Kinetics - Normal & Tangential Coordinates

Newton’s 2\textsuperscript{nd} Law \textit{still} governs: \[ \sum \vec{F} = m \vec{a} \]

In component form:

\[
\begin{align*}
\hat{e}_n & \quad \text{Normal} \\
\hat{e}_t & \quad \text{Tangential}
\end{align*}
\]

Remember:

\textbf{Note:} Occasionally we may need to add the "third" dimension and draw \textit{FBD} and \textit{KD} from \textit{two different} perspectives (viewpoints).

Example Problem 1.

\textbf{Given:} Your 50 kg instructor is flying a jet that pulls up into a vertical curve as shown. At $\theta = 30^\circ$ the \textit{speed} is 750 $km/h$ and the \textit{speed} is increasing at a rate of 30 $km/h$ per second. The radius of curvature $\rho$ is 1.5 $km$ at this point.

\textbf{Required:} The \textit{magnitude} of the force exerted by the seat \textit{onto} your instructor.
**Problem Cont.**

Step 2: Draw FBD & KD

---

**Problem Cont.**

What do we know?

\[ \rho = 1.5 \text{ km} \left( \frac{1000 \text{ m}}{\text{km}} \right) = 1500 \text{ m} \]

\[ v = 750 \frac{\text{km}}{\text{hr}} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{\text{hr}}{3600 \text{ s}} \right) = 208 \frac{\text{m}}{\text{s}} \]

\[ \dot{v} = 30 \frac{\text{km}}{\text{hr/s}} \left( \frac{1000 \text{ m}}{\text{km}} \right) \left( \frac{\text{hr}}{3600 \text{ s}} \right) = 8.33 \frac{\text{m}}{\text{s}^2} \]

Plug into the acceleration equation.

\[ \ddot{a} = \dot{v} \hat{e}_i + \frac{v^2}{\rho} \hat{e}_n = 8.33 \frac{\text{m}}{\text{s}^2} \hat{e}_i + \left( \frac{208 \frac{\text{m}}{\text{s}}}{1500 \text{ m}} \right)^2 \hat{e}_n \]
Example Problem 2

**Given:** An airplane is flying at a constant speed of 400 mi/hr and making a horizontal turn with a 2 mi radius.

**Required:** Find the proper bank angle $\theta$.

**Note:** The force exerted by the air is normal to the wing’s surface.
Step 2: Draw $FBD$ & $KD$

\[
\begin{align*}
\rho &= 2 \text{mi} \left( \frac{5280 \text{ ft}}{\text{mi}} \right) = 10560 \text{ ft} \\
v &= 400 \text{ mi/hr} \left( \frac{5280 \text{ ft}}{\text{mi}} \right) \left( \frac{\text{hr}}{3600 \text{ s}} \right) = 587 \frac{\text{ft}}{\text{s}}
\end{align*}
\]