Harmonic Motion

- $\omega$ - angular frequency
- $A$ - amplitude
- $k$ - stiffness
- $m$ - mass
- $\delta$ - phase angle
- $x_0$ - initial displacement
- $v_0$ - initial velocity
- $T$ - period
- $f$ - frequency

$x(t) = A \sin(\omega t + \delta)
= a_1 \sin(\omega t) + a_2 \cos(\omega t)$

$\nu(t) = A \omega \cos(\omega t + \delta)$
$= a_1 \omega \cos(\omega t) - a_2 \omega \sin(\omega t)$

$\alpha(t) = -A \omega^2 \sin(\omega t + \delta)$
$= -a_1 \omega^2 \sin(\omega t) - a_2 \omega^2 \cos(\omega t)$

$\omega = \frac{2\pi}{T}$
$A = \sqrt{a_1^2 + a_2^2}$
$\delta = \tan^{-1}\left(\frac{a_2}{a_1}\right)$

$\lambda = \frac{2L}{n}$
$a_1 = \frac{\nu_0}{\omega}$
$a_2 = x_0$

$T = \frac{2\pi}{\omega}$
$f = \frac{1}{T}$

Parallel Axis Theorem

$I = I_m + Md^2$

Pondulums

Simple: $\omega = \sqrt{\frac{k}{m}}$

Physical: $\omega = \sqrt{\frac{Mg}{I}}$

Speed of Sound in Air

$v = (331 + 0.607 T) m/s$
$T$ in $^\circ C$

Beat Frequency: $|f_2 - f_1|$

Natural Frequencies:
- $n$ - harmonic
- $T$ - tension
- $f$ - frequency
- $v$ - wave velocity in medium

Sound Level

- $I_0$ - reference intensity, $10^{-16}$ W/m$^2$
- $I$ - intensity
- $\beta$ (in dB) = $10 \log \frac{I}{I_0}$

Doppler Shift

- $f_0$ - frequency
- $f$ - shifted frequency
- $v$ - velocity of medium
- $v_0$ - velocity of source
- $v_s$ - velocity of observer

Moving source towards observer

$f' = f_0 \left(1 - \frac{v}{v_0}\right)$

Moving observer towards source

$f' = f_0 \left(1 + \frac{v}{v_0}\right)$

Opposite signs (+ -) used for source/observer moving away from observer/source.

String

- $\lambda = \frac{2L}{n}$
- $\nu_0 = \frac{n}{\lambda_0}$

Air Columns

- $\lambda = \frac{4L}{n}$
- $\lambda_0 = \frac{4L}{n}$

Wave Speed

Cables, Ropes, etc.

- $T$ - tension
- $\mu$ - mass per unit length
- $E$ - modulus of elasticity
- $\rho$ - mass density

Transverse: $v = \sqrt{\frac{T}{\mu}}$

Longitudinal: $v = \sqrt{\frac{E}{\rho}}$

Wave Energy, Power, Intensity

- $E$ - energy
- $I$ - intensity
- $P$ - power
- $P$ - average power

- $E = 2\pi \nu_0^2 \rho^2 A^2$
- $P = \int P(t) dt = 2\pi \rho \nu_0^2 A^2$
- $\lambda_0 = \frac{4\pi}{P}^\frac{1}{2}$

Wave Equations

$\nu = \nu_0 \sin(kx - \omega t)$
$\nu = A' f$

- $k = \frac{2\pi}{\lambda}$

- $v$ is positive if sign of $\omega$ is negative

1. (8 pts) An object in simple harmonic motion is described by $x = 11\sin(12t) + 5\cos(12t)$, where $x$ is measured in meters and $t$ in seconds. Determine the initial velocity of the object.

\[ v = \omega \sin(\omega t + \delta) = a_2 \cos(\omega t) \]

$\omega = 12 \text{ rad/sec}$

$a_2 = 5$

$v_0 = \omega a_2 = 11 \times 12 = 132 \text{ m/s}$

- 1 wrong approach/equation
- 2 math errors
- 2 units, sig figs

2. (8 pts) How long does it take sound to travel 3.2 km if the temperature of the air is 32°F?

\[ v = 331 + 0.607 \times 32 = 331.98 \text{ m/s} \]

\[ t = \frac{d}{v} = \frac{3200}{331.98} = 9.67 \text{ sec} \]

- 1 incorrect temp. units (1.6°F)
- 4 wrong approach/equation
- 1 math error
- 2 units, sig figs

EF 152 Exam #3, Spring, 2009
3. (14 pts) A 10 lb bowling ball is hung from a vertical spring (k=3.5 lb/in). The ball is pulled down from the static equilibrium position and released ($x_0=5.5$ inches, $v_0=0$) so that it oscillates in simple harmonic motion. Determine the maximum speed of the ball.

$$v_{\text{max}} = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.5 \text{ lb/in}}{10 \text{ lb}}} = 5.8 \text{ in/sec}$$

$$v = \frac{\omega}{\sqrt{1 - \frac{v^2}{v_{\text{max}}^2}}} = \frac{11.6}{\sqrt{1 - \frac{6.3^2}{5.8^2}}} = 8.1 \text{ ft/sec}$$

- 2 K calculated incorrectly
- 2w calculated as $\sqrt{\frac{k}{m}}$

- 2 test calculated $\text{Kg}$ as slugs.

4. (14 pts) A 12 meter long cable vibrates at a fundamental frequency of 53 Hz and has a mass of 42 g. Determine the tension in the cable.

$$f_n = \frac{1}{2L} \sqrt{\frac{T}{\mu}} = \frac{1}{2L} \sqrt{\frac{5660}{0.042}} = 566 \text{ N}$$

- 2 μ incorrect

5. (14 pts) Two organ pipes (one open and one closed) are producing their fundamental frequencies. Determine the beat frequency that will be heard if one pipe is 4.0 meters high and the other pipe is 4.2 meters high. The speed of sound in the 23°C room is 348 m/s.

- Both open, $f_h = \frac{48\pi}{4(4.0m)} = 21.7 \text{ Hz}$, $f_b = \frac{348 m/s}{4(4.2m)} = 20.7 \text{ Hz}$, $21.7\text{ Hz} - 20.7\text{ Hz} = 1.04 \text{ Hz}$

- Both closed, $f_h = \frac{48\pi}{4(4.2m)} = 22.8 \text{ Hz}$, $f_b = \frac{48\pi}{4(4.0m)} = 21.7 \text{ Hz}$, $22.8\text{ Hz} - 21.7\text{ Hz} = 1.1\text{ Hz}$

6. (14 pts) A large speaker emits its sound equally in all directions. At a distance of 1.5 meters the sound level is 120 dB. What is the sound level at a distance of 100 meters?

$$I_1 = 10 \log \left( \frac{1}{r_1^2} \right)$$

$$I_2 = 2.25 \times 10^{-4} \text{ W/m}^2$$

$$I_2 = I_1 \frac{r_1^2}{r_2^2}$$

$$I_2 = 10 \log \left( \frac{I_1}{100} \right) = 10 \log \left( \frac{1}{100} \right) = -20 \text{ dB}$$
7. (14 pts) A Gigantic Blue Octopus swimming at a constant speed toward a wall emits a wave pulse with a frequency of 42,000 Hz. It receives the reflected pulse with a frequency of 42,200 Hz. The velocity of sound in water is 1484 m/s. How fast is the GBO moving?

\[
\begin{align*}
\text{\textbf{f}}' &= \frac{f_0}{1 - \frac{v_{\text{obs}}}{v}} \\
\text{\textbf{f}}'' &= \frac{f_0}{1 + \frac{v_{\text{obs}}}{v}} \\
\text{\textbf{f}}'_n &= \frac{\text{f}}{1 + \frac{v_{\text{obs}}}{v}} \\
\text{\textbf{f}}'' &= \frac{\text{f}}{1 - \frac{v_{\text{obs}}}{v}} \\
\text{\textbf{f}}' &= \frac{\text{f}_0}{1 - \frac{v_{\text{obs}}}{v}} \\
\text{\textbf{f}}'' &= \frac{\text{f}_0}{1 + \frac{v_{\text{obs}}}{v}} \\
42,000 \text{ Hz} &= \frac{42,000 \text{ Hz}}{1 - \frac{v_{\text{obs}}}{1484 \text{ m/s}}} \\
42,200 \text{ Hz} &= \frac{42,200 \text{ Hz}}{1 + \frac{v_{\text{obs}}}{1484 \text{ m/s}}} \\
\Rightarrow \text{v}_{\text{obs}} &= \frac{42,000 \text{ Hz}}{42,000 \text{ Hz}} - 1 = \frac{v_{42,200}}{1484} \left(1 + \frac{42,000}{42,000}\right)
\end{align*}
\]

\[v_{\text{obs}} = 3.52 \text{ m/s}\]

8. (14 pts) A thin, rigid rod hangs from its end and oscillates with a period of 1.42 seconds. Determine the length of the rod. Thin rod: \(l_{\text{m}} = \frac{M}{l/2}\) and \(l_{\text{w}} = \text{mL}/3/3.

\[\omega = \frac{2\pi}{T} = \frac{2\pi}{1.42} = 4.42 \text{ rad/sec}\]

\[I = \text{about point point} = \frac{M^2}{3}\]

\[r = \frac{l}{2} \text{ (distance from center of mass)}\]

\[l = \left(\frac{3}{2}\right) \frac{1}{\omega^2} = \frac{3}{2} \left(9.81 \text{ m/s}^2\right) \left(4.42 \text{ rad/sec}^2\right)^{-2} = 0.7516 \text{ m}\]

\[= 2.47 \text{ ft}\]

- 10 wrong equation
- 2 math error