Module 4, Lecture 2 Beats, Doppler, and Sonic Boom

Beats:

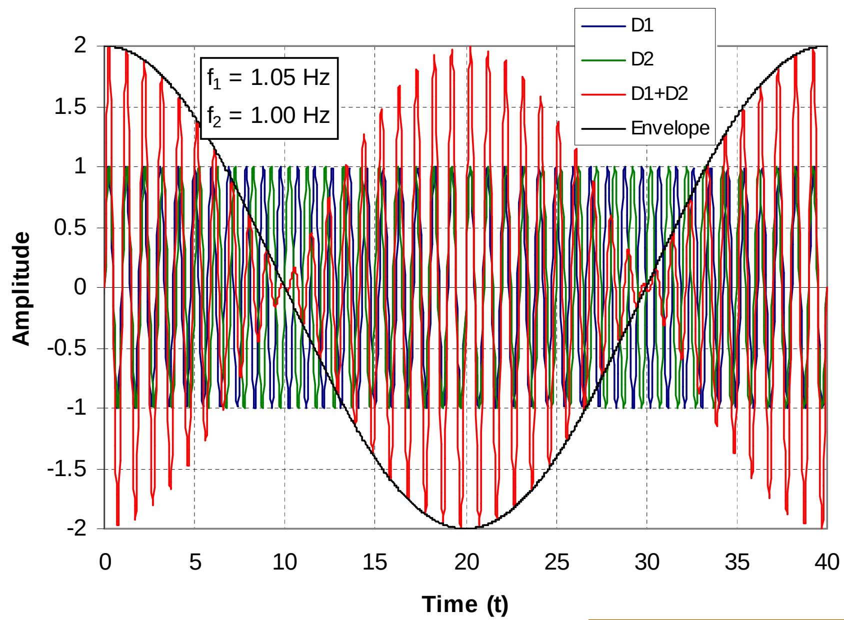
Beats occur when two waves of almost the same frequency are superimposed.

Wave equation: $z(x,t) = A\sin(kx - \omega t)$

At a fixed point in space: $D_i = A\sin(\omega_i t) = A\sin(2\pi f_i t)$

Add the two waves together, and use some trig identities:

$$D_1 + D_2 = \left[2A\cos\left(2\pi \frac{f_1 - f_2}{2} t\right)\right]\sin\left(2\pi \frac{f_1 + f_2}{2} t\right)$$



Hearing beat frequency

Example: Tuning Fork

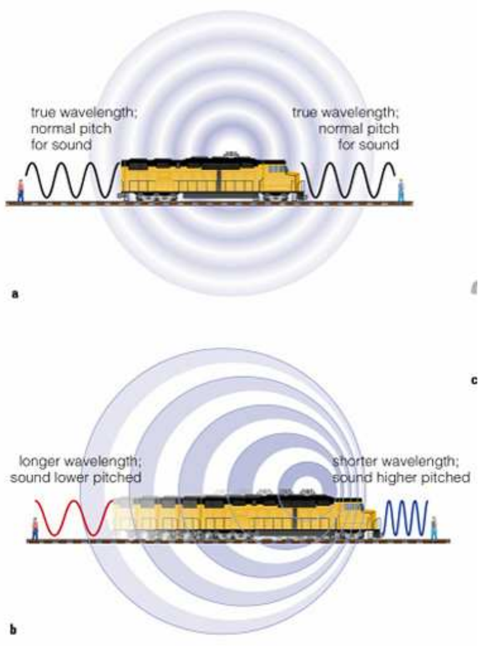
Given: A tuning fork produces a steady 400 Hz tone. When struck and held near a vibrating guitar string, twenty beats are counted in 5 seconds.

Required: Possible frequencies produced by guitar string.

Doppler Effect

- Frequency shift when sound source and/or observer is moving
- When source is moving towards observer, pitch appears _________
- When source is moving away from observer, pitch appears _________
- Example: NASCAR race car passing you

Doppler demonstration



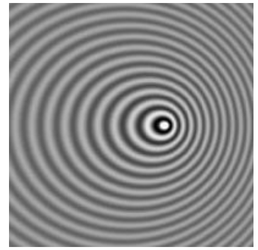
Doppler Case A: Moving Source

During one period, τ_o, source moves (v_s is speed of source):

$$v_s \tau_0 = v_s / f_o$$

Wavelength is decreased by:

$$\lambda' = \lambda - \frac{v_s}{f_o} = \left(\frac{v - v_s}{v}\right)$$



Wave speed remains unaffected; apparent frequency becomes:

$$f' = \frac{v}{\lambda} = f_0\left(\frac{v}{v - v_s}\right) = \frac{f_0}{1 - (v_s / v)}$$

With source moving towards observer, frequency increases. When source moving away, frequency decreases.

$$f' = \frac{f_0}{1 + (v_s / v)}$$

Doppler Case B: Moving Observer

If an observer moves towards a stationary source with speed v_r, the observer sees the wave crests with a speed:

$$v' = v + v_r$$

Wavelength remains the same, so the modified frequency is:

$$f' = \frac{v'}{\lambda_o} = \frac{v + v_r}{\lambda_o} = f_0\left(1 + \frac{v_r}{v}\right)$$

When the observer moves towards the source, the frequency increases. When the observer moves away from the source, the frequency decreases.

$$f' = f_0\left(1 - \frac{v_r}{v}\right)$$

Doppler Case C: Moving Source and Observer

Moving source: Wavelength changes, but not wave speed
Moving observer: Wave speed changes, but not wavelength

$$f' = f_0\left(\frac{v \pm v_r}{v \mp v_s}\right)$$

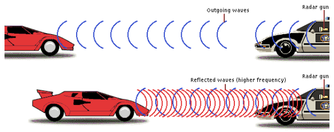
Signs: moving towards each other, frequency is higher, use upper signs; moving apart, frequency is lower, use lower signs.

If v_s and v_r are small with respect to v:

$$f' \cong f_0\left(1 - \frac{v_s - v_r}{v}\right)$$

Example: Speed of a car

Given: A 5000 Hz sound wave is emitted by a stationary source. The sound wave reflects off a car moving at 60 mph towards the source.
Required: Frequency of the wave as detected by a detector near the source.

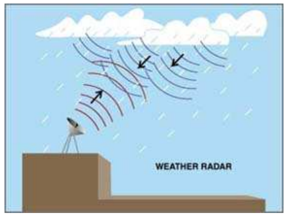


Example: Tuba on a Train

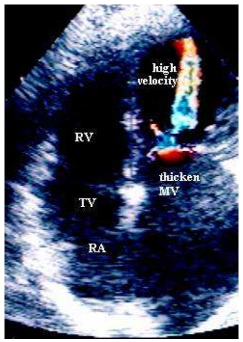
Given: In one of the original Doppler experiments, a tuba was played on a moving flat car at 75 Hz. An identical tuba was played at the train station. The train approached the station at 12 m/s.
Required: Beat frequency that was heard.

Doppler Effect: Applications

Weather radar: Developed in 1988, the Doppler shift is used to determine how fast a storm is moving by looking at the frequency shift as radar waves reflect off of raindrops.



Echocardiography: A test that uses ultrasound and Doppler technology to visualize the structure of the heart. Areas of the heart that have a high and low blood velocity can be seen in different hues of the resulting picture.



Sonic Booms

- Source of waves moves faster than the wave speed
- Source is "outrunning" the waves
- Waves pile up along the side
- Crests overlap and form one very large crest

Sonic boom demonstration YouTube Video