Combined Flexural and Axial Loads

- Interaction Diagram
  - Partially grouted bearing wall
- Bearing Walls: Slender Wall Design Procedure
  - Strength
  - Serviceability – Deflections
- Moment Magnification
- Example – Pilaster
- Bearing and Concentrated Loads
- Prestressed Masonry

Key Code Sections

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5.4  Pilasters
9.3.2  Design assumptions
9.3.4.1  Nominal strength
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  Section 4.3.3  Radius of gyration
9.3.5  Wall design for out-of-plane loads
  9.3.5.1  Scope
  9.3.5.2  Nominal axial and flexural strength
  9.3.5.3  Nominal shear strength
  9.3.5.4  P-delta effects
  9.3.5.5  Deflections
Concentric Axial Compression

\[
P_n = 0.8 \left[0.80 f'_m(A_n - A_{st}) + f_y A_{st} \left(1 - \left(\frac{h}{140r}\right)^2\right)\right]
\]

- \( h/r \leq 99 \)
- \( \phi = 0.9 \)

\[
P_n = 0.80 \left[0.80 f'_m(A_n - A_{st}) + f_y A_{st} \left(\frac{70r}{h}\right)^2\right]
\]

- \( h/r > 99 \)
- \( A_{st} = \text{area of laterally tied steel} \)

\[
P_{\text{euler}} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 EA_f r^2}{h^2} = \frac{\pi^2 (900f'_m) A_f r^2}{h^2} = A_n f'_m \left(94.2 \frac{r}{h}\right)^2
\]

- Equation above for CMU; for clay (\( E_m = 700f'_m \)), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling

Inclusion of wall weight

Wall weight provides uniform axial load over height of wall. Reasonable approximation is to use half the weight of wall acting at top.

Buckling Curve for \( A_{st} = 0 \)

\[
P_n = 0.8 \left[0.80 f'_m A_k \left(\frac{70r}{h}\right)^2\right]
\]

- \( P_n = 0.80 \left[0.80 f'_m A_k \left(\frac{70r}{h}\right)^2\right] \)
- \( h/r = 99 \)
- \( P_e = 0.80 \left[0.80 f'_m A_k \left(\frac{70r}{h}\right)^2\right] \)
4.3.3 Radius of gyration
Radius of gyration shall be computed using average net cross-sectional area of the member considered.

Questions:
• Is this a strict average or weighted average?
• What about different types of units (which changes block area)?
• What is the effect of bond beams?

• NCMA has tabulated values of average radii of gyration based on average of mortar bedded area and block area.
• Bennett often uses \( r = \sqrt{I_n/A_n} \) in the examples and spreadsheets.

Interaction Diagram

• Assume strain/stress distribution
• Compute forces in masonry and steel
• Sum forces to get axial force
• Sum moment about centerline to get bending moment
• Key points
  • Pure axial load
  • Pure bending
  • Balanced
Example – 8 in. CMU Bearing Wall

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; partial grout; $f'_{m} = 2000$ psi

Required: Interaction diagram in terms of capacity per foot

**Pure Moment:**

$A_s$

Nominal moment, $M_n$

$$M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8b f'_{m}} \right)$$

Design moment, $\phi M_n$

$$\phi M_n = 0.9 \left( 0.934 \frac{k - \frac{r}{h}}{f'_{m}} \right) = 0.840 \frac{k - \frac{r}{h}}{f'_{m}}$$

Check to make sure stress block is in face shell

$$a = \frac{A_s f_y}{0.8b f'_{m}} = \frac{0.05 \frac{A_s f_y}{0.8b f'_{m}} (60ksi)}{0.8(12 \frac{2}{3})(2.0ksi)} = 0.156 \text{in}$$

Combined Flexural and Axial Loads

Example – 8 in. CMU Bearing Wall

**Pure Axial:** NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

$r = 2.66 \text{ in.}$. $A_n = 40.7 \text{ in}^2/\text{ft}$ $I_n = 332.0 \text{ in}^4/\text{ft}$

Find $h/r$

Find $P_n$

$$P_n = 0.8 \left[ f_y 0.80 f'_{u} (A_n - A_p) + f_y A_n \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \right]$$

$$\phi P_n = 0.9 \left( 44.3 \frac{k}{f'_{m}} \right) = 39.9 \frac{k}{f'_{m}}$$

Using $r = \sqrt{\frac{I_n}{A_n}} = \sqrt{\frac{332.0 \text{ in}^4}{40.7 \text{ in}^2}} = 2.86 \text{in.}$. $h/r = 50.4$. $\phi P_n = 40.8 \text{ k/ft}$
Example – 8 in. CMU Bearing Wall

Balanced:

Find $C_m$

$C_{m, \text{face shell}} = \frac{0.8 f_m' (a)b}{12 \text{ in}}$

$C_{m, \text{web}} = \frac{0.8 f_m' (a)c}{12 \text{ in}}$

Find $T$

$T = f_y A_s = (60 \text{ ksi})(0.05 \text{ in}^2) = 3.0 \frac{k}{ft}$

Find $\phi P_n$

$\phi P_n = \frac{20.1}{k/ft}$

Find $\phi M_n$

$\phi M_n = \frac{5.96}{k-ft}$

Example – 8 in. CMU Bearing Wall

Below Balanced:

$c = 1.25 \text{ in.}$

Find $C_m$

$C_m = 0.8 f_m' (a)b = 0.8(2.0 \text{ ksi})(1.00 \text{ in})(12 \text{ in/ft}) = 19.2 \frac{k}{ft}$

Find $T$

$T = f_y A_s = (60 \text{ ksi})(0.05 \text{ in}^2) = 3.0 \frac{k}{ft}$

Find $\phi P_n$

$\phi P_n = \phi (C_m - T) = 0.9(19.2 - 3.0) \frac{k}{ft} = 14.6 \frac{k}{ft}$

Find $\phi M_n$

$\phi M_n = 0.9 \left[ 19.2 \frac{k}{ft} \left( 3.81 \text{ in} - \frac{0.8(1.25 \text{ in})}{2} \right) \right] = 57.2 \frac{k-in}{ft} = 4.77 \frac{k-ft}{ft}$

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### Example – 8 in. CMU Bearing Wall

**Combined Flexural and Axial Loads**

**Above Balanced:**

- \( c = 3.0 \) in.
- Strain: 0.0025
- 3.0 in.
- 0.00068
- 3.81 in.

Find \( C_m \)

\[
C_{m,\text{face shell}} = 0.80(2.0\text{ksi})(1.25\text{in})(12\frac{\text{in}}{\text{ft}}) = 24\frac{k}{\text{ft}}
\]

\[
C_{m,\text{web}} = 0.80(2.0\text{ksi})(2.40\text{in} - 1.25\text{in})(2.0\frac{\text{in}}{\text{ft}}) = 3.68\frac{k}{\text{ft}}
\]

Find \( T \)

\[
T = E_s \varepsilon_s A_s = 29000\text{ksi}(0.00068)(0.05\frac{\text{ksi}}{\text{ft}}) = 0.99\frac{k}{\text{ft}}
\]

Find \( \phi P_n \)

\[
\phi P_n = 0.9(24.0 + 3.68 - 0.99)\frac{k}{\text{ft}} = 24.0\frac{k}{\text{ft}}
\]

Find \( \phi M_n \)

\[
\phi M_n = 0.9\left[24.0\frac{k}{\text{ft}}\left(3.81\text{in} - \frac{1.25\text{in}}{2}\right) + 3.68\frac{k}{\text{ft}}\left(3.81 - 1.25 - \frac{2.40 - 1.25}{2}\right)\right] = 6.28\frac{k\cdot \text{ft}}{\text{ft}}
\]

**Example – 8 in. CMU Bearing Wall**

<table>
<thead>
<tr>
<th>Point</th>
<th>c (in)</th>
<th>( C_{m,fs} ) (kip/ft)</th>
<th>( C_{m,\text{web}} ) (kip/ft)</th>
<th>( T ) (kip/ft)</th>
<th>( \phi P_n ) (kip/ft)</th>
<th>( \phi M_n ) (kip-ft/ft)</th>
</tr>
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<tbody>
<tr>
<td>( a = d )</td>
<td>4.76</td>
<td>24.0</td>
<td>8.2</td>
<td>0</td>
<td>29.0</td>
<td>6.52</td>
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<td>( c = d )</td>
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<td>24.0</td>
<td>5.8</td>
<td>0</td>
<td>26.8</td>
<td>6.45</td>
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<tr>
<td></td>
<td>3.00</td>
<td>24.0</td>
<td>3.7</td>
<td>1.0</td>
<td>24.0</td>
<td>6.28</td>
</tr>
<tr>
<td>Balanced</td>
<td>2.09</td>
<td>24.0</td>
<td>1.3</td>
<td>3.0</td>
<td>20.1</td>
<td>5.97</td>
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<tr>
<td>( a = 1.25 \text{ in.} )</td>
<td>1.56</td>
<td>24.0</td>
<td>0</td>
<td>3.0</td>
<td>18.9</td>
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<td>14.6</td>
<td>4.77</td>
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<td></td>
<td>1.0</td>
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<td>3.0</td>
<td>11.1</td>
<td>3.93</td>
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<td>3.0</td>
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<td>3.0</td>
<td>5.6</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>6.1</td>
<td>0</td>
<td>3.0</td>
<td>2.8</td>
<td>1.68</td>
</tr>
<tr>
<td>Pure Moment</td>
<td>0.195</td>
<td>3.0</td>
<td>0</td>
<td>3.0</td>
<td>0.84</td>
<td></td>
</tr>
</tbody>
</table>

Combined Flexural and Axial Loads
Example – 8 in. CMU Bearing Wall

Interaction Diagram – Solid vs. Partial Grout
**Interaction Diagram – Below Balanced**

Tension, \( T = A_s f_y \)

Compression, \( C_m \)

\[ C_m = 0.8 f'_m b a \]

Nominal Axial Strength, \( P_n \)

\[ P_n = C_m - T = 0.8 f'_m b a - A_s f_y \]

Solve for \( a \)

\[ a = \frac{A_s f_y + P_n}{0.80 f'_m b} \]

Nominal Moment Strength, \( M_n \)

\[ M_n = 0.8 f'_m b a \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

\[ = \left( P_n + A_s f_y \right) \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

Can solve for \( M_n \) if \( P_n \) is known

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**Interaction Diagram – Below Balanced**

If we could only know one point on the interaction diagram, we would want to know the point corresponding to \( \phi P_n = P_u \)

\[ a = \frac{A_s f_y + P_u / \phi}{0.80 f'_m b} \]

\[ M_n = \left( P_u / \phi + A_s f_y \right) \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

These are equations in 9.3.5.2 commentary. They ignore any tension in a possible second layer of steel near the compression face.)

For centered bars:

\[ M_n = \left( P_u / \phi + A_s f_y \right) \left( d - \frac{a}{2} \right) \]
Design: Combined Bending and Axial Load

\[ a = d - \sqrt{d^2 - \frac{2}{\phi(0.8f'_m)b}} \left[P_u\left(d - t_{sp}/2\right) + M_u\right] \]

\[ c = \frac{a}{0.8} \]

Is \( c \geq c_{bal} \)?

For Grade 60 steel \( c_{bal} = 0.547 \)

YES

\[ A_s = \frac{0.8f'_m ba - P_u / \phi}{\varepsilon_{mu}E_s\left(d - c\right)c} \]

Compression controls

NO

\[ A_s = \frac{0.8f'_m ba - P_u / \phi}{f_y} \]

Tension controls

Example: Pilaster Design

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; \( f'_m = 2000 \) psi; Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Wind load of 26 psf (pressure and suction) and uplift of 8.1 kips (\( e = 5.8 \) in.); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Reinforcement

Solution:

Vertical Spanning

Horizontal Spanning

Vertical Spanning

Inside

Lateral Load

\[ w = 26 \text{psf}(16 \text{ft}) = 416 \text{lb/ft} \]

\[ d = 15.625 - 7.625/2 = 11.8 \text{ in} \]
Example: Pilaster Design

Weight of pilaster:
Weight of fully grouted 8 in wall (lightweight units) is 75 psf. Pilaster is like a
double thick wall. Weight is 2(75psf)(16in)(1ft/12in) = 200 lb/ft

**1.2D + 1.6S**

Critical location is top of pilaster. \( P_u = 26.9 \text{ kips} \quad M_u = 156.0 \text{ kip-in} \)

Find \( a \)

\[
a = d - \sqrt{\frac{2(P_u d - h/2) + M_u}{\phi(0.8 f_y b)}}
\]

\[
= 11.8 \text{in} - \sqrt{\left(11.8 \text{in}\right)^2 - \frac{2 \times 26.9 \text{kips}(11.8 \text{in} - 15.6 \text{in}/2) + 156 \text{kip-in}}{0.9(0.8)(2.0 \text{ksi})(15.6 \text{in})}} = 1.04 \text{in}
\]

Check \( c/d \)

\[
\frac{c}{d} = \frac{a/0.8}{d} = \frac{1.04 \text{in}/0.8}{11.8 \text{in}} = 0.110 \leq 0.547
\]

Tension controls

Find \( A_s \)

\[
A_s = \frac{0.8 f_y b a - P_u}{\phi} = \frac{0.8(2.0 \text{ksi})(15.6 \text{in})(1.04 \text{in}) - 26.9 \text{kips}/0.9}{60 \text{ksi}} = -0.066 \text{in}^2
\]

Combined Flexural and Axial Loads

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Example: Pilaster Design

Why the negative area of steel?
Sufficient area from just masonry to resist applied forces.
Determine \( a \) from just compression.

\[
a = \frac{P_u}{0.8 f_y b} = \frac{26.9 \text{kip}}{0.8(2.0 \text{ksi})(15.6 \text{in})} = 1.08 \text{in}
\]

Find the moment

\[
M = P_u \left( \frac{t}{2} - \frac{a}{2} \right) = 26.9 \text{kip} \left( \frac{15.6 \text{in}}{2} - \frac{1.08 \text{in}}{2} \right) = 195 \text{kip-in}
\]

Sufficient capacity from just masonry. No steel needed.
Example: Pilaster Design

0.9D + 1.0W  Check wind suction
At top of pilaster.  \( P_u = 0.9(9.6) - 1.0(8.1) = 0.54 \text{ kips} \)
\( M_u = 0.54(5.8) = 3.1 \text{ kip-in} \)

Location of maximum moment, \( x \)  
\[
x = \frac{L}{2} - \frac{M}{wL} = \frac{288 \text{ in}}{2} - \frac{3.1 \text{ kip-in}}{0.416 \text{ kip/ft}} = 143.7 \text{ in}
\]

Maximum moment, \( M_u \)  
\[
M_u = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2}
\]
Assume midheight moment is \( \sim M_u \).  
Third term in \( M_u \) adds 0.002 k-in.

Axial force, \( P_u \)  
\[
P_u = 0.54k + 0.9(0.20k / \text{ ft})(143.7 \text{ in})/12 \text{ in} = 2.69k
\]

Design for \( P_u = 2.7 \text{ kips}, M_u = 361 \text{ kip-in} \)

Example: Pilaster Design

0.9D + 1.0W  
At top:  \( P_u = 0.5 \text{ k} \)  \( M_u = 3 \text{ k-in} \)
\( x = 144 \text{ in} \)  \( P_u = 2.7 \text{ k} \)  \( M_u = 361 \text{ k-in} \)
\[
a = 1.49 \text{ in} \quad A_s = 0.57 \text{ in}^2
\]

1.2D + 1.0W + 0.5S  
At top:  \( P_u = 8.2 \text{ k} \)  \( M_u = 48 \text{ k-in} \)
\( x = 139 \text{ in} \)  \( P_u = 11.0 \text{ k} \)  \( M_u = 384 \text{ k-in} \)
\[
a = 1.74 \text{ in} \quad A_s = 0.52 \text{ in}^2
\]

1.2D + 1.6S + 0.5W  
At top:  \( P_u = 22.8 \text{ k} \)  \( M_u = 132 \text{ k-in} \)
\( x = 117 \text{ in} \)  \( P_u = 25.2 \text{ k} \)  \( M_u = 252 \text{ k-in} \)
\[
a = 1.41 \text{ in} \quad A_s = 0.12 \text{ in}^2
\]

Required steel = 0.57 in^2
Use 2-#5 each face, \( A_s = 0.62 \text{ in}^2 \)
Total bars, 4-#5, one in each cell
Example: Pilaster Design

Combined Flexural and Axial Loads

Design Strength

Axial (kip)

Moment (kip-ft)

Factored Loads

Balanced Point

Combined Flexural and Axial Loads