EF 151 Exam #3, Fall, 2005

Name: SOLUTION Section: __________

This exam consists of 10 short-answer questions, each worth 10 points

Be sure to:
- Show all of your work
- Include units for all answers
- Include the correct number of significant digits
- Include directions for all vectors
- Write your final answer in the box provided

Hints:
- Stay calm
- Glance over all problems, tackle the "easy" ones first
- Use reasonableness to guide you
- Allow yourself an average of 5 minutes per problem

Geometry/Trig

- Area of a circle = πr²
- Volume of a cylinder = πr²h
- Law of Sines: \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \)
- Law of Cosines: \( c^2 = a^2 + b^2 - 2ab \cos C \)

Useful Conversions
- 1 gallon = 231 cubic inches
- 1 gallon = 4 quarts
- 1 gallon = 128 fluid ounces
- 1 m³ = 1000 L
- 1 acre = 43,560 ft²
- 1 mile = 8 furlongs
- 1 fathom = 6 ft
- 1 rod = 15.5 ft
- 1 chain = 22 yards
- 1 inch = 25.4 mm
- 1 watt = 1 N m/sec
- 1 hp = 745.7 watts (approx.)
- 1 hp = 550 ft lb / sec
- 1 lb = 4.45 N (approx.)
- 1 m = 1000 mm
- 1 g = 32.2 ft/sec² = 9.81 m/sec²

Constant Acceleration

\[
\begin{align*}
v_2 &= v_1 + aΔt \\
s_2 &= s_1 + \left(\frac{v_1 + v_2}{2}\right)Δt \\
v_2 &= v_1 + v_1 Δt + \frac{1}{2}aΔt^2 \\
s_2 &= s_1 + \frac{v_1^2 - v^2_1}{2a}
\end{align*}
\]

Projectile Motion

\[
y = x \tan θ + \frac{x^2}{2v₀^2}\left[1 - \frac{g(1 + \tan² θ)}{2v₀^2}\right]
\]

Assumes origin at launch point
- \( θ \) - launch angle
- \( v₀ \) - launch velocity

Relative Motion

\[
v_{x'} = v_x + v_{x'}/J
\]

Uniform Circular Motion

- \( a_n \) - centripetal acceleration
- \( v \) - speed
- \( ρ \) - radius of curvature
- \( ω \) - rotational speed
- \( T \) - period
- \( f \) - frequency
- \( φ \) - angle

\[
\begin{align*}
\mathbf{a}_n &= \frac{v^2}{ρ} \text{ (any curve)} \\
\mathbf{a}_n &= ρ \omega^2 \\
v &= ωρ \\
T &= \frac{2π}{ω} \\
f &= \frac{1}{T} \\
Δs &= ρΔφ \\
ω &= 2πf
\end{align*}
\]
1. The Mini Cooper is driven horizontally off a 110 ft high cliff at 46 mph. How long is it before the Mini hits the ground?

\[ h = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 \]

\[ t = \sqrt{\frac{-2 \cdot y_0}{a_y}} = \sqrt{\frac{-2 \cdot (110 \text{ ft})}{-32.2 \text{ ft/s}^2}} \]

\[ t = 2.61 \text{ sec} \]

-60 using initial horizontal velocity in vertical direction
(additional -2 for inconsistent units)

2. A baseball is thrown from a height of 1.5m at 18.4 m/s at an angle of 30° from the horizontal on level ground. What is the maximum height of the ball above the ground?

\[ y_2 = y_1 + \frac{v_{y2}^2 - v_{y1}^2}{2a_y} \]

\[ v_{y1} = 18.4 \text{ m/s} \sin 30 = 9.2 \text{ m/s} \]

\[ v_{y2} = 0 \text{ at max y} \]

\[ y_2 = 1.5 \text{ m} + \frac{0 - (9.2 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)} = 1.5 \text{ m} + 4.31 \text{ m} \]

\[ y_2 = 5.81 \text{ m} \]

Alternate solution: find time, 0.938 sec

-2 forgetting to add 1.5 m
-2 using \( g = 32.2 \)
-2 using \( \cos \) instead of \( \sin \)
-4 using 18.4 m/s instead of 9.2 m/s
3. A cannonball is shot from the top of a building at a speed of 72.4 m/s and an angle of 35° up from the horizontal. 2.87 seconds later the cannonball hits the ground. How far away from the bottom of the building does the cannonball hit the ground?

\[ V_{x0} = 72.4 \text{ m/s} \cos 35° \]
\[ = 59.3 \text{ m/s} \]
\[ y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 \]
\[ x = (59.3 \text{ m/s})(2.87 \text{ s}) = 170.2 \text{ m} \]

- \( v_y = v_{y0} - gt \)
- \( - \) using \( \cos \) instead of \( \sin \)
- \( - \) finding \( y \) value, not \( x \) value
- \( - \) using \( a_y \neq 0 \)

4. A football kickoff leaves the tee at 25.2 m/s at an angle of 25° from the horizontal. How far does the football go?

\[ V_{y0} = 25.2 \text{ m/s} \cos 25° = 22.839 \text{ m/s} \]
\[ V_{y0} = 25.2 \text{ m/s} \sin 25° = 10.65 \text{ m/s} \]
\[ y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 \]
\[ x = \frac{V_{y0}}{\tan \theta} + x_0 \left( \frac{g}{2 V_{y0}^2} \right) (1 + \tan^2 \theta) \]

**Alternate solution:**

Find time from \( y \) direction (2.17 sec)

\[ t = \frac{v_{y0}}{g} - \frac{1}{2} (9.81) t^2 \]
\[ t = 2.17 \text{ sec} \]

\( x \)-direction:

\[ x = 22.839 \text{ m/s} \times (2.17) \]

- \( - \) using \( g = 32.2 \)
- \( - \) using only half the time (1.08 sec)
5. An accelerometer measures the acceleration of a car going around a curve to be 2.4 ft/s². The car is moving at a constant speed of 72 ft/sec. What is the radius of curvature of the curve?

\[ a_n = \frac{v^2}{\rho} \]
\[ \rho = \frac{v^2}{a_n} \]
\[ \rho = \frac{(72 \text{ ft/s})^2}{2.4 \text{ ft/s}^2} = 2160 \text{ ft} \]

Incorrect solution \( \rho = \frac{72}{2.4} = 30 \text{ ft} \) - 4 points if correct

Generic equation shown \( \rho = \frac{v^2}{a_n} \)

6. A bicyclist is pedaling such that the 27 inch diameter wheel is revolving at 94 rpm. What is the speed of the bicyclist in miles/hour?

\[ v = \omega r \]

\[ v = \frac{94 \text{ rev}}{\text{min}} \left( \frac{27 \text{ in}}{2} \right) \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 7973.5 \frac{\text{in}}{\text{min}} \]

\[ v = (7973.5 \frac{\text{in}}{\text{min}}) \left( \frac{\text{ft}}{12 \text{ in}} \right) \left( \frac{\text{mi}}{5280 \text{ ft}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) = 7.55 \frac{\text{mi}}{\text{hr}} \]

- 2 using \( r = 27 \text{ in} \)
- 4 forgetting \( 2\pi \text{ rad/rev} \) conversion
- 4 not converting inches to feet (94.6 mph, unreasonable)
7. A moving sidewalk has a velocity of 5 ft/sec to the east. A lazy dog is walking on the sidewalk at a velocity of 2 ft/sec to the east relative to the sidewalk. A flea flies by with a velocity of 9 ft/sec to the west. There is no wind. What is the flea’s velocity relative to the dog?

\[ 16 \text{ ft/s West} \]

\[ \vec{v}_D = 2 \text{ ft/s} \]

\[ \vec{v}_S = 5 \text{ ft/s} \]

\[ \vec{v}_F = \vec{v}_D + \vec{v}_F/ \vec{D} \]

\[ \vec{v}_F/ \vec{D} = \vec{v}_F - \vec{v}_D = -9 \text{ ft/s} - 7 \text{ ft/s} = -16 \text{ ft/s} \]

8. Professor F.B. Overholt’s cousin, W.B. Overholt, is going to row across a river. W.B. heads straight across the river towards point A rowing at constant speed of 3.24 ft/sec. The 232 ft wide river is flowing at 0.76 ft/sec. How far downstream from point A does W.B. land?

\[ 54 \text{ ft} \]

\[ 3 \text{24 ft/s} = V_{B/W} \]

\[ y \text{ dir: } V_{B/W} = \frac{\Delta y}{t} \]

\[ t = \frac{\Delta y}{V_{B/W}} = \frac{232 \text{ ft}}{3.24 \text{ ft/s}} = 71.6 \text{ sec} \]

\[ x \text{ dir: } V_W = \frac{\Delta x}{t} \]

\[ \Delta x = V_W t = (0.76 \text{ ft/s})(71.6 \text{ sec}) = 54.4 \text{ ft} \]
9  Professor F B Overholt is at position \((4\mathbf{i} + 5\mathbf{j})\) ft. 160 seconds later, the good professor is at position \((12\mathbf{i} - 3\mathbf{j})\) ft. Determine the magnitude of the average velocity of the professor

\[
\mathbf{V}_{\text{avg}} = \frac{\Delta \mathbf{s}}{\Delta t} = \frac{\mathbf{s}_2 - \mathbf{s}_1}{\Delta t}
\]

\[
\mathbf{V}_{\text{avg}} = \frac{[(12\mathbf{i} - 3\mathbf{j}) - (4\mathbf{i} + 5\mathbf{j})]}{16 \text{ sec}} = (8\mathbf{i} - 8\mathbf{j}) \text{ ft/s}
\]

\[
|\mathbf{V}_{\text{avg}}| = \sqrt{(8)^2 + (-8)^2} \text{ ft/s} = 0.7 \text{ ft/s}
\]

10 A particle has an acceleration of \((-4\mathbf{i}) \text{ m/s}^2\), an initial velocity of \((7\mathbf{j}) \text{ m/s}\), and an initial position of \((2\mathbf{i} - 8\mathbf{j}) \text{ m}\). What is the position (in \(\mathbf{fj}\) notation) of the particle at \(t=6\) seconds?

\[
\mathbf{s} = \mathbf{s}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2
\]

\[
\mathbf{s} = (2\mathbf{i} - 8\mathbf{j}) \text{ m} + (7\mathbf{j} \text{ m/s}) (6 \text{ s}) + \frac{1}{2} (-4 \mathbf{i} \text{ m/s}^2)(6^2)
\]

\[
\mathbf{s} = (2\mathbf{i} - 8\mathbf{j} + 42\mathbf{j} - 72 \mathbf{i}) \text{ m}
\]

\[
\mathbf{s} = (-70\mathbf{i} + 34\mathbf{j}) \text{ m}
\]