1) The simple harmonic motion of an object is shown in the graph below. Write an equation for the position of the object as a function of time using the sine function.

\[ x(t) = A \sin \left( \frac{\pi}{T} t + \delta \right) \]

\[ A = 4 \text{ m}, \quad T = 8 \text{ sec} \]

\[ T = \frac{2\pi}{\omega}, \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4} \text{ rad/sec} \]

Pick a point on graph: \( x = 4 \text{ m} \)

\[ x(0) = 2 \text{ m} \]

\[ \delta = \sin^{-1} \left( \frac{1}{4} \right) \approx 0.524 \]

2) The maximum speed of a 3.00-kg object in simple harmonic motion is 6.00 m/s. The maximum acceleration of the object is 5.00 m/s\(^2\). What is the period of the simple harmonic motion?

\[ v = A \omega \cos (\omega t + \delta) \]

\[ a = -A \omega^2 \sin (\omega t + \delta) \]

\[ v_{\text{max}} = A \omega = 6 \text{ m/sec} \]

\[ a_{\text{max}} = -A \omega^2 = 5 \text{ m/sec}^2 \]

\[ \omega = \frac{v_{\text{max}}}{A} = \frac{6}{3} = 2 \text{ rad/sec} \]

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = 3.1416 \text{ sec} \]

\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{7.54} \approx 0.8333 \text{ rad/sec} \]
3) Two simple pendula are made out of single 1.00 m piece of string that is cut into two pieces, one piece for each pendulum. The string is cut so that the period of one pendulum has a period that is one-half the period of the other pendulum. What is the length of the shorter piece of string?

\[
\ell_1 = 0.2 \text{ m}
\]

\[
\omega_1 = \frac{1}{2} \omega_2
\]

\[
\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}
\]

\[
\frac{k}{m} = \frac{4\pi^2}{T^2}
\]

\[
k = \frac{4\pi^2 \cdot m}{T^2}
\]

\[
m = \frac{4(0.002)^2 \cdot (1.5)}{(0.5)^2} = 0.316 \text{ kg}
\]

\[
\frac{k}{m} = \frac{0.316}{0.002} = 316 \text{ N/m}
\]

4) A clock is designed that uses a mass on the end of a spring as a timing mechanism. If the oscillation time needed is one-half second, what is the spring constant required if the mass is 2.00 g?

\[
k = 0.316 \text{ N/m}
\]

\[
\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}
\]

\[
\frac{k}{m} = \frac{4\pi^2}{T^2}
\]

\[
k = \frac{4\pi^2 \cdot m}{T^2} = \frac{4\pi^2 \cdot (0.002)}{(0.5)^2} = 316 \text{ N/m}
\]
5) A circular ring of radius \( R \) is suspended from a string attached to its outside edge. The length of the string is equal to the radius of the disk. If the disk swings in the plane of the disk, what is the period of its motion in terms of \( R \) and \( g \)?

\[
T = 2\pi \sqrt{\frac{5R}{2g}}
\]

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6) A wave described by the following function:
\[
y(x,t) = (2.00 \text{ m})\cos[(3.00 \text{ m}^{-1})x + (5.00 \text{ s}^{-1})t]
\]
What is the frequency of the wave? What is the wavelength?

\[
f = 0.796 \text{ Hz} \quad \lambda = 2.09 \text{ m}
\]

\[
f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{5 \text{ rad/s}}{2\pi} = 0.796 \text{ Hz}
\]

\[
\lambda = \frac{2\pi}{k} = \frac{2\pi}{3 \text{ m}} = 2.09 \text{ m}
\]
7) A spring has a spring constant 20.0 N/m and rests standing vertically on a hard surface. A 1.00-kg mass is dropped onto the spring. How much time does it take the spring to reach its maximum compression from the time the spring is initially contacted?

\[ x = 0.351 \, \text{m} \]

\[ k = 20 \, \text{N/m} \]

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20 \, \text{N/m}}{1 \, \text{kg}}} = 4.47 \, \text{rad/s} \]

\[ \text{min. at } \omega t = \frac{\pi}{2} \]

\[ x = \frac{\pi}{2\omega} = \frac{\pi}{4.47} \approx 0.351 \, \text{m} \]

8) What is the maximum kinetic energy of a 2.00-kg mass moving with simple harmonic motion with an amplitude 5.00 cm and period 0.300 s?

\[ KE_{\text{max}} = 1.10 \, \text{N} \cdot \text{m} \]

\[ m = 2.00 \, \text{kg} \]

\[ E = \frac{1}{2} kA^2 = U + KE \]

\[ U_{\text{max}} = KE \text{. K.E. when } U = V_{\text{max}} \text{, no spring stretch, } U = 0 \]

\[ KE_{\text{max}} = E = \frac{1}{2} kA^2 \]

\[ A = 0.05 \, \text{m} \]

\[ \omega = \sqrt{\frac{k}{m}} = 2.2 \, \text{rad/s} \]

\[ k = \frac{4\pi^2}{(0.3)^2} \approx 878.4 \, \text{N/m} \]

\[ KE_{\text{max}} = \frac{1}{2} \left( 878.4 \right) \frac{m}{m} (0.05)^2 \, \text{m}^2 = 1.095 \, \text{N} \cdot \text{m} \]