Lecture 3-5, Summary of Oscillations and Waves

• Simple Harmonic Motion in one dimension:
  \[ x(t) = A \sin(\omega t + \delta) \]  
  \((13-1a)\)

• Relation of acceleration and displacement:
  \[ a(t) = -\omega^2 x(t) \]  
  \((13-8)\)

• Example: movement of mass on a spring is simple harmonic, with angular frequency
  \[ \omega = \sqrt{\frac{k}{m}} \]  
  \((13-16)\)

Summary, cont.

• Potential energy of a mass on a spring:
  \[ U(x) = \frac{1}{2} kx^2 \]  
  \((13-18)\)

• Total energy is constant:
  \[ E = \frac{1}{2} kA^2 \]  
  \((13-23)\)

• Simple pendulum exhibits simple harmonic when angular displacement is small, with period:
  \[ T = 2\pi \sqrt{\frac{\ell}{g}} \]  
  \((13-26)\)
Summary, cont.

• Most small oscillations around an equilibrium point are simple harmonic
• Wave motion form of $F=ma$: \[
\frac{\partial^2 z(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 z(x, t)}{\partial t^2}
\]

• Solution is simple harmonic plus a traveling dimension, sinusoidal in both:
  \[z(x, t) = A \sin(kx - \omega t)\]

• Wave speed $v = \lambda f = \lambda/T = \omega/k$

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Example - A crank is being turned clockwise at 120 rev/min. The handle is 10 cm from the axle of the crank. An observer some distance away sees the crank edge-on so he only observes the vertical motion of the crank. What equation describes the motion of the handle seen by the observer if at $t=0$ the crank is at $y=0$ and moving in the negative $y$ direction?
Example - A solid circular disk of radius $R$ is suspended from a string attached to its outside edge. The length of the string is equal to the radius of the disk. If the disk swings in the plane of the disk, what is the period of its motion?

A wave is described by $y=A \sin(kx+\omega t)$ where $k=3.0 \; 1/m$ and $\omega=2.0 \; 1/s$, and $A=0.40 \; m$. Determine (a) the wavelength, (b) the frequency, (c) the maximum amplitude, (d) the period, and (e) the wave number.
A perfect triangular pulse travels along a string with a velocity of 4 m/s. The shape of the pulse at t=0 is shown in below. Plot the displacement and velocity of the segment at x=3 m as a function of time. Is such a pulse physically possible?