Direct Impact

Before impact - bodies approach

\[ v_A > v_B \]

Contact - increasing deformation

\[ v'_A \approx v'_B \]

Contact - decreasing deformation or restitution

After impact - bodies separate

\[ v_{A_2} < v_{B_2} \]

Note: \( \vec{F}_R \) MAY or MAY NOT equal \( \vec{F}_D \) BUT \( |\vec{F}_A| \) on \( A \) equals \( |\vec{F}_A| \) on \( B \).
Direct Impact Cont.

contact - increasing deformation from time 0 to \( t' \) the velocity of \( A \) changes from \( v_A \) to \( v_A' \) etc.

For \( A \):

\[ v_A' \approx v' \]

For \( B \):

\[ v_A' \approx v_b' \]

contact - decreasing deformation or restitution - from time \( t' \) to \( t_2 \) the velocity of \( A \) changes from \( v_A' \) to \( v_A \) etc.

Combine:

Simplify using \( v_A' = v_b' \):

Note: \( F_R \) MAY or MAY NOT equal \( F_D \) BUT \( |F_R| \) on \( A \) equals \( |F_R| \) on \( B \).

Direct Impact Cont.

contact - increasing deformation from time 0 to \( t' \) the velocity of \( A \) changes from \( v_A \) to \( v_A' \) etc.

If \( v_{B_i} - v_{A_k} = v_{A_i} - v_{A_k} \): \( e = 1 \)

If \( v_{B_i} = v_{A_k} \): \( e = 0 \)

like coefficients of friction, the coefficient of restitution is a property of BOTH objects.

Note: \( F_R \) MAY or MAY NOT equal \( F_D \) BUT \( |F_R| \) on \( A \) equals \( |F_R| \) on \( B \).
**Example**

**Given:** Car $B$ weighs 3000 lb and is traveling at 40 mph when it "rear ends" car $A$ which weighs 2500 lb and is traveling at 20 mph. The coefficient of restitution is 0.5.

**Required:** Calculate:
- a) The **velocities** of both cars after the collision.
- b) The **percentage of energy lost** during the collision.

Convert initial conditions to “workable” units:

\[
m_B = \frac{3000 \ lb}{32.2 \ ft/s^2} = 93.17 \ sl \quad \quad m_A = \frac{2500 \ lb}{32.2 \ ft/s^2} = 77.64 \ sl
\]

\[
v_{B1} = 40 \frac{mi}{hr} \left( \frac{5280 \ ft}{mi} \right) \left( \frac{hr}{3600 \ s} \right) = 58.67 \ \frac{ft}{s}
\]

\[
v_{A1} = 20 \frac{mi}{hr} \left( \frac{5280 \ ft}{mi} \right) \left( \frac{hr}{3600 \ s} \right) = 29.33 \ \frac{ft}{s}
\]

**Note:** "sl" is the abbreviation for slugs.
Conservation of momentum still holds: \( m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2} \)

Use:

Ans:

Calculate energies: 
\[
T_1 = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2 \\
T_2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2
\]