**Constant Acceleration**

\[
\begin{align*}
\omega_2 &= \omega_1 + \alpha \Delta t \\
\theta_2 &= \theta_1 + \left( \frac{\omega_1 + \omega_2}{2} \right) \Delta t \\
\theta_2 &= \theta_1 + \omega_1 \Delta t + \frac{1}{2} \alpha \Delta t^2 \\
\theta_2 &= \theta_1 + \frac{\omega_1^2 - \omega_2^2}{2\alpha}
\end{align*}
\]

**Uniform Circular Motion**

\[
\begin{align*}
\alpha_n &= \text{centripetal acceleration} \\
v &= \text{speed} \\
\rho &= \text{radius of curvature} \\
\omega &= \text{rotational speed} \\
T &= \text{period} \\
f &= \text{frequency} \\
\phi &= \text{angle} \\
a_n &= \frac{v^2}{\rho} \quad \text{(any curve)} \\
\tau &= \frac{\rho \omega^2}{\omega} \\
v &= \omega \rho \\
T &= \frac{2\pi}{\omega} \\
f &= \frac{1}{T} \\
\Delta \omega &= \rho \Delta \phi \\
\omega &= 2\pi f
\end{align*}
\]

**Parallel Axis Theorem**

\[
I = I_{cm} + Mh^2
\]

**Torque and Acceleration**

\[
\tau_{net} = I \ddot{\alpha}
\]

**Torque**

\[
\tau = Fr \sin \theta \\
\bar{\tau} = \bar{r} \times \bar{F}
\]

**Impulse / Momentum**

\[
\sum I \dot{\omega}' = \sum I \ddot{\omega} + \int \sum \vec{\tau} dt
\]

\[
\bar{L} = \bar{r} \times \bar{p} \\
\bar{L} = \text{angular momentum} \\
\bar{p} = \text{linear momentum}
\]

**Center of Mass**

\[
\bar{R} = \frac{m_1 \bar{r}_1 + m_2 \bar{r}_2 + \cdots + m_n \bar{r}_n}{m_1 + m_2 + \cdots + m_n}
\]

**Work and Power**

\[
W = \bar{r} \cdot \Delta \bar{r} \\
P = \bar{r} \cdot \frac{\Delta \bar{r}}{\Delta t}
\]
1. (14 pts) The head of the wooden mallet weighs 2.3 pounds and the handle (which goes through the head) weighs 1.6 pounds. Dimensions given in the drawing are in inches. Determine the location of the center of mass of the mallet from the right end of the mallet.

\[ X = \frac{2.3\text{lb}(10 + 2.2\text{in}) + 1.6\text{lb}(9.9\text{in})}{2.3\text{lb} + 1.6\text{lb}} = \frac{28.06 + 14.4}{3.9} = 10.89\text{in} \]

2. (14 pts) It takes 2 seconds for a motor to accelerate uniformly from stopped to an angular speed of 1725 rpm. There is a 4 inch diameter pulley connected to the motor that drives a 3 inch diameter pulley. Determine the angular acceleration of the 3 inch diameter pulley in rad/s².

\[ \omega = 1725 \text{ rpm} \times \frac{2\pi \text{ rad}}{60 \text{ sec}} = 180.6 \text{ rad/ sec} \]

\[ \alpha_m = \frac{\Delta \omega}{\Delta t} = \frac{180.6 \text{ rad/ sec}}{2 \text{ sec}} = 90.3 \text{ rad/ sec}^2 \]

\[ \alpha_m \Gamma_m = \alpha_p \Gamma_p \quad \Rightarrow \quad \alpha_p = \frac{\alpha_m \Gamma_m}{\Gamma_p} = \frac{90.3 \text{ rad/ sec} \times (4/2 \text{ in})}{(3/2 \text{ in})} = 120.4 \text{ rad/ sec}^2 \]
3. (14 pts) A barbell is made with two 1.2 kg, 0.2 m radius solid spheres attached to a rod of negligible mass. Determine the mass moment of inertia of the barbell around an axis through one end of the barbell (the dashed line shown in the figure).

\[ I = \frac{2}{5} m r^2 + m (0.2 m)^2 + \frac{2}{5} m r^2 + m (1.1 - 0.2 m)^2 \]
\[ = \frac{2}{5} (1.2 kg)(0.2 m)^2 + 1.2 kg(0.2 m)^2 + \frac{2}{5} (1.2 kg)(0.2 m)^2 + 1.2 kg (0.9 m)^2 \]
\[ = 0.0192 + 0.048 + 0.0192 + 0.972 = 1.0584 \text{ kg m}^2 \]

4. (14 pts) Determine the net torque of the two forces about the origin. Coordinates of the location of the forces are given in units of meters. Take CCW as positive.

\[ \tau = 26 (2) + 42 \sin(20^\circ)(1.5) - 42 \cos(20^\circ)(4) \]
\[ = 52 + 21.55 - 157.9 \]
\[ = -84.32 \text{ N m} \]
5. (14 pts) Twenty engineering professors are riding a merry-go-round that is initially spinning at 0.45 rad/sec. The merry-go-round has a mass moment of inertia of 18000 kg\cdot m^2. The average mass of the engineering professors is 75 kg and they are riding the outside horses which are at a distance of 5 m from the center. All at once the engineering professors move to the inside horses, and find that the merry-go-round is now spinning at 0.80 rad/sec. Determine the distance of the inside horses from the center.

\[
I = I_{mg} + 20I = 18000 + 20(75)(5^2) = 18000 + 37500 = 55500 \text{ kg} \cdot \text{m}^2
\]

\[
L_1 = I \omega_1 = 55500(0.45) = 24975 \text{ kg} \cdot \text{m}^2/\text{s}
\]

\[
L_2 = L_1 \Rightarrow I_2(0.80 \text{ rad/s}) = 24975
\]

\[
I_2 = 31219 \text{ kg} \cdot \text{m}^2
\]

\[
r = 2.971 \text{ m}
\]

6. (14 pts) The two forces are being used to start a disk spinning. The disk has a mass moment of inertia of 32 kg\cdot m^2 and it is desired that the angular acceleration have a magnitude of 2.5 rad/s^2. Determine the required force, F.

\[
F = 95 \text{ N}
\]

\[
\gamma = \frac{I}{r}
\]

\[
F(r_{0.4m}) + 60N(r_{0.7m}) = 32 \text{ kg} \cdot \text{m}^2 \cdot (2.5 \text{ rad/s}^2)
\]

\[
F = 95 \text{ N}
\]
7. (14 pts) Dean's band cart is rolling down a hill. The cart starts from at rest. The total weight of the cart (including the wheels) is 87 lb. The mass moment of inertia of each of the four wheels is 0.2 slug-ft² and the radius of the monster wheels is 1.2 ft. Determine the speed after the cart has gone 9 ft down the hill.

\[
8.18 \text{ ft/sec}
\]

\[
PE = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2
\]

\[
PE = mgh = 87 \text{ lb} \cdot 9 \text{ ft} \cdot \sin(8^\circ) = 108.97 \text{ ft-lb}
\]

\[
I = 4(0.2) = 0.8 \text{ slug-ft}^2
\]

\[
\omega = \frac{v}{r} = \frac{v}{1.2 \text{ ft}}
\]

\[
108.97 = \frac{1}{2} \left( \frac{32.25 \text{ lb}}{32.25 \text{ lb}} \right) v^2 + \frac{1}{2} (0.8 \text{ slug-ft}^2) \left( \frac{v}{1.2 \text{ ft}} \right)^2
\]

\[
108.97 = 1.351 v^2 + 0.278 v^2
\]

\[
v = 8.179 \text{ ft/sec}
\]

8. (2 pts) Two equal-mass particles (A and D) are located at some distance from each other. Particle A is held stationary while B is moved away at speed \( v \). What happens to the center of mass of the two-particle system?

A. it does not move
B. it moves away from A with speed \( v \)
C. it moves toward A with speed \( v \)
D. it moves away from A with speed \( \frac{1}{2}v \)
E. it moves toward A with speed \( \frac{1}{2}v \)