Module 4, Lecture 2  Beats, Doppler, and Sonic Boom

Beats:
Beats occur when two waves of almost the same frequency are superimposed.

Wave equation: $z(x,t) = A \sin(\omega x - \alpha t)$

At a fixed point in space: $D_i = A \sin(\omega_i t) = A \sin(2\pi f_i t)$

Add the two waves together, and use some trig identities:

$$D_1 + D_2 = 2A \cos \left( \frac{2\pi f_1 - f_2}{2} t \right) \sin \left( \frac{2\pi f_1 + f_2}{2} t \right)$$

Example: Tuning Fork

**Given:** A tuning fork produces a steady 400 Hz tone. When struck and held near a vibrating guitar string, twenty beats are counted in 5 seconds.

**Required:** Possible frequencies produced by guitar string.
Doppler Effect

- Frequency shift when sound source and/or observer is moving
- When source is moving towards observer, pitch appears ________
- When source is moving away from observer, pitch appears ________
- Example: NASCAR race car passing you

Doppler Case A: Moving Source

During one period, $\tau_0$, source moves ($v_s$ is speed of source):

$$v_s\tau_0 = v_s/f_o$$

Wavelength is decreased by:

$$\lambda' = \lambda - \frac{v_s}{f_o} = \left(\frac{v - v_s}{v}\right)$$

Wave speed remains unaffected; apparent frequency becomes:

$$f' = \frac{v}{\lambda'} = f_0\left(\frac{v}{v - v_s}\right) = \frac{f_o}{1 - (v_s/v)}$$

With source moving towards observer, frequency increases. When source moving away, frequency decreases.

Doppler Case B: Moving Observer

If an observer moves towards a stationary source with speed $v_r$, the observer sees the wave crests with a speed:

$$v' = v + v_r$$

Wavelength remains the same, so the modified frequency is:

$$f' = \frac{v'}{\lambda_0} = \frac{v + v_r}{\lambda_0} = f_0\left(1 + \frac{v_r}{v}\right)$$

When the observer moves towards the source, the frequency increases. When the observer moves away from the source, the frequency decreases.

Doppler Case C: Moving Source and Observer

Moving source: Wavelength changes, but not wave speed
Moving observer: Wave speed changes, but not wavelength

$$f' = f_0\left(\frac{v \pm v_s}{v \mp v_s}\right)$$

Signs: moving towards each other, frequency is higher, use upper signs; moving apart, frequency is lower, use lower signs.

If $v_s$ and $v_r$ are small with respect to $v$:

$$f' = f_0\left(1 - \frac{v_s - v_r}{v}\right)$$
Example: Speed of a car

**Given:** A 5000 Hz sound wave is emitted by a stationary source. The sound wave reflects off a car moving at 60 mph towards the source.

**Required:** Frequency of the wave as detected by a detector near the source.

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Example: Tuba on a Train

**Given:** In one of the original Doppler experiments, a tuba was played on a moving flat car at 75 Hz. An identical tuba was played at the train station. The train approached the station at 12 m/s.

**Required:** Beat frequency that was heard.

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**Doppler Effect: Applications**

- **Weather radar:** Developed in 1988, the Doppler shift is used to determine how fast a storm is moving by looking at the frequency shift as radar waves reflect off of raindrops.

- **Echocardiography:** A test that uses ultrasound and Doppler technology to visualize the structure of the heart. Areas of the heart that have a high and low blood velocity can be seen in different hues of the resulting picture.

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**Sonic Booms**

- Source of waves moves faster than the wave speed
- Source is “outrunning” the waves
- Waves pile up along the side
- Crests overlap and form one very large crest