**Today’s Topics:**
Rotational kinetic energy
Mass moment of inertia
Parallel axis theorem

**Rotational Kinetic Energy**
Ball spinning around in a circle at a constant speed:

\[
\text{KE} = \frac{1}{2} m v^2
\]

Replace \( v \) with \( \frac{r \omega}{2} \)

\[
\text{KE} = \frac{1}{2} m \left( \frac{r \omega}{2} \right)^2 = \frac{1}{2} [m r^2] \omega^2 = \frac{1}{2} I \omega^2
\]

Mass moment of inertia of a point mass: \( I = m r^2 \)

More than one point mass: \( I = \sum m_i r_i^2 \)

Distributed mass: \( I = \int r^2 \, dm \)

Mass moment of inertia describes resistance of a body to changes in angular acceleration

Units of \( I \): mass \( \times \) length\(^2\)
**Mass Moments of Inertia (from wikipedia.com)**

- **Thick-walled hollow sphere**
  \[ I = \frac{2}{3} m(r_2^2 - r_1^2) \]

- **Solid sphere**
  \[ I = \frac{2}{5} m r^2 \]

- **Hollow sphere**
  \[ I = \frac{2}{3} m r^2 \]

- **Solid rectangular box**
  \[ I_x = \frac{1}{12} m (h^2 + w^2) \]

- **Solid rectangular plate**
  \[ I_{	ext{center}} = \frac{1}{12} m (w^2 + h^2) \]

- **Thin-walled hollow cylinder**
  \[ I_x = I_y = \frac{1}{12} m (r_2^4 + r_1^4) \]

- **Solid cylinder**
  \[ I = \frac{1}{2} m r^2 \]

- **Thin rod**
  \[ I_{	ext{center}} = \frac{1}{12} m l^2 \]

- **Solid circular plate**
  \[ I_x = I_y = \frac{1}{4} m r^2 \]

- **Hollow plate (ring)**
  \[ I_x = I_y = \frac{1}{2} m r^2 \]

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**Example: Flywheel**

Calculate the rotational energy stored in a flywheel with an outer diameter of 2.0 m, an inner diameter 1.6 m, a mass of 330 kg, and a rotational speed of 1800 rpm (188.5 rad/s).

(neglect the inner portion of the wheel)

\[ KE = \frac{1}{2} I \omega^2 \]  
\[ I = \frac{1}{2} m (r_2^2 + r_1^2) = \frac{1}{2} (330 \text{ kg})(\frac{2}{2})^2 + (\frac{1.6}{2})^2 = 270.4 \text{ kg m}^2 \]

\[ KE = \frac{1}{2} (270.4 \text{ kg m}^2) (188.5 \text{ rad/s})^2 = 4807513 \frac{\text{J}}{\text{s}} = 4.81 \times 10^6 \text{ N m} \]

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Other applications:

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EF 151 Fall, 2017 Lecture 4-4
**Parallel Axis Theorem**

\[ I = I_{cm} + m d^2 \]

\[ d = \text{perpendicular distance between axis of interest and a parallel axis through the center of mass} \]

**Composite Shapes**

Body constructed of a number of simple shapes:
- Add moments of inertia of each shape about the desired axis to get the total moment of inertia.
- Use a negative value for \((I_{cm} + m d^2)\) to “subtract” a composite part from another, such as a hole from a plate.

\[ I = \Sigma(I_{cm} + m d^2) \]

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**Conservation of Energy with Rotational Kinetic Energy**

Identify things that are rotating and/or translating
- Identify center of mass and axis of rotation
- Use axis of rotation velocity for \(KE_i\) terms
- Use center of mass to determine \(h\) for \(PE_g\) terms
- Use Parallel Axis Theorem for \(I\) if rotation axis is not through the center of mass

\[ \sum PE_g + \sum PE_v + \sum KE_i + \sum KE_v + \sum E_{other} \]

\[ \sum mgh + \sum \frac{1}{2} kx^2 + \sum \frac{1}{2} mv^2 + \sum \frac{1}{2} I \omega^2 + \sum E_{other} \]
Example: Shape Rolling Down an Incline (COE)

\[ \text{No rotation} \]
\[ mgh = \frac{1}{2}mv^2 \]
\[ V = \sqrt{2gh} \]

\[ \text{With rotation} \]
\[ mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]
\[ mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[ \frac{2}{3}mr^2 \right] \left( \frac{V}{r} \right)^2 \]
\[ V = \sqrt{\frac{6}{5} gh} \]

The speed at the bottom of the hill is just a function of the shape of the object.

EF 151 Fall, 2017 Lecture 4-4
Example: Yardstick and Ball

A 0.13 lb, 2.5 in. diameter tennis ball is attached to the end of a 0.15 lb yardstick.

- What is the mass moment of inertia of the combination about the pivot point?

\[ I_{\text{tot}} = I_{YS} + I_{TB} \]

\[ I_{YS} = I_{YSCm} + md^2 \quad \text{thin rod} \]

\[ = \frac{1}{12} mL^2 + md^2 \quad d = 15\text{ in} \]

\[ = \frac{1}{12} \left( \frac{0.15}{32.2} \right)^2 + \left( \frac{0.15}{32.2} \right)^2 \]

\[ = 0.0107 \text{ slug ft}^2 \]

\[ I_{TB} = I_{TBcm} + md^2 \quad d = 33\text{ in} \]

\[ = \frac{2}{3} mr^2 + md^2 \]

\[ = \frac{2}{3} \left( \frac{0.13}{32.2} \right)^2 \left( \frac{1.25}{12} \right)^2 + \left( \frac{0.13}{32.2} \right)^2 \left( \frac{33}{12} \right)^2 \]

\[ = 0.03056 \text{ slug ft}^2 \]

\[ I_{\text{tot}} = 0.04133 \text{ slug ft}^2 \]

Example: Yardstick and Ball

A 0.13 lb, 2.5 in. diameter tennis ball is attached to the end of a 0.15 lb yardstick. The contraption is held horizontal.

- What is the speed of the ball when the stick is vertical?

\[ \text{COE} \]

\[ mgh_1 = \frac{1}{2} I \omega^2 + mgh_2 \]

\[ (0.13 + 0.15 \text{lb})(33\text{ in}) = \frac{1}{2} \left( 0.04133 \right) \omega^2 + (0.13 + 0.15 \text{ lb})(9.64 \text{ in}) \]

\[ \omega = 5.13 \text{ rad/s} \]

\[ V = r \omega = (5.13 \text{ rad/s})(\frac{33\text{ in}}{12}) = 14.1 \text{ ft/s} \]