Combined Flexural and Axial Loads

- Interaction Diagram
  - Solidly grouted bearing wall
  - Partially grouted bearing wall
- Bearing Walls: Slender Wall Design Procedure
  - Strength
  - Serviceability – Deflections
- Example – Pilaster
- Bearing and Concentrated Loads
- Prestressed Masonry

Key Code Sections

5.3  Columns
5.4  Pilasters
9.3.2  Design assumptions
9.3.4.1  Nominal strength
  9.3.4.1.1  Nominal axial and flexural strength
  Section 4.3.3  Radius of gyration
9.3.5  Wall design for out-of-plane loads
  9.3.5.1  Scope
  9.3.5.2  Nominal axial and flexural strength
  9.3.5.3  Nominal shear strength
  9.3.5.4  P-delta effects
  9.3.5.5  Deflections
Concentric Axial Compression

\[
P_n = 0.8 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \right] + f_y A_{st} \left( \frac{70r}{h} \right)^2
\]

- \( \frac{h}{r} \leq 99 \)
- \( \phi = 0.9 \)
- \( A_{st} \) = area of laterally tied steel

\[
P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 EA_r r^2}{h^2} = \frac{\pi^2 (900 f_m') A_r r^2}{h^2} = A_n f_m' \left( 94.2 \frac{r}{h} \right)^2
\]

- Equation above for CMU; for clay (\( E_m = 700 f_m' \)), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling

Inclusion of wall weight

Wall weight provides uniform axial load over height of wall. Reasonable approximation is to use half the weight of wall acting at top.

Concentric Axial Compression

\[
P_n = 0.8 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \right] + f_y A_{st} \left( \frac{70r}{h} \right)^2
\]

- \( \frac{h}{r} \leq 99 \)
- \( \phi = 0.9 \)
- \( A_{st} \) = area of laterally tied steel

Buckling Curve for \( A_{st} = 0 \)

![Buckling Curve for \( A_{st} = 0 \)](image)
4.3.3 *Radius of gyration*

Radius of gyration shall be computed using average net cross-sectional area of the member considered.

Questions:
- Is this a strict average or weighted average?
- What about different types of units (which changes block area)?
- What is the effect of bond beams?

- NCMA has tabulated values of average radii of gyration.
- Bennett often uses $r = \sqrt{I_n/A_n}$ in the examples and spreadsheets.

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**Interaction Diagram**

- Assume strain/stress distribution
- Compute forces in masonry and steel
- Sum forces to get axial force
- Sum moment about centerline to get bending moment
- Key points
  - Pure axial load
  - Pure bending
  - Balanced
**Example – 8 in. CMU Bearing Wall**

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; partial grout; $f'_{m} = 2000$ psi

**Required:** Interaction diagram in terms of capacity per foot

**Pure Moment:**

\[ M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8 f'_{m}} \right) \]

**Design moment, \( \phi M_n \)**

\[ \phi M_n = 0.9 \left( 0.934 \frac{k}{\text{ft}} \right) = 0.840 \frac{k}{\text{ft}} \]

Check to make sure stress block is in face shell

\[ a = \frac{A_s f_y}{0.8 f'_{m}} = \frac{0.05 \frac{\text{in}}{\text{ft}} (60 \text{ksi})}{0.8 (12 \frac{\text{in}}{\text{ft}}) (2.0 \text{ksi})} = 0.156 \text{in} \]

**Pure Axial:**

NCMA TEK 14-1B  Section Properties of Concrete Masonry Walls

\( r = 2.66 \text{ in.} \quad A_n = 40.7 \text{in}^2/\text{ft} \quad I_n = 332.0 \text{ in}^4/\text{ft} \)

Find \( h/r \)

Find \( P_n \)

\[ P_n = 0.8 \left[ 0.80 f'_{m} (A_n - A_s) + f_y A_s \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \right] \]

\[ \phi P_n = 0.9 \left( 44.3 \frac{k}{\text{ft}} \right) = 39.9 \frac{k}{\text{ft}} \]

Using \( r = \sqrt{\frac{I_n}{A_n}} = \sqrt{\frac{332.0 \text{in}^4}{40.7 \text{in}^2/\text{ft}}} = 2.86 \text{in.} \quad h/r = 50.4 \quad \phi P_n = 40.8 \text{ k/ft} \)
Example – 8 in. CMU Bearing Wall

**Balanced:**

\[ C_m \]
\[ 0.8f_m' = 1.2 \text{ ksi} \]
\[ a = 0.8c = 0.8(2.09\text{ in.}) = 1.67\text{ in.} \]

web length =

Find \( C_m \)

\[ C_{m,\text{face shell}} = \]

\[ C_{m,\text{web}} = \]

Find \( T \)

\[ T = f_y A_s = (60\text{ ksi})(0.05 \text{ in}^2/\text{ft}) = 3.0 \frac{k}{\text{ft}} \]

Find \( \phi P_n \)

\[ \phi P_n = \]

Find \( \phi M_n \)

\[ \phi M_n = \]

---

Example – 8 in. CMU Bearing Wall

**Balanced:**

\[ 0.8f_m = 1.6 \text{ ksi} \]
\[ a = 0.8c = 0.8(1.25\text{ in.}) = 1.00\text{ in.} \]

Find \( C_m \)

\[ C_m = 0.8 f'_m(a)b = 0.8(2.0\text{ ksi})(1.00\text{ in.})(12 \text{ in.}) = 19.2 \frac{k}{\text{ft}} \]

Find \( T \)

\[ T = f_y A_s = (60\text{ ksi})(0.05 \text{ in}^2/\text{ft}) = 3.0 \frac{k}{\text{ft}} \]

Find \( \phi P_n \)

\[ \phi P_n = \phi(C_m-T) = 0.9(19.2 - 3.0) \frac{k}{\text{ft}} = 14.6 \frac{k}{\text{ft}} \]

Find \( \phi M_n \)

\[ \phi M_n = 0.9 \left[ 19.2 \frac{k}{\text{ft}} \left(3.81\text{ in} - \frac{0.8(1.25\text{ in})}{2} \right) \right] = 57.2 \frac{k\cdot\text{in}}{\text{ft}} = 4.77 \frac{k}{\text{ft}} \]
Example – 8 in. CMU Bearing Wall

Above Balanced:
\[ c = 3.0 \text{ in.} \]

\[ \text{Strain} = 0.0025 \]

\[ 3.0 \text{ in.} \]

\[ 0.00068 \]

\[ 3.81 \text{ in.} \]

\[ 0.8f'_{m} = 1.6 \text{ ksi} \]

\[ C_m \]

\[ \text{Stress} \]

\[ T \]

\[ a = 0.8c = 0.8(3.0 \text{ in.}) = 2.4 \text{ in.} \]

\[ \text{web length} = \frac{8 \text{ in.}}{48 \text{ in.}} = \frac{2.0}{48} \text{ in} \]

\[ \frac{C_{m,\text{face shell}}}{24} = 0.80(2.0 \text{ ksi}) \left( 12 \frac{\text{ in.}}{\text{ ft}} \right) = 24 \frac{k}{\text{ ft}} \]

\[ C_{m,\text{web}} = 0.80(2.0 \text{ ksi}) \left( 2.40 \text{ in.} - 1.25 \text{ in.} \right) \left( 2.0 \frac{\text{ in.}}{\text{ ft}} \right) = 3.68 \frac{k}{\text{ ft}} \]

\[ T = E_s \varepsilon_s A_3 = 29000 \text{ ksi} \left( 0.00068 \right) \left( 0.05 \frac{\text{ in.}}{\text{ ft}} \right) = 0.99 \frac{k}{\text{ ft}} \]

\[ \phi P_n = 24.0 \frac{k}{\text{ ft}} \]

\[ \phi M_n = 6.28 \frac{k \cdot \text{ ft}}{\text{ ft}} \]

\[ \phi P_n = 0.9 \left( 24.0 + 3.68 - 0.99 \right) \frac{k}{\text{ ft}} = 24.0 \frac{k}{\text{ ft}} \]

\[ \phi M_n = 0.9 \left[ 24.0 \frac{k}{\text{ ft}} \left( 3.81 \text{ in.} - \frac{1.25 \text{ in.}}{2} \right) + 3.68 \frac{k}{\text{ ft}} \left( 3.81\text{ in} - 1.25\text{ in} - \frac{2.40 - 1.25}{2} \right) \right] = 6.28 \frac{k \cdot \text{ ft}}{\text{ ft}} \]

Combined Flexural and Axial Loads

Example – 8 in. CMU Bearing Wall

<table>
<thead>
<tr>
<th>c (in)</th>
<th>C_{m,\text{fs}} (kip/ft)</th>
<th>C_{m,\text{web}} (kip/ft)</th>
<th>T (kip/ft)</th>
<th>\phi P_n (kip/ft)</th>
<th>\phi M_n (kip-ft/ft)</th>
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<tbody>
<tr>
<td>a = d</td>
<td>4.76</td>
<td>24.0</td>
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<td>0</td>
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<tr>
<td>c = d</td>
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<td>24.0</td>
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<td>0</td>
<td>26.8</td>
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<tr>
<td>3.00</td>
<td>24.0</td>
<td>3.7</td>
<td>1.0</td>
<td>24.0</td>
<td>6.28</td>
</tr>
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<td>Balanced</td>
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<td>1.3</td>
<td>3.0</td>
<td>20.1</td>
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<tr>
<td>a = 1.25 in.</td>
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<td>18.9</td>
</tr>
<tr>
<td>1.25</td>
<td>19.2</td>
<td>0</td>
<td>3.0</td>
<td>14.6</td>
<td>4.77</td>
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<tr>
<td>1.0</td>
<td>15.4</td>
<td>0</td>
<td>3.0</td>
<td>11.1</td>
<td>3.93</td>
</tr>
<tr>
<td>0.8</td>
<td>12.3</td>
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<td>8.4</td>
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<td>0.6</td>
<td>9.2</td>
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<td>5.6</td>
<td>2.47</td>
</tr>
<tr>
<td>0.4</td>
<td>6.1</td>
<td>0</td>
<td>3.0</td>
<td>2.8</td>
<td>1.68</td>
</tr>
<tr>
<td>Pure Moment</td>
<td>0.195</td>
<td>3.0</td>
<td>0</td>
<td>3.0</td>
<td>0</td>
</tr>
</tbody>
</table>

Combined Flexural and Axial Loads
Example – 8 in. CMU Bearing Wall

Interaction Diagram – Below Balanced

Tension, $T$

$T = A_y f_y$

Compression, $C_m$

$C_m = 0.8 f_m' b a$

Nominal Axial Strength, $P_n$

$P_n = C_m - T = 0.80 f_m' b a - A_y f_y$

Solve for $a$

$a = \frac{A_y f_y + P_n}{0.80 f_m' b}$

Nominal Moment Strength, $M_n$

$M_n = 0.8 f_m' b a \left( \frac{t_{sp}}{2} - \frac{a}{2} \right) + A_y f_y \left( d - \frac{t_{sp}}{2} \right)$

$= \left( P_n + A_y f_y \left( \frac{t_{sp}}{2} - \frac{a}{2} \right) \right) + A_y f_y \left( d - \frac{t_{sp}}{2} \right)$

Can solve for $M_n$ if $P_n$ is known
If we could only know one point on the interaction diagram, we would want to know the point corresponding to $\phi P_n = P_u$

$$a = \frac{A_s f_y + P_u / \phi}{0.80 f_m' b}$$

$$M_n = \left( P_u / \phi + A_s f_y \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \right)$$

These are equations in 9.3.5.2 commentary. They ignore any tension in a possible second layer of steel near the compression face.

For centered bars:

$$M_n = \left( P_u / \phi + A_s f_y \left( d - \frac{a}{2} \right) \right)$$

Design: Combined Bending and Axial Load

Calculate

\[ a = d - \sqrt{d^2 - \frac{2 [P_u (d - t_{sp} / 2) + M_u]}{\phi (0.8 f_m' b)}} \]

\[ c = \frac{a}{0.8} \]

Is $c \geq c_{bal}$?

For Grade 60 steel

$c_{bal} = 0.547$

\[ c_{bal} = \frac{\varepsilon_{mu} d}{\varepsilon_{mu} + \varepsilon_y} \]

YES

\[ A_s = \frac{0.8 f_m' ba - P_u / \phi}{\varepsilon_{mu} E_s \left( \frac{d - c}{c} \right)} \]

Compression controls

NO

\[ A_s = \frac{0.8 f_m' ba - P_u / \phi}{f_y} \]

Tension controls
Interaction Diagram – Below Balanced

- Small _______ forces
  - Partially grouted walls act as _______ walls
  - Compression area is in_____________
- Strength design
  - Higher axial loads act as_________
  - Very high axial loads act as_________
  - Need to calculate $r$ based on grouted cross-section.

Interaction Diagram – Solid vs. Partial Grout
Walls: Slenderness Effects

1. _________________________  
   a. Axial capacity (and sometimes moment) reduced  
   b. Used to be in TMS 402 Code  
   c. deleted because it can be unconservative

2. ___________________________  
   a. Second-order moment directly added by P-δ  
   b. Usually requires iteration  
   c. Difficult for hand calculations for other than simple cases  
   d. Basis for second-order analysis in computer programs  
   e. Historical method used for masonry design

3. ___________________________  
   a. Added in 2013 TMS 402 Code  
   b. Very general, but a bit conservative.

Walls: Complementary Moment 9.3.5.4.2

- Assumes simple support conditions.  
- Assumes midheight moment is approximately maximum moment  
- Valid only for the following conditions:  
  - $\frac{P_u}{A_n} \leq 0.05 f_m'$ No height limit  
  - $\frac{P_u}{A_g} \leq 0.20 f_m'$ height limited by $\frac{h}{t} \leq 30$

**Moment:**  
$$M_u = \frac{W_u h^2}{8} + P_{uf} \frac{e_u}{2} + P_u \delta_u$$

**Deflection:**  
$$\delta_u = \frac{5M_u h^2}{48E_m I_n} \quad M_u < M_{cr}$$

$$P_u = P_{uw} + P_{uf}$$

$$P_{uf} = \text{Factored floor load}$$

$$P_{uw} = \text{Factored wall load}$$

$$\delta_u = \frac{5M_{cr} h^2}{48E_m I_n} + \frac{5(M_u - M_{cr}) h^2}{48E_m I_{cr}} \quad M_u > M_{cr}$$
Walls: Complementary Moment

Solve simultaneous linear equations:

\[ M_u > M_{cr} \]
\[
M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + \frac{5M_{cr}P_u h^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right)
\]
\[ 1 - \frac{5P_u h^2}{48E_m I_{cr}} \]

\[ \delta_u = \frac{5h^2}{48E_m I_{cr}} \left[ \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + M_{cr} \left( \frac{I_{cr}}{I_n} - 1 \right) \right]
\]
\[ 1 - \frac{5P_u h^2}{48E_m I_{cr}} \]

\[ M_u < M_{cr} \]
\[
M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2}
\]
\[ 1 - \frac{5P_u h^2}{48E_m I_{cr}} \]

\[ \delta_u = \frac{5h^2}{48E_m I_n} \left[ \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} \right]
\]
\[ 1 - \frac{5P_u h^2}{48E_m I_n} \]

Walls: Deflections

Depth to neutral axis, \( c \)
\[
c = \frac{A_s f_y + P_u}{0.64 f'_{m} b}
\]

Cracked moment of inertia, \( I_{cr} \)
\[
I_{cr} = n \left( \frac{A_s + P_u t_{sp}}{f_y} \right) (d - c)^2 + \frac{bc^3}{3}
\]

Centered bars:
\[
I_{cr} = n \left( \frac{A_s}{f_y} \right) (d - c)^2 + \frac{bc^3}{3}
\]

Cracking moment, \( M_{cr} \)
\[
M_{cr} = \frac{(P_u / A_n + f_r) I_n}{t_{sp} / 2}
\]

What axial load, \( P_u \), should be used to find \( M_{cr} \)?
Suggest using minimum \( P_u \).

Deflection Limit

\[ \delta_s \leq 0.007h \]

Calculated using allowable stress load combinations
Walls: Maximum Reinforcement

- Strain gradient of $\varepsilon_{mu}$ and $\alpha \varepsilon_p$, with $\alpha = 1.5$ for OOP loading
- $P_u$ determined from $D + 0.75L + 0.525Q_E$

\[ \rho = \frac{A_s}{bd} = \frac{0.64 f_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - P_u}{f_y - \min \left\{ \frac{d'}{d} (\varepsilon_{mu} + \alpha \varepsilon_y), \varepsilon_y \right\} E_s} \]

- Fully grouted with equal tension and compression reinforcement

\[ \rho = \frac{A_s}{bd} = \frac{0.64 f_m' \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - P_u}{f_y} \]

- Fully grouted with concentrated tension reinforcement, or partially grouted with neutral axis in face shell

\[ \rho = \frac{A_s}{bd} = \frac{0.64 f_m' \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) + 0.8 f_m' \left( \frac{b - b_w}{bd} \right) - P_u}{f_y} \]

- Partially grouted walls with concentrated tension reinforcement and neutral axis in web

Combined Flexural and Axial Loads

Out-of-Plane Loading: Wind Load

Wind Load on Parapet
- MWFRS (ASCE 7-10 27.4.5, 28.4.2)
  - Used for determining shear wall loads
  - Used for roof uplift load

- Components and Cladding (ASCE 7-10 30.7.1.2, 30.9):
  - Used for designing parapet
  - Used for designing wall-to-diaphragm connection

- Parapet pressure to use for designing wall?
  - Parapet wind load reduces midheight wall moment
  - Very conservative: parapet load is 0
  - Aggressive: full parapet C&C pressure
  - Moderate: extend wall pressure to parapet

Combined Flexural and Axial Loads
**Out-of-Plane Loading: Seismic Loading**

**Seismic Load on Parapet**
- In first mode, wall and parapet loads are in opposite directions
- Design forces for shear walls and wall-to-diaphragm connections
  - Suggest using wall and parapet loads in same direction
- Seismic parapet force to use for designing wall
  - Conservative: parapet load in opposite direction of wall
  - Aggressive: wall and parapet load in same direction
  - Moderate: no parapet load

\[ M_{\text{max}} = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2} \]

\[ x = \frac{L}{2} - \frac{M}{wL} \]

If \( x < 0 \), \( M_{\text{max}} = M \)
Example: Bearing Wall

**Given:** 8 in. CMU wall; Grade 60 steel; Type S masonry cement mortar; $f_{m} = 2000$ psi; roof forces act on 3 in. wide bearing plate at edge of wall.

**Required:** Reinforcement

**Solution:**

Estimate reinforcement

$$M = \frac{wh^2}{8} = \frac{(0.032ksf)(16ft)^2}{8} = 1.02 \frac{k-ft}{ft}$$

$a = 0.19$ in.

$A_s = 0.061$ in$^2$/ft

Try #4 @ 40 in.

**Summary of Strength Design Load Combination Axial Forces**

(wall weight is 40 psf for 40 in. grout spacing)

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>$P_{uf}$ (kip/ft)</th>
<th>$P_{uw}$ (kip/ft)</th>
<th>$P_u$ (kip/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D+1.0W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2D+1.0W+0.5L_r</td>
<td>$1.2(0.5)+1.0(-0.36)$ +$0.5(0.4)$ = 0.440</td>
<td>$1.2(0.040)(2.67+8)$ = 0.512</td>
<td>0.952</td>
</tr>
</tbody>
</table>

$P_{uf}$ = Factored floor load; just eccentrically applied load

$P_{uw}$ = Factored wall load; includes wall and parapet weight, found at midheight of wall between supports (8 ft from bottom)
Example: Bearing Wall

Find modulus of rupture; use linear interpolation between no grout and full grout

UngROUTed (Type S masonry cement): 51 psi
Fully grouted (Type S masonry cement): 153 psi

Find $M_{cr}$, cracking moment:

Commentary allows inclusion of axial load
Use minimum axial load (once wall has cracked, it has cracked)

$$M_{cr} = \frac{P_u / A_n + f_r}{t / 2} \cdot \frac{\left[ \left( \frac{474 \text{ in}}{ft} \right) \left( \frac{42.8 \text{ in}^2}{ft} \right) + 71 \text{ psi} \right]}{3.81 \text{ in}}$$

$$M_{cr} = 7.28 \frac{\text{kip-in}}{ft} = 0.606 \frac{\text{kip-ft}}{ft}$$

Wall properties determined from NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

Example: Bearing Wall

Find $c$

$$c = \frac{A_s f_y + P_u}{0.64 f_m' b}$$

Find $n$

$$n = \frac{E_s}{E_m}$$

Find $I_{cr}$

$$I_{cr} = n \left( A_y + \frac{P_u t_{sp}}{f_y} \right) \left( d - c \right)^2 + \frac{bc^3}{3}$$
Example: Bearing Wall

\( P_{u,\theta} \) is the moment at the top support of the wall, \( M_{u,\text{top}} \). It includes eccentric axial load and wind load from parapet.

\[
M_{u,\text{top}} = P_{u,\theta} e_u - w_{u,\text{parapet}} h_{\text{parapet}}^2 = 0.090 \text{ k-lbf} \left( \frac{2.8\text{ in}}{12\text{ in}} \right)^2 - \frac{0.032 \text{ ksf} \left( 2.67 \text{ ft} \right)^2}{2} = -0.093 \text{ k-ft/ft}
\]

Find \( M_u \):

\[
M_u = \frac{w_{u} h^2}{8} + P_{u,\theta} e_u + \frac{5 M_{u,\theta} h^2}{48 E_m} \left( \frac{1}{I_u} - \frac{1}{I_{cr}} \right)
\]

\[
= \frac{w_{u} h^2}{8} + \frac{5 P_{u,\theta} e_u}{48 E_m} \left( \frac{1}{I_u} - \frac{1}{I_{cr}} \right)
\]

\[
= \frac{0.032 \text{ ksf} \left( 16 \text{ ft} \right)^2}{8} + \frac{-0.093 \frac{k\text{-lbf}}{\text{ft}}}{2} + \frac{5(0.606 \frac{k\text{-lbf}}{\text{ft}})(0.474 \frac{\text{ft}}{\text{in}})(16 \text{ ft})^2}{48(1800 \text{ ksi})} \left( \frac{1}{337 \frac{\text{ksi}}{\text{in}^2}} - \frac{1}{13.8 \frac{\text{ksi}}{\text{in}^2}} \right) \left( 144 \text{ in}^2 \right) = 1.009 \frac{k\text{-lbf}}{\text{ft}}
\]

Combined Flexural and Axial Loads

Example: Bearing Wall

Check area of steel:

Find \( a \)

\[
a = A_s f_y + \frac{P_u}{\phi} \frac{f_y}{0.80 f_y} b
\]

Find \( \phi M_n \)

\[
\phi M_n = \phi \left( \frac{P_u}{\phi} + A_s f_y \right) \left( d - \frac{a}{2} \right)
\]

Compare

\[
M_u = 1.01 \frac{k\text{-lbf}}{\text{ft}} \leq 1.15 \frac{k\text{-lbf}}{\text{ft}} = \phi M_n \quad \text{OK}
\]

Check other load combinations:

1.2D+1.0W+0.5L_{n}, \quad M_u = 1.09 \text{ k-ft/ft} \quad \text{and} \quad \phi M_n = 1.29 \text{ k-ft/ft} \quad \text{OK}
Example: Deflections

Check Deflections: Use ASD Load Combinations

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>D+0.6W</th>
<th>0.6D+0.6W</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$ (k/ft)</td>
<td>0.5+0.6(-0.36) = 0.284</td>
<td>0.6(0.5)+0.6(-0.36) = 0.084</td>
</tr>
<tr>
<td>$P_w$ (k/ft)</td>
<td>40(2.67+8) = 0.427</td>
<td>0.6(0.427) = 0.256</td>
</tr>
<tr>
<td>$P$ (k/ft)</td>
<td>0.711</td>
<td>0.340</td>
</tr>
<tr>
<td>$c$ (in)</td>
<td>0.280</td>
<td>0.256</td>
</tr>
<tr>
<td>$I_{cr}$ (in^4/ft)</td>
<td>14.5</td>
<td>13.4</td>
</tr>
<tr>
<td>$M_{top}$ (k-ft/ft)</td>
<td>-0.002</td>
<td>-0.049</td>
</tr>
<tr>
<td>$\delta$ (in)</td>
<td>0.066</td>
<td>0.044</td>
</tr>
<tr>
<td>$M$ (k-ft/ft)</td>
<td>0.617 (cracked)</td>
<td>0.565 (uncracked)</td>
</tr>
</tbody>
</table>

Deflection Limit

$\delta_s \leq 0.007h = 0.007(192in) = 1.34in \quad \text{OK}$

Deflections, Sample Calculations (D+0.6W):
Replace factored loads with service loads

$$\delta = \frac{5h^2}{48E_m I_{cr}} \left[ \frac{wh^2}{8} + \frac{P}{2} + M_{cr} \left( \frac{I_{cr}}{I_m} - 1 \right) \right]$$

$$= \frac{5(16ft)^2}{48(1800ksi)(14.5 \frac{in^4}{ft})} \left[ \frac{(0.6)(0.032ksi)(16ft)^2}{8} + \frac{-0.002 \frac{ksi}{ft}}{2} + 0.606 \frac{ksi}{ft} \left( \frac{14.5 \frac{in^4}{ft}}{336.7 \frac{ksi}{in^2}} - 1 \right) \right] \frac{1728in^3}{1ft^3}$$

$$= \frac{5(0.711 \frac{ksi}{ft})(16ft)^2}{48(1800ksi)(14.5 \frac{in^4}{ft})} \frac{144in^3}{1ft^3}$$

$$= 0.066\text{in}.$$

Check that $M > M_{cr}$: $M = 0.617 \text{ k-ft/ft} > 0.606 \text{ k-ft/ft} = M_{cr}$
Example: Maximum Reinforcement

Check Maximum Reinforcement:
• neutral axis is in face shell
• \( P_u \) is just dead load = 0.5 + 0.04(2.67 + 8) = 0.927 k/ft

\[
\rho_{\text{max}} = \frac{A_s}{bd} = \frac{0.64 f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{f_y}}{bd}
\]

\[
0.64(2.0\text{ksi}) \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) - \frac{0.927 k}{60\text{ksi}} = 0.00918
\]

\[
\rho = \frac{A_s}{bd} = \frac{0.06 \frac{f^2}{\pi}}{12 \frac{\text{in}}{\text{ft}} (3.81\text{in})} = 0.00131 \quad \text{OK}
\]

Example: Seismic Loads

Given: 8 in. normal weight (125 pcf) CMU wall; Grade 60 steel; Type S PCL mortar (special reinforced wall); \( f'_m = 2000 \text{psi} \); roof forces act at 7.32 in. eccentricity; SDS = 1.43, \( I = 1.0 \).

Required: Reinforcement

Solution: Estimate amount of steel
• Lateral load directly proportional to wall weight
• Need to know grout spacing to determine wall weight
• Wall is primarily in flexure
• Check different grout spacings in flexure and determine reasonable amount of steel

\[
w_u = 0.4S_{DS}w_w
\]
Example: Seismic Loads

Summary of Strength Design Load Combination Axial Forces

- 0.9D+1.0E = (0.9-0.2SDS)D = (0.9-0.2*1.43)D = 0.614D
- 1.2D+1.0E = (0.9+0.2SDS)D = (1.2+0.2*1.43)D = 1.486D

<table>
<thead>
<tr>
<th>Grout spacing</th>
<th>w_u (psf)</th>
<th>w_e (psf)</th>
<th>M_u (k-ft/ft)</th>
<th>a (in.)</th>
<th>A_s (in^2/ft)</th>
<th>Bar size</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in.</td>
<td>4.54</td>
<td></td>
<td>0.944</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 in.</td>
<td>2.86</td>
<td></td>
<td>0.562</td>
<td>0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48 in.</td>
<td>2.47</td>
<td></td>
<td>0.479</td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* FEMA P-751 uses 65 psf to account for bond beams and additional grouted cells (27% more)

\[
a = d - \sqrt{d^2 - \frac{2M_u}{\phi f_y' b}}
\]

\[
A_s = \frac{0.8f_y' b a - P_u / \phi}{f_y}
\]

Try #6 @ 24 in., use 56 psf for wall weight (10% increase)

\[w_u = 32.0 \text{ psf}\]
Example: Seismic Loads

Find modulus of rupture; use linear interpolation between no grout and full grout

Ungrouted (Type S PCL): 84 psi
Fully grouted (Type S PCL): 163 psi

\[ f_r = 84 \text{ psi} \left( \frac{2 \text{ ungrouted cells}}{3 \text{ cells}} \right) + 163 \text{ psi} \left( \frac{1 \text{ grouted cell}}{3 \text{ cells}} \right) = 110 \text{ psi} \]

Find \( M_{cr} \), cracking moment:

\[
M_{cr} = \frac{P_u}{A_n} + f_r \left( \frac{673 \frac{\text{lb}}{\text{ft}}}{51.3 \frac{\text{in.}^2}{\text{ft}}} + 110 \text{ psi} \right) \left( \frac{355.3 \frac{\text{in.}^2}{\text{ft}}}{3.8 \text{ lin}} \right) \left( \frac{1}{t_{sp}/2} \right) = 11.48 \frac{k-\text{in}}{\text{ft}} = 0.957 \frac{k-\text{ft}}{\text{ft}}
\]

Wall properties determined from NCMA TEK 14-1B Section
Properties of Concrete Masonry Walls

<table>
<thead>
<tr>
<th>Example: Seismic Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strength</strong></td>
</tr>
<tr>
<td>Load Combination</td>
</tr>
<tr>
<td>c (in.)</td>
</tr>
<tr>
<td>( I_{cr} ) (in^4/ft)</td>
</tr>
<tr>
<td>( M_u ) (k-ft/ft)</td>
</tr>
<tr>
<td>Second Order / First Order</td>
</tr>
<tr>
<td>a (in.)</td>
</tr>
<tr>
<td>( \phi M_n ) (k-ft/ft)</td>
</tr>
<tr>
<td>( M_u/\phi M_n )</td>
</tr>
</tbody>
</table>

| **Deflections**        |
| Load Combination       | 0.6D+0.7E | D+0.7E |
| \( \delta \) (in.)     | 3.44      | 4.07   |
| \( \delta/\delta_{allow} (\delta_{allow} = 2.35\text{in.}) \) | 1.46 | 1.73 |
Example: Seismic Loads

- Load combinations with higher axial forces control due to P-δ effects
- FEMA P-751 does not check deflections; reasoning is standard does not have mid-height deflection limit for walls; TMS 402 does have a deflection limit

<table>
<thead>
<tr>
<th>Option</th>
<th>$M_{u}/M_{n}$</th>
<th>$\delta/\delta_{allow}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#7@ 24 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7 @ 24 in. LW units ($w_{wall} = 51$ psf; $w_{u} = 29$ psf)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7 @ 24 in. $f'_{m}=2500$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#5 @ 8 in. Fully grout; $f'_{m}=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6 @ 8 in. Fully grout; $f'_{m}=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-#5 @ 24 in, 2 in. cover; $f'_{m}=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 in. CMU, #6 @ 32 in. $w_{wall} = 75$ psf; $w_{u} = 43$ psf</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example: Seismic Loads

Check Maximum Reinforcement: NA in face shell; $P_u$ is just dead load

$$
\rho_{max} = \frac{0.64 f'_{m} \left( \frac{e_{mu} + \alpha e_{y}}{e_{mu} + \alpha e_{y}} \right) - \frac{P_u}{b d}}{f_y} = 0.64 \left( \frac{2.0 \text{ksi}}{0.0025 + (0.00027)} \right) - \frac{1.10 \text{k/ft}}{12 \text{ in/ft} \left(3.81 \text{in} \right)} = 0.00911
$$

$$
\frac{A_s}{b} = \rho_{max} d = 0.00911 \left(3.81 \text{in} \right) \left(\frac{12 \text{in}}{\text{ft}} \right) = 0.42 \text{in}^2/\text{ft}
$$

<table>
<thead>
<tr>
<th>Grout spacing</th>
<th>$P_u$ (kip/ft)</th>
<th>$\rho_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in. CMU; 24 in.</td>
<td>0.2+0.056(2+14) = 1.10 k/ft</td>
<td>0.00911 = 0.42 in²/ft #5 @ 24 in. = 0.16 in²/ft</td>
</tr>
<tr>
<td>8 in. CMU; full grout</td>
<td>0.2+0.081(2+14) = 1.50 k/ft</td>
<td>0.00897 = 0.42 in²/ft #6 @ 8 in. = 0.66 in²/ft</td>
</tr>
<tr>
<td>12 in. CMU; 32 in</td>
<td>0.2+0.075(2+14) = 1.40 k/ft</td>
<td>0.00918 = 0.64 in²/ft #6 @ 32 in. = 0.16 in²/ft</td>
</tr>
</tbody>
</table>
Moment Magnification Method

Complementary Moment

\[ M_u = \frac{w_u h^2}{8} + P_{wf} \frac{e_u}{2} + \frac{5M_{cr}P_u h^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right) \]

\[ 1 - \frac{5P_u h^2}{48E_m I_{cr}} \]

\[ M_u = \psi M_{u,0} \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} \]

\[ M_u < M_{cr} : I_{eff} = 0.75I_n \]

\[ M_u \geq M_{cr} : I_{eff} = I_{cr} \]

\[ P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \]

Moment Magnification: Deflections

First-order deflection (simply supported wall):

\[ \delta_0 = \frac{5wh^4}{384E_m I_e} + \frac{P_f eh^2}{16E_m I_e} \]

Rewriting TMS 402 OOP equations:

Same as proposed by Bischoff, P. (2005).
Example: Moment Magnification

Given: CMU wall; Grade 60 steel; Type S PCL mortar (special reinforced wall); \( f'_{m} = 2000 \text{psi} \); roof dead load of 400lb/ft at 3.5 in. outside face of wall; \( S_{DS} = 1.43 \), \( I = 1.0 \). 12 ft high x 12 ft wide door openings. See elevation on next page.

Required: Reinforcement

Solution:

After 2009 NEHRP Recommended Seismic Provisions:
Design Examples FEMA P-751 / September 2012
Example: Moment Magnification

- Load has approximately doubled
- Use 12 inch block, two layers of bars, 2 inch cover, d ~ 9.2 in.
- Normal weight units, 24 in. bar spacing, $w_w = 75$ psf, say 85 psf
- $w_u = 0.4S_{DS}l w_w = 0.4(1.43)(1)(2*85\text{psf}) = 97$ psf
- $M_u = 0.097\text{ksf}(28\text{ft})^2/8 = 9.51\text{k-ft/ft}$

\[
a = 9.2\text{in} - \sqrt{(9.2\text{in})^2 - \frac{2(9.51 \frac{\text{ksf}}{\text{ft}})(12 \frac{\text{in}}{\text{ft}})}{0.9(0.8)(2\text{ksi})(12 \frac{\text{in}}{\text{ft}})}} = 0.748\text{in.}
\]

\[
\frac{A_s}{f_y} = \frac{0.8A_s'}{f_y} = \frac{0.8(2\text{ksi})(12 \frac{\text{in}}{\text{ft}})(0.748\text{in})}{60\text{ksi}} = 0.24 \frac{\text{in}^2}{\text{ft}}
\]

- Over 8 ft pier width, $A_s = 8\text{ft}(0.24\text{in}^2/\text{ft}) = 1.92\text{in}^2$
- $1.92\text{in}^2 = 6.2$ #5 bars or 4.4 #6 bars.
- Try 5 - #6 bars; two at each jamb, and one in center.
- $d = 11.62 - 2 - 0.31 = 9.31\text{in.}$
- $e = 11.62/2 + 3.5 = 9.31\text{in.}$
- 85 psf is reasonable wall weight
Example: Moment Magnification

Shear and Moment Diagrams for Pier: 0.9D+1.0E

\[ w_u = 0.4(1.43)(0.085)(8) = 0.389k/ft \]

\[ w_u = 0.4(1.43)(0.085)(20) = 0.972k/ft \]

Analyze at point of maximum moment:
- \( M_{u,0} = 77.6 \text{ k-ft} \)
- \( P_u = 0.614(8k+0.085\text{ksf}(20\text{ft})(12.3\text{ft})) = 17.8 \text{ kips} \)

Find \( c \)
\[ c = \frac{A_y f_y + P_u}{0.64 f_{m}^3 b} \]

(\( c \) is in face shell)

Find \( I_{cr} \)
\[ I_{cr} = n \left( A_s + \frac{P_u t_{sp}}{f_y} \right) \left( d - c \right)^2 + \frac{bc^3}{3} \]
**Example: Moment Magnification**

**Buckling load,** \( P_e \)

\[
P_e = \frac{\pi^2 E_m I_{eff}}{h^2} =
\]

**Moment magnifier,** \( \psi \)

\[
\psi = \frac{1}{1 - \frac{P_u}{P_e}} =
\]

**Factored moment,** \( M_u \)

\[
M_u = \psi M_{u,0} =
\]

---

**Example: Moment Magnification**

**Check capacity:**

*Find a*

\[
a = \frac{A_y f_y + P_u / \phi}{0.80 f_m' b} = \frac{2.2in^2(60ksi) + 17.8k / 0.9}{0.80(2ksi)(96in)} = 0.988in.
\]

*Find nominal moment,** \( M_n \)

\[
M_n = \left( P_u / \phi + A_y f_y \left( \frac{t_w - a}{2} \right) \right) + A_y f_y \left( \frac{d - L_w}{2} \right)
\]

\[
= \left( 17.8k / 0.9 + 2.2in^2(60ksi) \right) \left( \frac{11.62in - 0.99in}{2} \right) + 2.2in^2(60ksi) \left( 9.3in - \frac{11.62in}{2} \right)
\]

\[
= 1269k - in = 105.8k - ft
\]

*Check capacity*

\[
\phi M_n = 0.9(105.8k - ft) = 95.2k - ft > M_u = 81.1k - ft \quad \text{OK}
\]

*Check other load combinations*

For \(1.2D+1.0E\), \( M_u = 89.4k-ft \quad \phi M_n = 104.9k-ft \quad \text{OK} \)
Example: Moment Magnification

Check Maximum Reinforcement:
- neutral axis is in face shell
- \( P_u \) is just dead load = 8.0k + 0.085ksf(20ft)(12.3ft) = 28.9k

\[
\rho_{\text{max}} = \frac{A_s}{bd} = \frac{0.64 f'_y \left( \frac{\epsilon_{\text{mu}}}{\epsilon_{\text{mu}} + \alpha \epsilon_y} \right) - \frac{P_u}{bd}}{f_y - \min \left\{ \frac{d}{d} \left( \frac{\epsilon_{\text{mu}}}{\epsilon_{\text{mu}} + \alpha \epsilon_y} \right) \epsilon_y \right\} E_s}
\]

\[
= \frac{0.64(2\text{ksi}) \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) - \frac{28.9k}{96\text{in}(9.31\text{in})}}{60\text{ksi} - \min \left\{ \frac{2.3\text{lin}}{9.3\text{lin}}(0.0025 + 1.5(0.00207)), 0.00207 \right\} \text{29000ksi}} = 0.0194
\]

\[
\rho = \frac{A_s}{bd} = \frac{2.2\text{in}^2}{96\text{in}(9.31\text{in})} = 0.0025 < 0.0194 \quad \text{OK}
\]

Example: Moment Magnification

Check deflections:
\[0.6D + 0.7E\]

Net area, \( A_n \)
\[A_n = 2.5\text{in}(96\text{in}) + 5(8\text{in})(11.62\text{in} - 2.5\text{in}) = 605\text{in}^2\]

Net moment of inertia, \( I_n \)
\[I_n = \frac{1}{12} (96\text{in})(11.62\text{in})^3 - \frac{1}{12} (96\text{in} - 5(8\text{in}))(11.62\text{in} - 2.5\text{in})^3 = 9022\text{in}^4\]

Modulus of rupture, \( f_r \)
\[f_r = 84\text{psi} \left( \frac{7\text{ ungrouted cells}}{12\text{ cells}} \right) + (163\text{psi} \left( \frac{5\text{ grouted cells}}{12\text{ cells}} \right) = 117\text{psi}\]

Cracking moment, \( M_{cr} \)
\[M_{cr} = \frac{\left( \frac{17.8k}{605\text{in}^4} + 0.117\text{ksi} \right)(9022\text{in}^4)}{5.81\text{in}} = 227.4\text{kip} - \text{in} = 18.9k - \text{ft}\]
Example: Moment Magnification

\[ c = \frac{A_f + P}{0.64 f'_{w}, b} = \frac{2.2in^2 (60 ksi) + 11.6 k}{0.64 (2.0 ksi) 96 in} = 1.17 in \]

Cracked moment of inertia, \( I_{cr} \)

\[ I_{cr} = n \left( A + \frac{P}{f'_{w} \frac{d - c}{2d}} \right) (d - c)^2 + \frac{bc^3}{3} \]

\[ = 16.1 \left[ 2.2in^2 + \frac{11.6 k}{60 ksi} \frac{11.62in}{2(9.31in)} \right] \left( 9.31in - 1.17in \right)^2 + \frac{96 in (1.17 in)}{3} = 2528in^4 \]

Effective moment of inertia, \( I_e \)

\[ I_e = \frac{I_{cr}}{1 - \frac{M_{cr}}{M} \left( \frac{1 - I_{cr}}{I_n} \right)} = \frac{2528in^4}{1 - \frac{18.9k - ft}{54.2k - ft} \left( 1 - \frac{2528in^4}{9022in^4} \right)} = 3375in^4 \]

<table>
<thead>
<tr>
<th>Deflections</th>
<th>Load Combination</th>
<th>0.6D+0.7E</th>
<th>D+0.7E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{max} ) (k-ft)</td>
<td></td>
<td>54.2</td>
<td>57.0</td>
</tr>
<tr>
<td>( P ) (kip)</td>
<td></td>
<td>11.6</td>
<td>34.2</td>
</tr>
<tr>
<td>( I_{cr} ) (in^4)</td>
<td></td>
<td>2528</td>
<td>2687</td>
</tr>
<tr>
<td>( I_e ) (in^4)</td>
<td></td>
<td>3375</td>
<td>3502</td>
</tr>
<tr>
<td>( \delta_0 ) (in)</td>
<td></td>
<td>1.22</td>
<td>1.25</td>
</tr>
<tr>
<td>( \Psi )</td>
<td></td>
<td>1.030</td>
<td>1.088</td>
</tr>
<tr>
<td>( \delta ) (in)</td>
<td></td>
<td>1.26</td>
<td>1.36</td>
</tr>
</tbody>
</table>

\[ \delta_{all} = 0.007h = 0.007(28ft)(12in/ft) = 2.35in. \]  OK
Example: Pilaster Design

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; \( f_m' = 2000 \text{ psi} \); Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Wind load of 26 psf (pressure and suction) and uplift of 8.1 kips (\( e = 5.8 \text{ in.} \)); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Reinforcement

Solution:

\[ e = 5.8 \text{ in.} \]

Vertical Spanning

\[ d = 15.625 - 7.625/2 = 11.8 \text{ in.} \]

Lateral Load

\[ w = 26 \text{psf}(16\text{ft}) = 416\text{lb/ft} \]

Combined Flexural and Axial Loads

Critical location is top of pilaster.

\[ P_u = 26.9 \text{ kips} \quad M_u = 156.0 \text{ kip-in} \]

1.2D + 1.6S

Find \( a \)

\[ a = d - \sqrt{d^2 - \frac{2[P_u(d - b/2) + M_u]}{\phi(0.8f_{m}'b)}} \]

\[ = 11.8\text{in} - \sqrt{(11.8\text{in})^2 - \frac{2[26.9kL(11.8\text{in} - 15.6\text{in}/2) + 156k - in]}{0.9(0.8)(2.0\text{ksi})(15.6\text{in})}} = 1.04\text{in} \]

Find \( A_s \)

\[ A_s = \frac{0.8f_{m}'b - P_u}{\phi} = \frac{0.8(2.0\text{ksi})(15.62\text{in})(1.04\text{in}) - 26.9k/0.9}{60\text{ksi}} = -0.066\text{in}^2 \]
Example: Pilaster Design

Why the negative area of steel?
Sufficient area from just masonry to resist applied forces.
Determine \( a \) from just compression.

\[
a = \frac{P_u}{0.8 f'_m b} = \frac{26.9 \text{kip}}{0.8(2.0 \text{ksi})15.6 \text{in}} = 1.08 \text{in}
\]

Find the moment

\[
M = P_u \left( \frac{t}{2} - \frac{a}{2} \right) = 26.9 \text{kip} \left( \frac{15.6 \text{in}}{2} - \frac{1.08 \text{in}}{2} \right) = 195 \text{kip-in}
\]

\( M_u = 156 \text{ kip-in} \)

Sufficient capacity from just masonry. No steel needed.

---

Example: Pilaster Design

0.9D + 1.0W

Check wind suction

At top of pilaster. \( P_u = 0.9(9.6) - 1.0(8.1) = 0.54 \text{ kips} \)
\( M_u = 0.54 \text{kips}(5.8\text{in}) = 3.1 \text{ kip-in} \)

Location of maximum moment, \( x \)

\[
x = \frac{L}{2} - \frac{M}{wL} = \frac{288 \text{in}}{2} - \frac{3.1 \text{kip-in}}{0.416 \text{kip/ft}(24 \text{ft})} = 143.7 \text{in}
\]

Maximum moment, \( M_u \)

\[
M_u = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2}
= \frac{3.1 \text{kip-in}}{2} + \frac{0.0347 \text{kip/in}(288 \text{in})^2}{8} + \frac{(3.1 \text{kip-in})^2}{2(0.0347 \text{kip/in})(288 \text{in})^2} = 361.0 \text{kip-in}
\]

Axial force, \( P_u \)

\[
P_u = 0.54k + 0.9(0.20 \text{ kip/ft})(143.7 \text{in})\text{ft}/12 \text{in} = 2.69k
\]

Design for \( P_u = 2.7 \text{ kips} \), \( M_u = 361 \text{ kip-in} \)
Example: Pilaster Design

### 0.9D + 1.0W
- At top: \( P_u = 0.5 \text{ k} \) \( M_u = 3 \text{ k-in} \)
- \( x = 144 \text{ in} \) \( P_u = 2.7 \text{ k} \) \( M_u = 361 \text{ k-in} \)
- \( a = 1.49 \text{ in} \) \( A_s = 0.57 \text{ in}^2 \)

### 1.2D + 1.0W + 0.5S
- At top: \( P_u = 8.2 \text{ k} \) \( M_u = 48 \text{ k-in} \)
- \( x = 139 \text{ in} \) \( P_u = 11.0 \text{ k} \) \( M_u = 384 \text{ k-in} \)
- \( a = 1.74 \text{ in} \) \( A_s = 0.52 \text{ in}^2 \)

### 1.2D + 1.6S + 0.5W
- At top: \( P_u = 22.8 \text{ k} \) \( M_u = 132 \text{ k-in} \)
- \( x = 117 \text{ in} \) \( P_u = 25.2 \text{ k} \) \( M_u = 252 \text{ k-in} \)
- \( a = 1.41 \text{ in} \) \( A_s = 0.12 \text{ in}^2 \)

Required steel = 0.57 \text{ in}^2
Use 2-#5 each face, \( A_s = 0.62 \text{ in}^2 \)
Total bars, 4-#5, one in each cell

Combined Flexural and Axial Loads 75

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Example: Pilaster Design

![Design Strength Graph](graph.png)

- Factored Loads
- Balanced Point

Combined Flexural and Axial Loads 76
Columns

- Structural member, not built integrally into a wall, designed primarily to resist compressive loads parallel to its longitudinal axis.
- Minimum side dimension is 8 in. (5.3.1.1(b))
- Distance between lateral supports ≤ 99r \{h/r ≤ 99\} (5.3.1.1(a))
- Minimum reinforcement is 0.0025A_n (5.3.1.3)
- Maximum reinforcement is 0.04A_n (5.3.1.3)
  - Additional maximum reinforcement requirements in strength design
- Minimum of 4 bars (5.3.1.3)
- Fully grouted (5.3.1.2)

**Ties:** (5.3.1.4)
- ≥ 1/4 in. diameter; located in mortar joint or grout
- spacing ≤ 16 longitudinal bar diameter, 48 tie diameter, or least cross-sectional dimension

Bearing Walls

**Location of Reaction:**
Wall section

Members that rotate will cause reaction to shift towards edge

Members that experience little rotation (deep truss)

**Bearing area (4.3.4):**

\[ A_1 \sqrt{\frac{A_2}{A_1}} \leq 2A_1 \]

Wall section

A2 ends at edge of member or head joint in stack bond

Plan view

**Strength Design:**

\[ \varphi = 0.6 \ (9.1.4.2) \]

\[ B_n = 0.8f_m A_{br} \ (9.1.8) \]
Bearing Walls

Distribution of Concentrated Loads Along Wall:  (5.1.3)
Load is dispersed along a 2 vertical: 1 horizontal line.

(a) Distribution of concentrated load through bond beam

If the spacing of the load is less than half the wall height, then the load acts like a distributed load.

(b) Distribution of concentrated load in wall
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Combined Flexural and Axial Loads

Load indicating washer (LIW)

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Combined Flexural and Axial Loads