EF 151 Final Exam, Spring, 2009

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Name: **SOLUTION**

Section: __________

Guidelines:

- Assume 3 significant figures for all given numbers unless otherwise stated.
- Show all of your work — no work, no credit.
- Write your final answer in the box provided.
- Include units for all answers.
- Include directions for all vectors.

Geometry:

- Area of a circle = \( \pi r^2 \)
- Volume of a cylinder = \( \pi r^2 h \)
- Law of Sines: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)
- Law of Cosines: \( c^2 = a^2 + b^2 - 2ab \cos C \)

Useful Conversions:

- 1 gallon = 231 cubic inches
- 1 gallon = 4 quarts
- 1 gallon = 128 fluid ounces
- 1 m\(^2\) = 1000 L
- 1 acre = 43,560 ft\(^2\)
- 1 mile = 8 furlongs
- 1 mile = 5280 ft
- 1 fathom = 6 ft
- 1 rod = 16.5 ft
- 1 chain = 22 yards
- 1 inch = 25.4 mm
- 1 watt = 1 N m/ sec
- 1 hp = 745.7 watts (approx.)
- 1 hp = 550 ft lb/ sec
- 1 lb = 4.45 N (approx.)
- 1 m = 1000 mm
- 1 g = 32.2 ft/sec\(^2\) = 9.81 m/sec\(^2\)

Rocket Propulsion:

- \( v = \frac{m_0}{m_f} \) or \( v = \frac{r}{m_f} \)
- Thrust = \( m_0 \) (deviation)

Conservation of Energy:

- \( mgh + \frac{1}{2} v^2 + \frac{1}{2} kx^2 + W_F = mgh + \frac{1}{2} v^2 + \frac{1}{2} kx^2 + E_{int} \)

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Constant Angular Acceleration:

- \( \alpha = \frac{d^2 \Theta}{dt^2} \)
- \( \Theta = \Theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \)
- \( \omega = \omega_0 + \alpha t \)

Relative Motion:

- \( v_x = v_{x0} + v_{x1} \)
- \( v_y = v_{y0} + v_{y1} \)
- \( v_z = v_{z0} + v_{z1} \)

Circular Motion:

- \( v = \sqrt{\frac{2a}{c}} \)
- \( \alpha = \frac{a}{c} \)

Projectile Motion:

- \( x = x_0 + v_{x0} t + \frac{1}{2} a_{x0} t^2 \)
- \( y = y_0 + v_{y0} t + \frac{1}{2} a_{y0} t^2 \)
- \( z = z_0 + v_{z0} t + \frac{1}{2} a_{z0} t^2 \)

Circular Motion:

- \( v = \sqrt{\frac{2a}{c}} \)
- \( \alpha = \frac{a}{c} \)

Force and Acceleration:

- \( F_{net} = ma \)

Friction:

- \( F_{friction} = \mu N \)
- \( F_{friction} = \mu N \)

Work:

- \( W = F \cdot \Delta \theta = F \theta \cos \theta \)

Power:

- \( P = F \cdot v \)
- \( P = \frac{dW}{dt} \)

Spring force:

- \( F = kx \)

Impulse / Momentum / Restitution:

- \( \sum m_i v_i = \sum m_i v_i + \int F dt \)
- \( m_0 v_0 + m_1 v_1 = m_0 v'_0 + m_1 v'_1 \)
- \( v = \frac{v'_0 - v'_1}{v_0} \)

Torque:

- \( \tau = r \times F \)

Torque and Acceleration:

- \( \tau = I \alpha \)

Angular impulse / momentum:

- \( I_0 \alpha_0 + I_1 \alpha_1 = I_0 \alpha_0 + I_1 \alpha_1 \)

Parallel Axis Theorem:

- \( I = I_{cm} + Mb^2 \)
1. The position of a car driving in the African safari is described by the following equation: 
\[ s(t) = \left[-4t + \left(5 + 10t\right)t^2 + 3\right] \text{ meters. What is the magnitude of the velocity of the car at } t = 3 \text{ sec?} \]

\[
\begin{align*}
\vec{v} &= \frac{ds}{dt} = 3(-4t^2 + 2(5 + 10t)t) \\
\vec{v}_t = & 3(-4t^3) + 2(5 + 10t)(3) \\
&= -108t + 30t + 60 = -48t + 30t \\
\text{Used constant acc...} & \text{ do not know how to work with vectors...} \\
\text{calc error} & \text{ and hit } \vec{v} \text{ wrong...} \\
\text{velocity vector and not mag...} & \text{ is } -4 \\
\end{align*}
\]

2. An elevator is moving upward at a constant speed of 7.5 m/s. At 23 m above the level ground, a tennis ball is shot from the elevator at a speed of 34 m/s and angle of 20° above horizontal, relative to the elevator. How long (time) does it take for the ball to hit the ground? (Neglect air resistance)

\[
\begin{align*}
\vec{v}_{y_G} &= \vec{v}_y + vy_{y_G} \\
&= 34 \sin 20 + 7.5 \\
&= 19.13 \\
\gamma &= \gamma_0 + vy_{y_0} + \frac{1}{2}at^2 \\
&= 23 + 19.13t + \frac{1}{2}(-9.81)t^2 \\
t &= 4.86 \text{ sec or } -0.96 \text{ sec} \\
\text{Wrong } \vec{v}_{y_G} & \text{ used } a=0 \text{ in the y comp...} \\
\text{Used } v_y=0 & \text{ -3} \\
\end{align*}
\]

3. How fast is Zach going if he is moving around a 48 ft radius curve with a total acceleration of 15 ft/s² and he is slowing down at a rate of 11 ft/s²?

\[
\begin{align*}
\frac{\text{22.1 ft/s}}{5} & \text{ or } 0.441 \text{ rad/sec} \\
\frac{\text{22.1 ft/s}}{5} & \text{ or } 0.441 \text{ rad/sec} \\
\frac{\text{10.7 ft/s}}{5} & \text{ or } 2.14 \text{ rad/sec} \\
\frac{\text{22.1 ft/s}}{5} & \text{ or } 0.441 \text{ rad/sec} \\
\end{align*}
\]

4. Shane sits in a wagon (total weight 185 lb) and rolls down a 10° incline. He drags his feet to provide a force of 45 lb in an attempt to slow the wagon. What is the acceleration of the wagon? (FBD = KD required, use positive parallel to and down the hill)

\[
\begin{align*}
\Sigma F_x &= \text{max} \\
185 \sin 10 - 40 &= \frac{185}{32.2} a_x \\
a_x &= -1.37 \frac{ft}{s^2} \\
\end{align*}
\]

5. How hard must Nate push to keep the box moving at a constant speed? (FBD required)

\[
\begin{align*}
\Sigma P &= \Sigma F \\
706 N & \text{ or } 0.6 \text{ of } 120 \text{ lb} \\
\Sigma N &= 0.6 (120 \times 32) N \\
706 N & \text{ added } \Sigma x \text{ and } \Sigma y \text{ together...} \\
\text{used } a_y & \text{ wrong sign...} \\
\text{used } \Sigma F_x & \text{ not added...} \\
\text{used } g & \text{ -1} \\
\end{align*}
\]
6. A 140 hp crane lifts a 3500 lb load at a constant speed to a height of 60 ft in 5 seconds. How efficient is the crane in this situation?

\[ P_{tot} = \frac{140 \text{ hp} \times 550 \text{ ft-lb}}{5 \text{ sec}} = 77000 \text{ ft-lb/sec} \]
\[ \eta = \frac{P_{tot}}{P_{eff}} = \frac{77000}{35000} = 54.5\% \]

7. A varying magnitude force is applied to a 6.5 kg object that has an initial speed of 5.0 m/s. The force is applied parallel to the direction of motion and is depicted as a half of a circle with a radius of 8. What is the object's speed after 8 seconds?

\[ F_{tot} = \frac{1}{2} m v^2 \]
\[ F_{tot} = \frac{1}{2} (6.5 \text{ kg}) \times (5.0 \text{ m/s})^2 = 63.75 \text{ N} \]
\[ \text{Area} = \frac{1}{4} \pi r^2 = \frac{1}{4} \pi (8 \text{ m})^2 = 53.73 \text{ Nm} \]
\[ \text{Velocity} = \sqrt{13.3} \text{ m/s} \]

8. A grocery store paid a monthly power bill of $475. Electricity costs 9 cents per kW·hr. How many Joules of energy were used in that month?

\[ E = \frac{475 \text{ $/kw·hr}}{0.09 \text{ $/J}} \times 1000 \text{ kW} \times 3600 \text{ sec} = \frac{5}{6} \text{ J} = 1.9 \times 10^6 \text{ J} \]

9. (A) A very light spring hangs vertically in equilibrium with no mass at its end. (B) A 2.0 kg mass hangs from the spring in equilibrium. (C) The mass is pulled down a distance c and released. (D) The mass is at the reference height moving with a speed of v=4.0 m/s. How far down (c) was the mass pulled before it was released? (Hint -- you can calculate the spring constant with the given information)

\[ k = \frac{m \cdot g}{c} = \frac{2 \text{ kg} \times 9.81 \text{ m/s}^2}{0.34 \text{ m}} = 59.24 \text{ N/m} \]

\[ \frac{1}{2} k c^2 = m g c + \frac{1}{2} m v^2 \]
\[ \frac{1}{2} (59.24 \text{ N/m})(c)^2 = 2(9.81 \text{ m/s}) c + \frac{1}{2} (2 \text{ kg})(4 \text{ m/s})^2 \]
\[ c = 0.34 \text{ m} \text{ or } 0.24 \text{ m} \]

\[ 0.34 \text{ m} \text{ or } 0.24 \text{ m} \]

\[ \text{Wrong } k = 59.24 \text{ N/m} \]
10. Two identical cars collide and their bumpers interlock. Each car was moving with speed of 15 mph, one coming from the south and the other from the east. What is the speed of the cars after the collision?

15.6 mph

\[ \vec{v} = (15 \hat{i}) + \vec{v}(15 \hat{j}) = 2\vec{v}(\hat{v}) \]

\[ \vec{v} = (-7.5 \hat{i} + 7.5 \hat{j}) \text{ mph} \]

\[ |\vec{v}| = 10.6 \text{ mph} \]

11. A basketball hits a wall with the velocity shown. The coefficient of restitution between the ball and the wall is 0.85. What is the speed of the ball when it leaves the wall?

\[ v^1 = v_y = -12.3 \sin 33 \]

\[ = -6.7 \text{ ft/s} \]

\[ v^1 = -e v_y = -0.85(12.3 \cos 33) \]

\[ = -8.07 \text{ ft/s} \]

\[ v = \sqrt{(6.7)^2 + (8.07)^2} \]

\[ = 11 \text{ ft/s} \]

12. A horizontal disk (I = 6.5 kg·m²) spins at 16 rad/sec. A 1.1 kg glob of clay is dropped vertically onto the disk and sticks, resulting in a new angular speed of 13 rad/sec. How far is the clay from the center of the disk?

\[ \theta = \frac{\text{v}_1}{\text{w}_1} \]

\[ = \frac{6.5}{13} \]

\[ \theta = 0.5 \text{ rad} \]

\[ \text{r} = \frac{I\omega_2 - I\omega_1}{I} \]

\[ = \frac{6.5 \times 13 - 1.1 \times 16}{6.5} \]

\[ = 1.17 \text{ m} \]

13. A disk (I = 6.5 kg·m²) starts from rest and rotates freely about its center. Two constant forces are applied as shown. What is the angular acceleration of the disk? (use CCW as the positive direction)

\[ \sum \tau = I\alpha \]

\[ -(44)(2) + (33 \sin 50)(2.5) = 6.5 \alpha \]

\[ \alpha = \frac{6.5}{7.12} \]

\[ = 0.90 \text{ rad/s}^2 \]
Concept Questions – 1 point each – circle the correct answer.

C1. What is the dot product of the following vectors: \((3\mathbf{i} + 4\mathbf{j})\mathbf{b}\cdot(-2\mathbf{i} + \mathbf{j} + 5\mathbf{k})\mathbf{b}\? 
A. \((-6\mathbf{i} + 4\mathbf{j})\mathbf{b}\cdot\mathbf{ft}\) 
B. \((-2\mathbf{j})\mathbf{b}\cdot\mathbf{ft}\) 
C. \((-2\mathbf{i} + 5\mathbf{j})\mathbf{b}\cdot\mathbf{ft}\) 
D. \((-7.2\mathbf{j})\mathbf{b}\cdot\mathbf{ft}\) 
E. None of the above

C2. If \(\mathbf{A}\) and \(\mathbf{B}\) are vectors, which of the following is NOT possible?
A. \(\mathbf{A} + \mathbf{B} = |\mathbf{A}| + |\mathbf{B}|\)
B. \(\mathbf{A} + \mathbf{B} < |\mathbf{A}| + |\mathbf{B}|\)
C. \(\mathbf{A} + \mathbf{B} > |\mathbf{A}| + |\mathbf{B}|\)

C3. Two objects are dropped from the roof of a 20 foot tall building. Both start from rest. Object A is 20 lb and object B is 100 lb. Neglecting air resistance, which of the following is true?
A. Object A will reach the ground first
B. Object B will reach the ground first
C. They will reach the ground at the same time
D. Not enough information

C4. Which of the following is NOT a possible unit for momentum?
A. \(\text{kg} \cdot \text{m}/\text{sec}\)
B. \(\text{N} \cdot \text{sec}\)
C. \(\text{N} \cdot \text{sec}\)
D. \(\text{slug} \cdot \text{ft}/\text{sec}\)

C5. A particle is initially at rest when it suddenly explodes into 2 separate, equal mass halves. After the explosion one half has a velocity of \((5\mathbf{i} - 3\mathbf{j})\mathbf{m}/\mathbf{s}\). What is the velocity of the second half?
A. \((5\mathbf{i} - 3\mathbf{j})\mathbf{m}/\mathbf{s}\)
B. \((-5\mathbf{i} + 3\mathbf{j})\mathbf{m}/\mathbf{s}\)
C. \((3\mathbf{i} - 5\mathbf{j})\mathbf{m}/\mathbf{s}\)
D. \((-3\mathbf{i} + 5\mathbf{j})\mathbf{m}/\mathbf{s}\)
E. Not enough information

C6. A projectile has a speed of 5 ft/s at its highest point. If it was launched at an angle of 30° above horizontal, what was its launch speed?
A. 0 ft/s
B. 2.5 ft/s
C. 4.3 ft/s
D. 5 ft/s
E. 5.8 ft/s
F. 10 ft/s

C7. A block is pushed as shown. The block does not slide. What is the magnitude of the friction force between the block and the floor?
A. 0 N
B. 10 N
C. 12 N
D. 20 N
E. 30 N
F. 36 N

C8. A small train (1 engine and 10 coal cars) is accelerating at 0.6 ft/s² on level ground with a tractive force of 100,000 lb. How does the tractive force (T) of the engine compare to the coupling force (C) between the engine and first coal car?
A. \(T < C\)
B. \(T = C\)
C. \(T > C\)

C9. Initially, a 2-kg mass is whirling at the end of a string (in a circular path with a radius of 0.750 m) on a horizontal frictionless surface with a tangential speed of 5 m/s. As the mass rotates the string wraps around a vertical rod, and a few seconds later the length of the string has shortened to 0.250 m. How does the instantaneous speed at \(r = 0.250\text{ m}\) compare to the original speed at \(r = 0.750\text{ m}\)?
A. New speed < Original speed
B. New speed = Original speed
C. New speed > Original speed