Today’s Topics:
Angular Momentum

• Angular momentum
• Conservation
• Impulse
• Precession

• Objects rotating about a fixed axis
  \[ \vec{L} = I \vec{\omega} \]

• Can relate to Newton’s second law
  \[ \sum \vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d(I \vec{\omega})}{dt} = I \frac{d(\vec{\omega})}{dt} = I \cdot \alpha \]

Conservation of momentum

• Total angular momentum remains \[ \text{constant} \] if net external torque acting on system is \[ \text{zero} \].

Units:

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Angular momentum
Conservation
Impulse
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Examples:

A clutch disc with \( I = 1.2 \text{ kg-m}^2 \) is brought into contact with a spinning flywheel having \( I = 1.8 \text{ kg-m}^2 \) and \( \omega = 72 \text{ rad/s} \).

What is the angular velocity after the disc is moving with the flywheel?

A person stands at the edge of a rotating circular platform and starts walking towards the center. What happens?

Angular Momentum - Vector Cross Product Representation

Objects rotating about a fixed axis
\[ \vec{L} = I \vec{\omega} \quad \text{or} \quad \vec{L} = \vec{r} \times \vec{p} \]

\[ \vec{r} \quad \text{or} \quad \vec{p} \quad \text{or} \quad \vec{L} \]

Direction of \( \vec{L} \)
\[ |\vec{L}| = |\vec{r} \times \vec{p}| = \]

Example: Conservation of Angular Momentum

A 2 m long 90 N plank hangs vertically from a hinge. The plank is struck 1.5 m below the hinge by a 3 kg ball traveling at 10 m/s. The ball rebounds at 6 m/s. What is the angular velocity of the plank? Why is linear momentum not conserved?
Spinning bicycle wheel

Mr. Physics is holding the spinning bicycle wheel while standing on a platform that can rotate freely. Suddenly he flips the bicycle wheel over so it is spinning in the opposite direction. What happens?
A. Platform starts rotating in direction wheel was originally rotating
B. Platform starts rotating in opposite direction of original wheel rotation
C. Platform stays at rest
D. Platform turns only while Mr. P is flipping the wheel

Now he moves the wheel only 90° so it is vertical. What happens?
A. Platform starts rotating in same direction and speed as before
B. Platform starts rotating in same direction as before, but at a slower speed
C. Platform stays at rest
D. Platform turns only while Mr. P is flipping the wheel

Angular Impulse

Linear Impulse: Change in
\[ \Delta p = p_2 - p_1 \]

Angular Impulse: Change in
\[ \Delta L = \tau \cdot \Delta t \]

Precession

Linear motion
- Parallel force:
- Perpendicular force:

Angular motion
- Parallel torque:
- Perpendicular torque:

Weight: series of impulses
Causes wheel to spin around a vertical axis

Precession angular speed:
\[ \Omega = \frac{\tau}{I} \]

Conceptual Example: Spinning bicycle wheel

Prof. McCord is holding a rapidly spinning bicycle wheel so the axis is horizontal. The first time the rope is at the end of the axle. The second time the rope is near the hub. How does this affect the motion?
A. Increase in wheel’s angular speed
B. Decrease in wheel’s angular speed
C. Increase in precession speed
D. Decrease in precession speed
E. No change in either speed
F. Prof. McCord throws the wheel at Prof. Schleter
A thin rectangular plate is rotating about its center of mass with an angular velocity of $\omega_1 = 14.4$ rad/s in the CCW direction as viewed from above. The mass moment of inertia of a thin rectangular plate about its center of mass is \((1/12)m(a^2+b^2)\).

What is the plate's angular velocity just after the hook on the corner strikes the peg P without rebounding and the plate starts to rotate about P? What percentage of the initial kinetic energy is lost?

\[
a = 0.10 \text{ meters} \\
b = 0.27 \text{ meters}
\]