Partially Grouted Walls

\[ b = \text{effective compressive width per bar} = \min\{s, 6t, 72 \text{ in}\} \] (5.1.2)

A. Neutral axis in flange:
   a. Almost always the case
   b. Design for solid section

B. Neutral axis in web
   a. Design as a T-beam section

C. Often design based on a 1 ft width

Minimum reinforcement:
No requirements

Maximum reinforcement:
Same requirement as beams

Partially Grouted Walls: Design Aid

<table>
<thead>
<tr>
<th>Spacing (inches)</th>
<th>Steel Area in²/ft</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
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<td>0.66</td>
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<td>0.23</td>
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<td>0.020</td>
<td>0.031</td>
<td>0.044</td>
<td></td>
</tr>
</tbody>
</table>
Example - Partially Grouted Walls

**Given:** 8 in. CMU wall; 12 ft high; Grade 60 steel, $f'_{m} = 2000$ psi; Wind load of $w_u = 30$ psf

**Required:** Reinforcing (place in center of wall)

**Solution:**

Determine $M_u$

$$M_u = \frac{w_u h^2}{8} = \frac{30 \frac{lb}{in^2} \left( \frac{12 \text{ in}}{12 \text{ ft}} \right)^2}{8} = 6480 \frac{lb\cdot in}{in^2} = 0.540 \frac{k-ft}{in^2}$$

Use # ___ @ ___ inches ($A_S = ____ in^2/ft$)

---

Example - Partially Grouted Walls

**Problem:** 72 inch is greater than effective compression width of $6t = 48$ in.

**Solution:** Use reduced value of $b$: $b = \frac{\text{48 in} \cdot \text{12 in}}{\text{72 in} \cdot \text{ft}} = 8 \text{ in/ft}$

Determine $a$

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.8f'_{m}b}}$$

$$= 3.81 \text{ in} - \sqrt{(3.81 \text{ in})^2 - \frac{2(6480 \frac{lb\cdot in}{in^2})}{0.8(0.9)(2000 \text{ psi})(8 \frac{\text{in}}{\text{ft}})}} = 0.1505 \text{ in}$$

Determine $A_S$

$$A_s = \frac{0.8f'_{m}ba}{f_y} = \frac{0.8(2000 \text{ psi})(8 \frac{\text{in}}{\text{ft}})(0.1505 \text{ in})}{60000 \text{ psi}} = 0.0321 \frac{\text{in}^2}{\text{ft}^2}$$

0.7% increase in required area of steel

- Do not worry about it
- If worried, use $b = 6 \text{ in./ft}$ in design, which is close enough
Example - Partially Grouted Walls

Horizontal spanning masonry between bars:

- Some treat as unreinforced masonry, although debate as to whether you can mix unreinforced and reinforced masonry.
- There is arching occurring, so not truly a simply supported flexural member between vertical bars.
- Can use joint reinforcement

<table>
<thead>
<tr>
<th>Wire Size</th>
<th>$d_b$ (in.)</th>
<th>$A_s$ (in.²)</th>
<th>Spacing (in.)</th>
<th>$\phi M_n$ (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1.7 (9 gage)</td>
<td>0.148</td>
<td>0.017</td>
<td>8</td>
<td>0.921</td>
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<tr>
<td></td>
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<td>16</td>
<td>0.462</td>
</tr>
<tr>
<td>W2.8 (3/16)</td>
<td>0.187</td>
<td>0.028</td>
<td>8</td>
<td>1.506</td>
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<tr>
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<td>16</td>
<td>0.841</td>
</tr>
</tbody>
</table>

$f_y = 70$ ksi; $f'_m = 2000$ psi; $d = t_{sp} - 5/8 - d_y/2$

Example: 30 psf; 72 in. spacing; $M_u = 0.135$ k-ft/ft

Partially Grouted Walls - Tolerances

Placement tolerances: (3.4.B.11)

- $d \leq 8$ in.  ± 1/2 in.
- 8 in. $< d \leq 24$ in.  ± 1 in.
- $d > 24$ in.  ± 1 1/4 in.

Along wall: ± 2 in.

$8$ in. CMU; $f'_m = 2000$ psi; Grade 60
8 in. CMU wall, fully grouted, bars in the center, Grade 60 steel, \( f'_m = 2000 \text{ psi} \).

Following table lists the maximum reinforcement for various axial loads.

<table>
<thead>
<tr>
<th>( P/(bdf'_m) )</th>
<th>( A_s ) (in(^2) per ft)</th>
<th>Spacing (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#4</td>
<td>#5</td>
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<tr>
<td>0</td>
<td>0.436</td>
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<tr>
<td>0.05</td>
<td>0.359</td>
<td>8 (6.7)</td>
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<td>0.10</td>
<td>0.283</td>
<td>16 (8.5)</td>
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<tr>
<td>0.15</td>
<td>0.207</td>
<td>16 (11.6)</td>
</tr>
<tr>
<td>0.20</td>
<td>0.130</td>
<td>24 (18.4)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.054</td>
<td>48 (44.3)</td>
</tr>
</tbody>
</table>

Requirements can be difficult to meet for heavily reinforced shear walls and at jambs.

Shear - Strength Design (9.3.4.1.2)

\[
V_n = (V_{nm} + V_{ns})\gamma_g \\
V_{nm} = 4 - 1.75\left(\frac{M_u}{V_u d_v}\right)A_{nv}\sqrt{f'_m} \\
V_{ns} = 0.5\left(\frac{A_s}{s}\right)f_v d_v
\]

- \( \gamma_g \) = partially grouted shear wall factor (\( \gamma_g = 1.0 \) for beams)
- \( d_v \) = actual depth of masonry
- \( A_{nv} \) = net shear area = \( bd_v \)
  - Many designers use \( d \) instead of \( d_v \) for beams
- \( M_u/(V_u d_v) \) need not be taken \( > 1.0 \)
- \( \phi = 0.8 \)
- Maximum \( V_n \)
  - \( V_n \leq 6A_{nv}\sqrt{f'_m} \) where \( \frac{M_u}{V_u d_v} \leq 0.25 \)
  - \( V_n \leq 4A_{nv}\sqrt{f'_m} \) where \( \frac{M_u}{V_u d_v} \geq 0.25 \)
  - Linearly interpolate between 0.25 and 1.0

Conservative approximation

\[
M_u/(V_u d_v) = 1.0 \\
V_{nm} = 2.25 A_{nv}\sqrt{f'_m}
\]
**Shear – Transverse Reinforcement (9.3.4.2.3)**

Detailing of shear reinforcement for beams:
- a) Single bar with 180-degree hook at each end
- b) Hook shear reinforcement around flexural reinforcement
- c) Minimum area of shear reinforcement is $0.0007bd_v$
- d) First bar within $d_v/4$
- e) Maximum spacing is $d_v/2$ or 48 in.

- c) Reasonable interpretation is $0.0007bd_v$ over $d_v$. This becomes $\frac{A_v}{s} \geq 0.0007b$

ACI 318: $\frac{A_v}{s} \geq 50 \frac{b}{f_y} = 0.00083b$ for Grade 60 steel

Sections within $d/2$ from face of support can be designed for shear at $d/2$
(8.3.5.4): (moved to Chapter 5 in 2022 TMS 402)

A. Noncantilever beam
B. Reaction introduces compression into end region of member
C. No concentrated load between $d/2$ and face of support

---

**Deflections**

- Deflection of beam or lintels supporting unreinforced masonry is limited to $L/600$ (5.2.1.4.1)
- Deflections of approximately $L/300$ needed to be visible.
- Deflections do not need to be checked when $L \leq 8d$ (5.2.1.4.3).
- End restraint of masonry beams reduces deflections from 20 to 45% of those of simply supported beams.

\[
I_{eff} = I_n \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \leq I_n
\]  
(Equation 5-1)

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2
\]  
Singly reinforced beam

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 + (n - 1)A_s'(kd - d')^2
\]  
Doubly reinforced beam
Flexural Members – Determination of k

\[
\rho = \frac{A_s}{bd}, \quad n = \frac{E_s}{E_m}, \quad k = \sqrt{(n\rho)^2 + 2n\rho - n\rho}
\]

Doubly reinforced beam

\[
k = \sqrt{(n\rho)^2 (1 + r)^2 + 2n\rho \left(\frac{1 + rd'}{d}\right) - n\rho (1 + r)} \quad r = \frac{(n-1)A_s'}{nA_s}
\]

Example: Beam

Given: 10 ft. opening; dead load (excluding beam) of 1.5 kip/ft; live load of 1.5 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; \( f_m' = 2000 \text{ psi} \)

Required: Design beam

Solution:

Length of bearing: Assume to be 8 in.

Span length: \( L = 10 \text{ ft} + 2(4 \text{ in.}) = 10.67 \text{ ft} \)

Lateral support of compression face:

- \( 32b = 32(7.625 \text{ in.}) = 244 \text{ in.} = 20.3 \text{ ft} \)
- \( 120b^2/d = 120(7.625 \text{ in.})^2 / (20 \text{ in.}) = 349 \text{ in.} = 29.1 \text{ ft} \)
Example: Beam, Flexure

**Factored Load**

Weight of fully grouted normal weight: 83 psf

\[ w_u = 1.2D + 1.6L = 1.2\left(1.5 \frac{k}{ft} + 0.083\frac{k}{ft^2}(2 \text{ ft})\right) + 1.6\left(1.5 \frac{k}{ft} \right) = 4.40 \frac{k}{ft} \]

**Factored Moment**

\[ M_u = \frac{w_u L^2}{8} = \frac{\left(4.40 \frac{k}{ft}\right)(10.67 \text{ ft})^2}{8} = 62.6k – ft \]

**Find a**

Depth of equivalent rectangular stress block

\[ a = d - \sqrt{d^2 - 2\frac{M_u}{0.8f_m^*b}} = 20in - \sqrt{(20in)^2 - \frac{2(62.6k – ft)(12 in)}{0.8(0.9)(2.0ksi)(7.625in)}} = 3.78in \]

**Find \( A_s \)**

Area of steel

\[ A_s = \frac{0.8f_m^*ba}{f_y} = \frac{0.8(2.0ksi)(7.625in)(3.78in)}{60ksi} = 0.77in^2 \]

Use 2 - #6 \( (A_s = 0.88 \text{ in}^2) \)

\[ M_n = 78.5 \text{ k-ft} \]

\[ \phi M_n = 70.6 \text{ k-ft} \]

---

**Example: Beam, Flexure**

**Alternate Design:**

Creatively place 1-#6 bar in bottom two courses to facilitate construction.

\( d = \) distance to centroid of tension reinforcement; \( d = 18 \) in.

**Find a**

Depth of equivalent rectangular stress block

\[ a = d - \sqrt{d^2 - 2\frac{M_u}{0.8f_m^*b}} = 18in - \sqrt{(18in)^2 - \frac{2(62.6k – ft)(12 in)}{0.8(0.9)(2.0ksi)(7.625in)}} = 4.32in \]

**Find \( A_s \)**

Area of steel

\[ A_s = \frac{0.8f_m^*ba}{f_y} = \frac{0.8(2.0ksi)(7.625in)(4.32in)}{60ksi} = 0.878in^2 \]

Check that middle layer has yielded

\[ \epsilon_x = \frac{\epsilon_{mu}(d_s - c)}{c} = \frac{0.0025}{5.40in.} \]

\( 0.00491 \geq 0.00207 = \epsilon_y \)
**Example: Beam, Min and Max Reinforcement**

**Minimum Reinforcement Check:**  \[ f_r = 160 \text{ psi} \quad \text{(parallel to bed joints in running bond; fully grouted)} \]

Section modulus \[ S_n = \frac{bh^2}{6} = \frac{(7.625\text{in})(24\text{in})^2}{6} = 732\text{in}^3 \]

Cracking moment \[ M_{cr} = S_n f_r = (732\text{in}^3)(160 \text{psi}) = 117.1k-in = 9.76k-ft \]

Check 1.3\(M_{cr} \)

\[ 1.3M_{cr} = 1.3(9.76k-ft) = 12.7k-ft \leq M_n = 78.5k-ft \]

**Maximum Reinforcement Check:**

\[ f_m' = 2 \text{ ksi} \quad \rho_{max} = 0.00952 \]

\[ \rho = \frac{A_s}{bd} = \frac{0.88\text{in}^2}{(7.625\text{in})(20\text{in})} = 0.00577 \]

---

**Example: Beam, Development**

\[ l_{de} = \frac{0.13d_b^2 f_y \gamma}{K \sqrt{f_m'}} \]

\[ \gamma = 1.3 \quad \#6, \#7 \]

\[ K = \min\{\text{masonry cover, clear spacing between adjacent bars, } 9d_b\} \]

Cover = 1.25 (face shell thickness) + 0.5 (coarse grout) + 0.375 (assumed #3 stirrup) = 2.125 in.

Determine \(K\)

\[ K = \min\{2.125, 9(0.75)\} = 2.125\text{in} \]

Determine \(l_{de}\)

\[ l_{de} = \frac{0.13(0.75\text{in})^2(60000\text{ psi})(1.3)}{2.125\text{in} \sqrt{2000\text{ psi}}} = 60.0\text{in} = 5.00\text{ft} \]
Example: Beam, Shear

Shear area \( A_{nv} \)

Use \( d \) instead of \( d_v \)

\[
A_{nv} = bd_v = (7.625\text{in})(20\text{in}) = 152.5\text{in}^2
\]

Masonry shear strength

Use \( M_s/(V_d d_v) = 1.0 \)

\[
V_{nm} = 2.25A_{nv}\sqrt{f'_m} = 2.25\left(152.5\text{in}^2\right)\sqrt{2000\text{psi}} = 15.34\text{kips}
\]

Design strength

\[
\phi V_{nm} = 0.8(15.34\text{kips}) = 12.27\text{kips}
\]

Check max \( V_n \)

\[
\phi V_n_{\text{max}} \leq 4A_{nv}\sqrt{f'_{m}} = 0.8(4)(152.5\text{in}^2)\sqrt{2000\text{psi}} = 21.82\text{kips}
\]

Shear at \( d/2 \) from face of support

\[
V_u = 18.33\text{kips} < 21.82 \text{ kip} \text{ OK}
\]

Requirement for shear at \( d/2 \) from face of support is in ASD, not SD, but assume it applies.
Example: Beam, Shear Reinforcement

Req’d $V_{ns}$

$$V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{18.33 - 12.27}{0.8} = 7.58 \text{kips}$$

Determine $A_v$ for a spacing of 8 in.

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_vd_v \Rightarrow A_v = \frac{V_{ns}s}{0.5f_vd_v}$$

$$A_v = \frac{7.58k(8\text{in.})}{0.5(60\text{ksi})(20\text{in.})} = 0.101 \text{in}^2$$

Use #3 stirrups

Determine $d$ so that no shear reinforcement would be required.

$$d = \frac{V_u}{b(\phi 2.25\sqrt{f'_m})} = \frac{18.33k}{7.625\text{in.}(0.8(2.25)\sqrt{2000\text{psi}})} = 29.9\text{in.}$$

Use 4 courses ($h = 32\text{in.}$) and inverted bond beam unit to get $d = 30\text{ in.}$

Reinforced Masonry - Flexural Members 44

Example: Beam, Shear Details

9.3.4.2.3 Transverse reinforcement

When transverse reinforcement is required, the following provisions shall apply:
(a) Transverse reinforcement shall be a single bar with a 180° hook at each end.
(b) Transverse reinforcement shall be hooked around longitudinal reinforcement.
(c) The minimum area of transverse reinforcement shall be $0.0007bd_v$.
(d) The first transverse bar shall not be located no more than one-fourth of the beam depth, $d_v$, from the end of the beam.
(e) The maximum spacing shall not exceed 1/2 the depth of the beam nor 48 in.

Intent of provision (c) is area over a length of $d_v$.

$$\frac{A_v}{s} = \frac{0.11 \text{in.}^2}{8\text{in.}} = 0.0138\text{in.} \geq 0.0007b = 0.0007(7.625\text{in.}) = 0.0053\text{in.}$$

Development length

$$l_{dc} = \frac{0.13d_v^2f_v}{K\sqrt{f'_m}} = \frac{0.13(0.375)^2(60000)(1.0)}{\min[9(0.375),1.25 + 0.5\sqrt{2000}]} = 14.0\text{in.}$$

With hook:

$$l_{dc} = 14.0 - 13(0.375) = 9.1\text{in.}$$

Reinforced Masonry - Flexural Members 44
Example: Beam, Shear Details

Check width:
- 2(1.25 in.) Face shells
- 2(0.75 in.) #6 bars
- 2(0.5 in.) coarse grout space
- 2(0.375 in.) #3 stirrups
- 1.0 in. space between bars
- Total 6.75 in. OK

Need stirrups over first 2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.

Example: Beam, Alternate Shear Details

Check width:
- 2(1.25 in.) Face shells
- 2(0.75 in.) #6 bars
- 2(0.5 in.) coarse grout space
- 2(0.375 in.) #3 stirrups
- 1.0 in. space between bars
- Total 6.75 in. OK

Need stirrups over first 2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.
Example: Beam, Alternate Shear Details

6.1.7.2.2 Pairs of U-stirrups or ties placed to form a closed unit shall be considered properly spliced when length of laps are $1.7l_y$. In grout at least 18 in. deep, such splices with $A_yf_y$ not more than 9,000 lb per leg shall be permitted to be considered adequate if legs extend the full available depth of grout.

$$A_yf_y = (0.11in^2)(60000 \text{ psi}) = 6600 \text{ lb}$$

Example: Shear, Sharpen the Pencil

$$V_{nm} = 4 \cdot 1.75 \left( \frac{M_u}{V_{u}d_v} \right) A_m \sqrt{f_m'}$$

Simply supported beam; uniform load

$$V_u = w_u \left[ \frac{L}{2} - x \right]$$

$$M_u = w_u \left[ \frac{L}{2} x - \frac{x^2}{2} \right]$$

$$(\phi V_{ns})_{max} = 4.73 \text{ kips at } x = 1.64 \text{ ft}$$

Previous was 7.58 kips; 38% reduction
Example: Beam, Deflections

Check L/d ratio

\[ \frac{L}{d} = \frac{10.67 \text{ ft}}{20 \text{ in}(12 \text{ in} / \text{ ft})} = 6.4 \]

Since \( \frac{L}{d} < 8 \), deflections are OK; check to show process.

ASD load

\[ w = D + L = \left(1.5 \frac{L}{d} + 2 \text{ ft}\left(0.083 \frac{L}{d}\right)\right) + 1.5 \frac{L}{d} = 3.17 \frac{L}{d} \]

Find \( M_a \)

\[ M_a = \frac{wL^2}{8} = \left(3.17 \frac{L}{d}\right)^2 \left(10.67 \text{ ft}\right)^2 \frac{8}{8} = 45.03k - \text{ft} \]

Modulus ratio

\[ n = \frac{E_s}{E_m} = \frac{29000 \text{ksi}}{900f_m'} = \frac{29000 \text{ksi}}{900(2.0 \text{ksi})} = \frac{29000 \text{ksi}}{1800 \text{ksi}} = 16.11 \]

Reinforcement ratio

\[ \rho = 0.00577 \quad n \rho = 0.09296 \]

Find \( k \)

\[ k = \sqrt{(n \rho)^2 + 2n \rho - n \rho} = \sqrt{(0.0930)^2 + 2(0.0930) - 0.0930} = 0.348 \]

\[ kd = 0.348(20 \text{ in}) = 6.96 \text{ in} \]

Example: Beam, Deflections

Net moment of inertia, \( I_n \)

\[ I_n = \frac{bh^3}{12} = \frac{7.625 \text{ in}(24 \text{ in})^3}{12} = 8784 \text{ in}^4 \]

Cracked moment of inertia, \( I_{cr} \)

\[ I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 \]

\[ = \frac{7.625 \text{ in}(6.96 \text{ in})^3}{3} + (16.11)(0.88 \text{ in}^2)(20 \text{ in} - 6.96 \text{ in})^2 = 3268 \text{ in}^4 \]

Effective moment of inertia, \( I_{eff} \)

\[ I_{eff} = I_n \left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr} \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \]

\[ = 8784 \text{ in}^4 \left(\frac{9.76k - \text{ft}}{45.03k - \text{ft}}\right)^3 + 3268 \text{ in}^4 \left[1 - \left(\frac{9.76k - \text{ft}}{45.03k - \text{ft}}\right)^3\right] = 3324 \text{ in}^4 \]

Deflection, \( \delta \)

\[ \delta = \frac{5WL^4}{384EI} = \frac{5(3166 \frac{L}{d})(10.67 \text{ ft})^4}{384(1800000 \text{ psi})(3324 \text{ in}^4)} = \frac{1728 \text{ in}^3}{1 \text{ ft}^3} = 0.154 \text{ in} \]

Allowable \( \delta \)

\[ \frac{L}{600} = \frac{(10.67 \text{ ft})(12 \text{ in})}{600} = \frac{0.213 \text{ in}}{1 \text{ ft}} \]

OK
Deep Beams, 5.2.2

- Lintels in which there is a large height of masonry above the opening
- Walls not continuously supported (supported on pier foundations)

- Plane sections do not remain plane
- Internal level arm smaller than computed from linear strain distribution

Effective span length, $l_{eff}$, smaller of:
- center-to-center distance between supports
- 1.15 multiplied by the clear span

$$ \frac{l_{eff}}{d_v} \leq \begin{cases} 
3 & \text{continuous spans} \\
2 & \text{simple spans} 
\end{cases} $$

<table>
<thead>
<tr>
<th>Simple spans</th>
<th>Continuous spans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \leq \frac{l_{eff}}{d_v} &lt; 2$</td>
<td>$1 \leq \frac{l_{eff}}{d_v} &lt; 3$</td>
</tr>
<tr>
<td>$z = 0.2(l_{eff} + 2d_v)$</td>
<td>$z = 0.2(l_{eff} + 1.5d_v)$</td>
</tr>
<tr>
<td>$\frac{l_{eff}}{d_v} &lt; 1$</td>
<td>$\frac{l_{eff}}{d_v} &lt; 1$</td>
</tr>
<tr>
<td>$z = 0.6l_{eff}$</td>
<td>$z = 0.5l_{eff}$</td>
</tr>
</tbody>
</table>

Deep Beams

- Flexural reinforcement
  - distributed flexural reinforcement for half beam depth
  - maximum spacing of one-fifth $d_v$ or 16 in.
  - joint reinforcement can be used as flexural reinforcement
  - horizontal reinforcement anchored to develop yield strength at face of supports

- Shear reinforcement (when required)
  - minimum area of vertical reinforcement is 0.0007$bd_v$
  - horizontal shear reinforcement shall have area $\geq$ half vertical shear reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.

- Total reinforcement: sum of horizontal and vertical reinforcement shall be at least 0.001$bd_v$. 

Reinforced Masonry - Flexural Members 51

Reinforced Masonry - Flexural Members 52
Deep Beams – Example 1

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 3.0 kip/ft; live load of 2.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f_m' = 2000$ psi

Required: Design beam

Solution:

Center-to-center between supports = 10 ft + 2(4 in.) = 10.67 ft
1.15(clear span) = 1.15(10 ft) = 11.5 ft
Effective span length, $l_{eff} = \min(10.67, 11.5) = 10.67$ ft

Factored moment, $M_u$

$$M_u = \frac{1.2(3.0 \frac{\text{kips}}{\text{ft}}) + 1.6(2.0 \frac{\text{kips}}{\text{ft}})(10.67 \text{ ft})^2}{8} = 96.7 \text{kips} \cdot \text{ft}$$

Span ratio, $l_{eff}/d_v$

$$\frac{l_{eff}}{d_v} = \frac{10.67 \text{ ft}}{6 \text{ ft}} = 1.78$$

Internal lever arm, $z$

$$z = 0.2(l_{eff} + 2d_v) = 0.2(10.67 + 2(6 \text{ ft})) = 4.53 \text{ ft} = 54.4 \text{ in}$$

Using standard beam theory, $A_{s,req} = 0.320 \text{ in}^2$ (19% less)

Flexural reinforcement requirements:
- distributed flexural reinforcement for half beam depth
  - need flexural reinforcement over bottom 36 inches.
- maximum spacing of one-fifth $d_v$ or 16 in.
  - $(1/5)(72) = 14.4 \text{ in.}$
- Use W1.7 (9 gage) joint reinforcement every 8 in. in bottom 5 bed joints (as a practical matter, use in every bed joint in beam)
Deep Beams – Example 1, Shear

Factored shear, $V_u$

$$V_u = \frac{[1.2(3.0 + 1.6)(10.67ft)]}{2} = 36.3\text{kip}$$

Masonry shear strength, $V_{nm}$

$$V_{nm} = 4 - 1.75\left(\frac{M_u}{V_u d_v}\right) A_m \sqrt{f'_m}$$

$$= 2.25[\left(7.625\text{in}\right)\left(68\text{in}\right)] \sqrt{2000\text{psi}} = 52.2\text{kips}$$

Design shear strength, $\phi V_{nm}$

$$\phi V_{nm} = 0.8(52.2\text{kip}) = 41.7\text{kips}$$

No shear reinforcement required

Total reinforcement

$$0.001bd_v = 0.001(7.625\text{in})(72\text{in}) = 0.55\text{in}^2$$

$$2\#4 (0.40\text{in}^2) + 5(2)(0.017\text{in}^2) = 0.57\text{in}^2$$

OK

joint reinforcement

Deep Beams – Example 1, Development

Clear cover

1.25 (face shell) + 0.5 (coarse grout) = 1.75 in.

Development length, $l_{de}$

$$l_{de} = \frac{0.13d^2 f_{y} \gamma}{\min\{9d_b, \text{clear cover}\} \sqrt{f'_m}}$$

$$= \frac{0.13(0.5in)^2(60000\text{psi})1.0}{\min\{9(0.5in), 1.75in\} \sqrt{2000\text{psi}}} = 24.9\text{in.}$$

Extend bars 24 in. beyond face of support

(close enough; cover of 1.82 in., 1/16 in. more than 1.75 in., results in $l_{de} = 24$ in.)
Deep Beams – Example 2

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 6.0 kip/ft; live load of 3.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000$ psi

Required: Design beam

Solution:

Factored moment, $M_u$

$$M_u = \frac{1.2(6.0 \frac{ft}{kip}) + 1.6(3.0 \frac{ft}{kip})}{8}(10.67 \text{ ft})^2 = 170.7 \text{ kip-ft}$$

Req'd $A_s$

$$A_s = \frac{(M_u / \phi)}{zf'_y} = \frac{(170.7 \text{ kip-ft} / 0.9)}{4.53 \text{ ft}(60 \text{ ksi})} = 0.697 \text{ in}^2$$

Use 2-#4 bars in each of bottom 2 courses (4 total bars)

Using standard beam theory, $A_{s,req} = 0.607 \text{ in}^2$ (13% less)

Use joint reinforcement in every course for distributed reinforcement

Deep Beams – Example 2, Shear

Factored shear, $V_u$

$$V_u = \frac{1.2(6.0 \frac{ft}{kip}) + 1.6(3.0 \frac{ft}{kip})}{2}(10.67 \text{ ft}) = 64.0 \text{ kip}$$

Max shear

$$\phi V_{n,max} = 0.8 \left[ 4A_n \sqrt{f'_m} \right] = 0.8 \left[ 4(518 \text{ in}^2) \sqrt{2000 \text{ psi}} \right] = 74.1 \text{ kips}$$

Req'd shear strength, $V_{ns}$

$$\phi V_{ns,req} = 64.0 - 41.7 = 22.3 \text{ kips}$$

Req'd shear steel, $A_v$

$$A_v = \frac{V_{ns} s}{0.5 f'_s d_v} = \frac{(22.3 \text{ kips} / 0.8)(8 \text{ in})}{0.5(60 \text{ ksi})(72 \text{ in})} = 0.103 \text{ in}^2$$

Use 1-#3 at 8 in. vertical shear reinforcement
Deep Beams – Example 2, Shear Details

- Minimum area of vertical reinforcement is $0.0007b d_v$
  - $0.0007b d_v = 0.0007(7.625)(72) = 0.384 \text{ in}^2$
  - Total vertical is $0.11\text{in}^2(15) = 1.65 \text{ in}^2$

- Horizontal shear reinforcement shall have area $\geq$ half vertical shear reinforcement
  - Use 1-#4 each course
  - Do not need joint reinforcement

- Maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.
  - Maximum spacing is 14.4 in.