Today’s Topics
- Drag force
- Terminal speed
- Projectile motion and drag

Drag Forces
(e.g. Air Resistance)

\[ F_D = \frac{1}{2} \rho A C_D v^2 \]

Simple Approximation
\( \rho \) – density of medium
\( A \) – cross-sectional area
\( C_D \) – Coefficient of drag
  - streamlined auto 0.25
  - can be greater than 1.0
\( v \) – velocity
**Drag Example**

Dr. McCord is driving her Prius at 60 mph. What is the drag force?

\[ C_d = 0.25, \quad A = 23.4 \text{ ft}^2 \quad \text{(http://ecomodder.com/wiki/index.php/Vehicle_Coefficient_of_Drag_List)} \]

\[ \rho_{\text{air}} = 2.33 \times 10^{-3} \text{ slug/ft}^3 \text{ at } 70^\circ \text{F} \quad \text{(http://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html)} \]

\[
\begin{align*}
F_d &= \frac{1}{2} \rho A V^2 C_d = \frac{1}{2} \left( 2.33 \times 10^{-3} \frac{\text{slug}}{\text{ft}^3} \right) \left( 23.4 \text{ ft}^4 \right) \left( \frac{23.4 \text{ ft}}{0.78 \text{ ft}^3/\text{slug}} \right) \left( 0.25 \right) \\
&= 52.8 \text{ lb}
\end{align*}
\]

\[ \frac{S \ell}{ft^2} \cdot \frac{ft}{s^2} \cdot \frac{ft}{s^2} = \frac{S \ell \cdot ft^4}{ft^3 \cdot s^2} = \frac{S \ell \cdot ft}{s^2} = \frac{\text{mass}}{\text{accel}} \]

What is the drag force at 30 mph?

Car Physics for Games:
http://www.asawicki.info/Mirror/Car%20Physics%20for%20Games/Car%20Physics%20for%20Games.html

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**Terminal Speed**

**Terminal Speed**
Drag force and gravity equal

\[ v_t = \sqrt{\frac{2mg}{\rho AC_D}} \]

**Typical terminal speeds**
- Parachutist (chute closed): 130 mph
- Parachutist (chute open): 10 mph
- Baseball: 90 mph
- iPhone: 27 mph (flat); 95 mph (edge)

**Raindrop**

Determine the terminal velocity of a 2.5 mm diameter raindrop. Assume $C_d = 0.50$ and $\rho_{\text{air}} = 1.22 \text{ kg/m}^3$.

\[
V_t = \sqrt{\frac{2mg}{\rho AC_D}}
\]

\[
A = \pi r^2 = \pi \left(\frac{0.0025\text{m}}{2}\right)^2 = 4.91\times10^{-6} \text{ m}^2
\]

\[
m = \rho_{\text{water}} V = 4.91\times10^{-6} \text{ m}^3
\]

\[
m = \frac{1000 \text{kg}}{\text{m}^3} \left(8.18\times10^{-6} \text{ m}^3\right) = 8.18\times10^{-6} \text{ kg}
\]

\[
V_t = \sqrt{\frac{2(8.18\times10^{-6} \text{ kg}) (9.81 \text{ m/s}^2)}{1.22 \text{ kg/m}^3 \cdot (4.91\times10^{-6} \text{ m}^2)(0.50)}} = 7.32 \text{ m/s} \approx 16 \text{ mph}
\]

What is the terminal velocity of a 1.0 mm diameter raindrop?

\[
V_t = 4.63 \text{ m/s}
\]

A decreases faster, $V$ decreases faster.
Building Codes in the United States

International Building Code

ASCE 7 Loads  Structural Steel  Reinforced Concrete  Masonry

Seismic  Wind  Live
**Projectile Motion and Drag**

\[ F_0 \text{ opposite direction of } V \]

\[ X: \quad -F_0 \cos \theta = \text{max} \quad \Rightarrow \quad -kV_x \frac{V_x}{V} = \text{max} \]

\[ Y: \quad -W - F_0 \sin \theta = \text{max} \quad \Rightarrow \quad -kVV_x = \text{max} \]

\[ F_0 = \frac{1}{2} \rho AC_0 V^2 = kV^2 \]

\[ -mg - kV \frac{V_y}{V} = \text{max} \]

What is the direction of the drag force?

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Assume: Acc is constant over small interval of Δt.

**Projectile Motion and Drag**

Initial conditions: \( x_0, y_0, \vec{v}_0, A, \rho, C_d \)

Choose Δt:

1. Calculate
   \[
   F_0 = \frac{1}{2} \rho A C_d v^2, \quad a_x = -\frac{k v x}{m}, \quad a_y = -\frac{k v y}{m} - g
   \]
   \[
   v_{x_i} = v_{x_0} + a_x \Delta t = v_{x_0} - \frac{k v x}{m} \Delta t
   \]
   \[
   v_{y_i} = v_{y_0} + a_y \Delta t = v_{y_0} - \frac{k v y}{m} \Delta t - g \Delta t
   \]
   \[
   \Delta t \sim \frac{1}{100} \text{s}
   \]

2. Assume constant acceleration
   \[
   x_i = x_0 + v_{x_0} t + \frac{1}{2} a_x (\Delta t^2)
   \]
   \[
   y_i = y_0 + v_{y_0} t + \frac{1}{2} a_y (\Delta t^2)
   \]

3. Go back to step 1 and repeat