Given: A rope is being moved up and down sinusoidally with a frequency of 2 Hz and an amplitude of 0.075 m. The wave speed is 12 m/s. At time t=0, the end has a maximum positive displacement and is instantaneously at rest.

Required:
A) Find the wavelength, angular frequency, period, and wave number of the wave.
B) Write the wave equation for this wave
C) Find the maximum velocity and acceleration of a particle 2 m from the end; determine the first time that the maximum velocity and acceleration occur.

Solution:
A) 
\[ \lambda = \frac{v}{f} = \frac{12 \text{ m/s}}{2 \text{ Hz}} = 6 \text{ m} \]
\[ \omega = \frac{2\pi \text{ rad}}{\text{cycle}} \times \frac{2 \text{ cycles}}{\text{sec}} = 12.57 \text{ rad/sec} \]
\[ T = \frac{1}{f} = \frac{1}{2 \text{ cycles/sec}} = 0.5 \text{ sec} \]
\[ k = \frac{2\pi}{\lambda} = \frac{2\pi}{6 \text{ m}} = \frac{1.047}{\text{m}} \quad \text{or} \quad k = \frac{\omega}{v} = \frac{12.57 \text{ rad/sec}}{12 \text{ m/s}} = \frac{1.047}{\text{m}} \]

B) 
\[ y(x,t) = A \cos(kx - \omega t) = (0.075m) \cos\left((1.047/m)x - (12.57 \text{ rad/sec})t\right) \]
\[ y(x = 2 \text{ m},t) = (0.075m) \cos\left((1.047/m)(2 \text{ m}) - (12.57 \text{ rad/sec})t\right) \]
\[ = (0.075m) \cos(2.094 \text{ rad} - (12.57 \text{ rad/sec})t) \]
\[ \hat{y}(x = 2 \text{ m},t) = (-12.57 \text{ rad/sec}(0.075m) \sin(2.094 \text{ rad} - (12.57 \text{ rad/sec})t) \]
\[ = (0.9425 \text{ m/sec}) \sin(2.094 \text{ rad} - (12.57 \text{ rad/sec})t) \]

Maximum velocity = 0.9425 m/sec. This will occur when \( \sin(\theta) = 1 \), or \( \theta = \pi/2 \) rad.
\[ \pi/2 = 2.094 \text{ rad} - (12.57 \text{ rad/sec})t \]
\[ t = \frac{2.094 \text{ rad} - \pi/2}{12.57 \text{ rad/sec}} = 0.0417 \text{ sec} \]

\[ \hat{y}(x = 2 \text{ m},t) = -(12.57 \text{ rad/sec})^2(0.075m) \cos(2.094 \text{ rad} - (12.57 \text{ rad/sec})t) \]
\[ = -\left(11.84 \text{ m/s}^2\right) \cos(2.094 \text{ rad} - (12.57 \text{ rad/sec})t) \]

Maximum acceleration = 11.84 m/s^2. This will occur when \( \cos(\theta) = 1 \), or \( \theta = 0 \).
\[ 0 = 2.094 \text{ rad} - (12.57 \text{ rad/sec})t \]
\[ t = \frac{2.094 \text{ rad}}{12.57 \text{ rad/sec}} = 0.1667 \text{ sec} \]