional to velocity: $F_{\text{damping}} = -b \cdot v$

$$\begin{cases} < \sqrt{4mk} & \text{underdamped} \\ = \sqrt{4mk} & \text{critically damped} \\ > \sqrt{4mk} & \text{overdamped} \end{cases}$$

A shock absorber is:
1. underdamped
2. critically damped
3. overdamped

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Harmonic Motion: Underdamped Motion

\[ F_D = -bv = -b \frac{dx}{dt} \]

\[ m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \]

\[ x = Ae^{-\alpha t} \sin(\omega' t + \delta) \]

\[ \alpha = \frac{b}{2m} \quad \omega' = \omega \sqrt{1 - \frac{b^2}{4mk}} \]

What decreases with time?
1. Frequency
2. Amplitude
3. Wavelength
4. Period
Harmonic Motion: Driven Motion

External harmonic force is applied at its own frequency, $\omega$

\[ F = F_0 \sin(\omega t) \]

$\omega$ = forcing frequency
$\omega_0$ = natural frequency

\[
A = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + b^2 \omega^2}}
\]