EF 102 – Fundamentals of Engineering Mechanics
Physical Homework 7 – Projectile Motion

Lazydog Projectile Motion Equations
coordinate system: x – horizontal; y – vertical up

<table>
<thead>
<tr>
<th>Vector Equations</th>
<th>Magnitude</th>
<th>Direction</th>
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<tr>
<td>( \vec{a} = 0\hat{i} + (-g)\hat{j} )</td>
<td>g</td>
<td>down</td>
</tr>
<tr>
<td>( \vec{v} = v_{x0}\hat{i} + (v_{y0} - gt)\hat{j} )</td>
<td>( \sqrt{(v_{x0})^2 + (v_{y0} - gt)^2} )</td>
<td>Tangent to motion</td>
</tr>
<tr>
<td>( \vec{x} = (x_{0} + v_{x0}t)^\hat{i} + (y_{0} + v_{y0}t - 0.5gt^2)^\hat{j} )</td>
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Scalar Equations – horizontal direction
\( a_x = 0 \)
\( v_x = v_{x0} \)
\( x = x_{0} + v_{x0}t \)

Scalar Equations – vertical direction
\( a_y = 0 \)
\( v_y = v_{y0} - gt \)
\( y = y_{0} + v_{y0}t - 0.5gt^2 \)

Solving projectile motion problems: There are seven unknowns: \( x_0, y_0, x, y, v_{x0}, v_{y0}, \) and \( t \). The origin is arbitrary, so \( x_0 \) and \( y_0 \) can be set. We have two other equations, \( x \) and \( y \). Thus, three of the unknowns have to be given. An alternate way to write \( v_{x0}, v_{y0} \) are \( v_0\cos \theta \) and \( v_0\sin \theta \) respectively, where \( v_0 \) is the launch speed and \( \theta \) is measured from the horizontal.

A. Determine where the projectile is at a given time:
Set the launcher to an angle of the launcher to 30°. The launch height is 39.5 inches. Launch at the medium range setting, which gives the projectile a speed of 22.6 ft/sec. Determine the location of the projectile at \( t = 0.75 \) seconds. Set up a target at this location, and see if you can hit it.

B. Determine where the projectile will hit:
For the launcher at an angle of 30° and on the medium range setting, determine where the ball will hit the floor and the time it will take to hit the floor. Check your result for the location.
Bonus question: What is maximum height reached by the projectile?

C. Determine the angle of launch:
Place the PVC tube target 12.0 ft from the launcher. Determine the angle to set the launcher (for medium range setting) so that the ball goes in the tube. Check your results by shooting the ball in the tube.

Hints for solution of angle.
Solve for the time from \( x = x_{0} + v_0(\cos \theta)t \)

Substitute this into \( y = y_{0} + v_0(\sin \theta)t - 0.5gt^2 \)

Divide the trig identity of \( \sin^2 \theta + \cos^2 \theta = 1 \) by \( \cos^2 \theta \) to get \( \tan^2 \theta + 1 = 1/\cos^2 \theta \)
Use this identity to obtain a quadratic equation in \( \tan \theta \).

Some fun (and perhaps instructive) web sites:
Blow up the tenured professors: jersey.uoregon.edu/vlab/Cannon/
Other nice applets at www.phy.ntnu.edu.tw/ntnujava/index.php
Projectile motion: www.walter-fendt.de/ph11e/projectile.htm