1. (10 pts) A string of negligible mass is wrapped around a 0.4 kg, 5.0 cm diameter hollow cylinder. One end of the string is held and the cylinder is released from rest. What is the rotational speed of the cylinder after it has fallen 2.0 meters? Assume the cylinder axis remains horizontal.

\[ \text{Torque} \ 
\tau = mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]

\[ \text{Impulse} \ 
I_1\,\ddot{\theta}_1 + I_2\,\ddot{\theta}_2 = I_1\,\ddot{\theta}_1 + I_2\,\ddot{\theta}_2 + \int \tau \, dt \]

\[ L = I\,\bar{\omega} \]

\[ \text{Impulse / Momentum} \]

\[ \text{Angular momentum} \]

\[ \text{Linear momentum} \]

2. (10 pts) Determine the torque of the 25 N force about the origin. The bracketed numbers are the coordinates (in meters) of the point where the force is applied.
3. (16 pts) Determine the center of mass of this object. All dimensions are in cm. Use a coordinate system located at the lower left corner and assume uniform thickness and density.

\[
\bar{x} = \frac{(16 \cdot 10) - (4 \cdot 3)(1 + 1.5) - (10 \cdot 3^2)}{(16 \cdot 10) - (4 \cdot 3) - (10 \cdot 3^2)} = 7.61 \text{ cm}
\]

\[
\bar{y} = \frac{(16 \cdot 10) - (4 \cdot 3)(3 + 3) - (10 \cdot 3^2)}{(16 \cdot 10) - (4 \cdot 3) - (10 \cdot 3^2)} = 5 \text{ cm}
\]

4. (16 pts) An object rotates with a constant angular acceleration of 0.25 rad/sec\(^2\) CW. If its initial speed is 7.0 rad/sec CW, how many revolutions does the object make before it changes direction?

\[
15.6 \text{ rev}
\]

5. (16 pts) Determine the mass moment of inertia of the system (A and B) about the pivot point (C).

Hollow sphere: \(I = \frac{2}{3}mr^2\)

\[
\text{about } \alpha: \frac{3}{8}mr^2 + \Delta = \frac{3}{8} \cdot 0.25 \text{ g}(0.05 \text{ cm})^2 + 0.25 \text{ g}(1 \text{ cm})^2 = 1.585 \text{ g cm}^2
\]

\[
\text{rod(cm)} \quad I = \frac{1}{12}ml^2, \quad I_{\text{rod}} = \frac{1}{12} \cdot 0.05 \text{ g}(0.1 \text{ cm})^2 = 0.195 \text{ g cm}^2
\]

\[
I_{\text{total}} = (1.585 + 0.195) \text{ g cm}^2 = 1.780 \text{ g cm}^2
\]

6. (16 pts) Shane (70 kg) has a location of \(\mathbf{r} = \langle 3i + 6j - 4k \rangle \text{ m}\) and a velocity of \(\mathbf{v} = \langle 3i + 6j - 5k \rangle \text{ m/s}\). What is the \(i\) component of his angular momentum about the point \((2i - 9j + 6k) \text{ m})?

\[
-4\text{ kg m}^2/\text{s}
\]

\[
\mathbf{r} \times \mathbf{v} \rightarrow \mathbf{\hat{r}} \times \mathbf{\hat{v}} \rightarrow \langle -4 \rangle
\]

\[
\mathbf{P} = \mathbf{m} \mathbf{v} = 70 \text{ kg} \langle 3i + 6j - 5k \rangle = (210 \text{ kg m/s} + 420 \text{ kg m/s} - 350 \text{ kg m/s})
\]

\[
\mathbf{r} = \langle 2i + 9j + 4k \rangle - \langle 3i - 9j + 7k \rangle = \langle 6i + 18j + 11k \rangle
\]

\[
\mathbf{\hat{r}} \times \mathbf{\hat{p}} = \left| \begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 6 & 4 \\
6 & 4 & 2
\end{array} \right| = (-350 \cdot 9 - 4 \cdot 420) \hat{j} - 930 \hat{k}
\]
7. (16 pts) A merry-go-round (I=29.11 slug*ft²) is spinning as shown. Nate (m=5.59 slugs) runs and grabs on the edge. What is the new angular speed of merry-go-round and Nate? Treat Nate as a point mass on the edge of the ride.

\[ \omega = \frac{3.5 \text{ rev}}{5 \text{ min}} \cdot \frac{2 \text{ rad}}{1 \text{ rev}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 0.367 \text{ rad/s} \]

\[ \vec{L}_1 = \vec{L}_2 \]

\[ \vec{L}_1 = \vec{I}_{mg} + \vec{L}_{Nate} = I_w + r \times m \vec{v} \]

\[ = (29.11 \text{ slug*ft}^2) \left( 0.367 \text{ rad/s} \right) + (2.5 \text{ ft} \cdot 5.59 \text{ slugs} \cdot 3.88 \text{ ft/s}) \]

\[ = 64.97 \text{ slug*ft} \]

\[ \vec{V}_1 = 44 \text{ ft/s} \cdot \sin(62) \]

\[ = 3.88 \text{ ft/s} \]

\[ \vec{L}_2 = I_w = \left[ 29.11 \text{ slug*ft}^2 + 5.59 \text{ slugs} \cdot (2.5 \text{ ft})^2 \right] \vec{w} \]

\[ = (64.04 \text{ slug ft}^2) \cdot \vec{w} \]

\[ \text{Used } \vec{L}_{Nate} = 0 \text{ (before)} \]

\[ \Rightarrow 64.04 \text{ slug ft}^2 = 64.04 \text{ slug ft}^2 \cdot \vec{w} \text{ wrong w} \]

\[ \Rightarrow \omega = 1.04 \text{ rad/s} \]