**Venturi Meter Example**

Apply the Bernoulli equation across the meter

\[ \frac{P_1}{\rho} + \frac{1}{2} \rho v_1^2 = \frac{P_2}{\rho} + \frac{1}{2} \rho v_2^2 \text{ or} \]

\[ P_1 - P_2 = \frac{1}{2} \rho (v_1^2 - v_2^2) \]

Using continuity

\[ v_2 = \frac{v_1 A_1}{A_2} \]

∴

\[ P_1 - P_2 = \frac{1}{2} \rho v_1^2 \left( \frac{A_1}{A_2} \right) \]

\[ \left(16 - 25\right) \]

The pressure difference is calculated from \( h \) in the manometer, 16 - 25 is used to get \( v_1 \) and the flow rate is calculated using \( A_1 \).

**Example**

A tank of cross-sectional area 0.07 m\(^2\) is filled with water. A tightly fitting piston with a total mass of 10 kg, rests on top of the water. A circular hole of diameter 1.5 cm is opened at a depth of 60 cm below the water level of the tank. What is the initial rate of flow of water out of the hole? How would you maximize the distance the stream would flow?

**Solution**

\[ P_{atm} + P_1 + \rho gh = P_{atm} + \frac{1}{2} \rho v^2 \]

\[ P_1 = \frac{mg}{A} \]

\[ v^2 = \frac{(\frac{v_1}{2})}{\rho} = 2g \left( \frac{\rho}{\rho} \right) \]

\[ v^2 = 2 \left( \frac{\rho}{\rho} \right) \left[ \frac{10 \text{ kg}}{27} \left( \frac{1}{10 \text{ kg}} \right) \left( \frac{1}{0.07 \text{ m}^2} \right) \right] = 15 \text{ m}^2/\text{s}^2 \]

\[ \Phi = vA - v \pi \left( \frac{d}{2} \right)^2 = v \pi \left( \frac{7.5 \times 10^{-3} \text{ m}}{2} \right)^2 = 0.7 \frac{\text{L}}{s} \]
Real Fluids

- Have viscosities greater than zero which results in shear, velocity gradients, and friction.
- Osborne Reynolds 1883 experiment
- Steamlines vs laminar vs turbulence
- Corrections required to basic equations

Bernoulli Equation for Real Fluids

\[ \frac{P_1}{\rho} + \frac{gh_1}{g_c} + \frac{\alpha_1 v_1^2}{2g_c} + \xi W_p = \frac{P_2}{\rho} + \frac{gh_2}{g_c} + \frac{\alpha_2 v_2^2}{2g_c} + f_F \]

where
\( \alpha \) is the correction for turbulent flow
\( \bar{v} \) is the average fluid velocity across the crossection
\( \xi \) is the efficiency of the pump
\( W_p \) is the work produced by the pump
\( f_F \) is the friction correction
\( \frac{\Delta}{h_c} = 1.00 \)

Real Bernoulli Example

In the equipment shown below a pump draws a solution of Specific gravity = 1.84 from a storage tank through a 3 in. Schedule 40 steel pipe. The efficiency of the pump is 60%. The velocity in the suction line is 3 ft/s. The pump discharges Through a 2 in. schedule 40 pipe to an overhead tank. The end Of the discharge pipe is 50 ft above the level of the solution In the feed tank. Friction losses in the entire piping system are 10 ft-lb/lb. What pressure must the pump develop? What is The power of the pump?
**Solution**

Take the datum as point a, \( \nu_{\text{tank}} = 0, \alpha = 1.0 \)

\[
\xi W_p = \frac{g}{g_c} h_b + \frac{v_b^2}{2g_c} + f_v
\]

\( \Lambda_3 = 0.0513 \text{ ft}^2 \) & \( \Lambda_2 = 0.0233 \text{ ft}^2 \)

\( v_b = \frac{3x0.0513}{0.0523} = 6.61 \text{ ft/s} \)

\( 0.60 W_p = 50 \frac{g}{g_c} + 6.61^2 \frac{64.34}{g_c} + 10 = 60.68 \)

\( W_p = \frac{60.68}{0.06} = 101.1 \frac{\text{ft-lb}}{\text{lb}} \)

Now apply Bernoulli around the pump

\[
P_{\text{out}} - P_{\text{in}} = 1.84 \times 62.37 \left( \frac{1}{2} \cdot \frac{60.68}{0.06} \right) - 6.61^2 \frac{64.34}{g_c} + 60.68
\]

\( P_{\text{out}} - P_{\text{in}} = 6902 \frac{\text{ft-lb}}{\text{ft}^2} = 47.9 \frac{\text{lb}-\text{in}}{\text{in}^2} = 330 \text{ kN-m} \)

\( 1 \text{ hp} = 550 \frac{\text{ft-lb}}{\text{sec}} \)

\( \eta = \nabla A \rho = 0.0513 \times 3 \times 1.84 \times 62.37 = 17.66 \frac{\eta}{\text{sec}} \)

Power = \( \frac{\eta A}{\text{sec}} = \frac{17.66 \times 0.0513}{3} = 3.25 \text{ hp} = 2.42 \text{ kW} \)

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**Venturi Meter for Real Fluids**

\[
v = \frac{C_v}{v} \sqrt{\left( \frac{2}{\rho} \right) (P_1 - P_2)} \quad \text{(SI)}
\]

\[
v = \frac{C_v}{v} \sqrt{\left( \frac{2 g_c}{\rho} \right) (P_1 - P_2)} \quad \text{(US)}
\]

Essentially the same equation as derived in the examples from the 3/10 lecture. \( C_v \) depends on flow conditions and the specific meter normally 0.92 - 0.99 for liquids. 1 and 2 refer to Venturi figure presented on slide two.

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**Reynolds Number**

\[
N_{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\frac{v^2}{L}}{\frac{\mu v}{L^2}} = \frac{Lv\rho}{\mu}
\]

Where \( L \) is a generic distance, for a pipe \( L = D \) the pipes diameter.
Why \( N_{re} \) is so Important

- It is physically meaningful - relating the forces which control this process
- It is essential for scale-up and design
- It is the first of many (~17) dimensionless groups engineers use to relate fundamental forces in various processes
- It is a very useful correlating variable
- \( N_{re} \) is dimensionless, its magnitude is independent of the units used, provided the units are consistent.

Example

Calculation of \( N_{re} \) Determination of Flow Regime

Water at 303K is flowing at 0.2917 m/s in a 0.0525 m pipe. Calculate the Renyolds number and determine the flow regime. The specific gravity is 0.996 and \( \mu = 0.800 \text{ cp at 303K} \).

\[
\rho = 0.996 \times 1000 \quad \frac{\text{kg}}{\text{m}^3} = 996 \quad \frac{\text{kg}}{\text{m}^3}
\]

\[
\mu = 0.800 \text{ cp} \times 0.001 \quad \frac{\text{kg}}{\text{m} \cdot \text{s}} = 0.800 \times 10^{-4} \quad \frac{\text{kg}}{\text{m} \cdot \text{s}}
\]

\[
N_{re} = \frac{D \cdot v \cdot \rho}{\mu} = \frac{0.0525 \times 0.2917 \times 996}{0.800 \times 10^{-4}} = 19,050
\]

The water is in turbulent flow as \( N_{re} > 4,000 \).

Rotameter

Used for measuring the flow rate of fluids. Describe the meter and draw a FBD of the float. There are three forces:

Drag force \( D_f \propto f(V_{\text{fluid}},H_{\text{fluid}} \text{ and } D_{\text{float}}) \)

\[
F_D = \rho_{\text{fluid}} V_{\text{fluid}} g
\]

\[
F_{\text{float}} = V_{\text{float}} p_{\text{fluid}} g
\]