Module 2, Lecture 1
Oscillations – harmonic motion

- Most problems have been constant force
  - CONSTANT acceleration
- Non-constant force
  - example: SPRING cons. of energy
  - solved with ENERGY methods
  - no information on TIME
- Will solve with time
  - Simple harmonic motion

\[ F = kx \]

Conservation of Energy

\( \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} \)

x measured from UNDEFORMED position

Where is the maximum speed? \( x = 0 \) (undeformed)
Where is the minimum speed? \( x = x_{\text{max}} \)
Where is the maximum acceleration? \( x = x_{\text{max}} \)
Spring Force: Include Time

How do we relate \( x \) and \( a \)?

\[
\begin{align*}
\dot{x} &= \text{velocity} = \frac{dx}{dt} \\
\ddot{x} &= \text{accel} = \frac{d\dot{x}}{dt} = \frac{d^2x}{dt^2}
\end{align*}
\]

\[
F_s = kx
\]

\[
-m\ddot{x} = \frac{d^2x}{dt^2}
\]

\[
\begin{align*}
F_s &= \max \\
-kx &= m\frac{d^2x}{dt^2} \\
\text{Differential Equation}
\end{align*}
\]

Try:

\[
x(t) = A\sin(\omega t)
\]

\[
\frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t)
\]

\[
kx(t) = \omega^2 A\sin(\omega t)
\]

\[
\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}
\]

\[
\omega = \text{natural frequency in rad/sec}
\]
Simple Harmonic Motion:

\[ \omega = \text{angular natural frequency} \quad \omega = \sqrt{\frac{k}{m}} \]

\[ f = \text{frequency, Hz} \quad f = \frac{\omega}{2\pi}; \quad T = \text{period, sec} \quad T = \frac{1}{f} \]

\[ x(t) = A \sin(\omega t + \delta) \]

Trig identity:

\[ \sin(x + y) = \sin x \cos y + \cos x \sin y \]

Initial conditions:

\[ a_1 = \frac{\text{initial velocity}}{\omega} = A \cos \delta \]

\[ a_2 = \text{initial displacement} = A \sin \delta \]

\[ x(t) = a_1 \sin(\omega t) + a_2 \cos(\omega t) \]

A = amplitude

\[ A = \sqrt{a_1^2 + a_2^2} \]

\[ \delta = \text{phase angle} \]

\[ \delta = \tan^{-1} \left( \frac{a_2}{a_1} \right) \]
Harmonic or Oscillatory Motion

\[ x(t) \]

Initial displacement

Initial velocity

\[ T = \frac{2\pi}{\omega} \]

\[ A \]

EF 152 Spring, 2014 Lecture 2-1
# Oscillatory Motion: Velocity, Acceleration

<table>
<thead>
<tr>
<th>Displacement</th>
<th>( x(t) = A \sin(\omega t + \delta) )</th>
<th>Initial Conditions</th>
<th>( x(t) = a_1 \sin(\omega t) + a_2 \cos(\omega t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_{\text{max}} = A )</td>
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</table>

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<tr>
<th>Velocity</th>
<th>( v = \frac{dx}{dt} = \omega A \cos(\omega t + \delta) )</th>
<th></th>
<th>( v = a_1 \omega \cos(\omega t) - a_2 \omega \sin(\omega t) )</th>
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<tbody>
<tr>
<td></td>
<td>( v_{\text{max}} = \omega A )</td>
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<tr>
<th>Acceleration</th>
<th>( a = \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \delta) )</th>
<th></th>
<th>( a = -a_1 \omega^2 \sin(\omega t) - a_2 \omega^2 \cos(\omega t) )</th>
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<td>( a_{\text{max}} = \omega^2 A )</td>
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<tr>
<td></td>
<td>( a = -\omega^2 x(t) )</td>
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</table>

\* calc must be in radians
Example: Earthquake

After the shock of an earthquake, a building will vibrate in simple harmonic motion. A particular building has a natural frequency of 4.2 Hz (26.4 rad/sec), a displacement of 0.86 inches and a velocity of -22 in/sec after the earthquake shock. Write an equation for the displacement vs. time plot shown. Determine the maximum acceleration.

\[ x(t) = a_1 \sin(\omega t) + a_2 \cos(\omega t) = A \sin(\omega t + \delta) \]

\[ \omega = 26.4 \, \text{rad/sec} \]

\[ x_0 = a_2 = 0.86 \, \text{inches} \]

\[ v_0 = -22 \, \text{in/sec} \]

\[ a_1 = \frac{v_0}{\omega} = \frac{-22 \, \text{in/sec}}{26.4 \, \text{rad/sec}} \approx -0.834 \, \text{in/sec}^2 \]

\[ x(t) = 0.834 \sin(26.4t) + 0.86 \cos(26.4t) \]

*Calc in radians

\[ A = \sqrt{a_1^2 + a_2^2} = \sqrt{(-0.834)^2 + (0.86)^2} \approx 1.198 \, \text{in} \]

\[ \frac{a_{\text{max}}}{x_0} = \frac{\omega^2 A}{x_0^2} = (26.4 \, \text{rad/sec})^2 \times 1.198 \, \text{in} = 834.1 \, \text{in/sec}^2 \approx 2.169 \]
**Example: Loudspeaker**

The cone of a loudspeaker oscillates at 262 Hz (middle C). The amplitude of motion is 0.15 mm. The mass of the cone is 225 g. Determine the stiffness of the cone and the maximum velocity and acceleration of the cone.

\[
\omega = 262 \text{ cycle/sec} \times \frac{2\pi \text{ rad}}{\text{cycle}} = 1646 \text{ rad/sec}
\]

\[
\omega = \sqrt{\frac{k}{m}} \quad k = \omega^2 m = (1646 \text{ rad/sec})^2 \times 225 \text{ kg}
\]

\[
\text{vel}_{\text{max}} = \omega A = 1646 \text{ rad/sec} \times 0.15 \text{ mm} = 247 \text{ mm/s}
\]

\[
\text{acc}_{\text{max}} = \omega^2 A = 406.5 \text{ m/s}^2 \times 41.4 \text{ g}
\]
Example: Car supported on springs

Constant force in direction of spring—the weight.
- State 1: weight is supported such that spring is undeflected
- State 2: spring is deflected under the weight
- State 3: maximum deflection during oscillation

Define $x_1$ as the deflection under the weight and $x$ as the additional deflection from $x_1$ as the car oscillates.

About which state does the weight oscillate?

Equilibrium position (state 2)

$$-kx = mx^*$$

$$w - k(x_1 + x) = mx^*$$

$$kx_1 - k(x_1 + x) = mx^*$$