Shear Walls

- Load Distribution to Shear Walls
  - Shear wall stiffness
  - Shear walls with openings
  - Diaphragm types
- Types of Masonry Shear Walls
- Maximum Reinforcement Requirements
- Shear Strength
- Example: simple building

Shear Walls: Stiffness

- Lateral Force Resisting System

<table>
<thead>
<tr>
<th>Condition</th>
<th>Stiffness Predominates</th>
</tr>
</thead>
<tbody>
<tr>
<td>h/d &lt; 0.25</td>
<td>Both shear and bending stiffness are important</td>
</tr>
<tr>
<td>0.25 &lt; h/d &lt; 4.0</td>
<td></td>
</tr>
<tr>
<td>h/d &gt; 4.0</td>
<td></td>
</tr>
</tbody>
</table>

(d and h as shown in diagram)
Shear Wall Stiffness

9.1.5.2 Deflection calculations shall be based on cracked section properties. Assumed properties shall not exceed half of gross section properties, unless a cracked-section analysis is performed.

Cantilever wall

\[ k_{\text{cant}} = \frac{E_m t}{\left(\frac{h}{L}\right) \left(\frac{h}{L}\right)^2 + 3} \]

\[ \Delta_{\text{cant}} = \]

Fixed wall (fixed against rotation at top)

\[ k_{\text{fixed}} = \frac{E_m t}{\left(\frac{h}{L}\right) \left(\frac{h}{L}\right)^2 + 3} \]

\[ \Delta_{\text{fixed}} = \]

Real wall is probably between two cases; diaphragm provides some rotational restraint, but not full fixity.

T- or L- Shaped Shear Walls

Section 5.1.1 Wall intersections designed either to:

a)___________________:

b)___________________:

Connection that transfers shear: (must be in running bond)

a) Fifty percent of masonry units interlock

b) Steel connectors at max 4ft.

c) Intersecting bond beams at max 4 ft. Reinforcing of at least 0.1in² per foot of wall
Effective Flange Width (5.1.1.2.3)

Effective flange width on either side of web shall be smaller of actual flange width, distance to a movement joint, or:

- Flange in compression: \(6t\)
- Flange in tension:
  - Unreinforced masonry: \(6t\)
  - Reinforced masonry: 0.75 times floor-to-floor wall height

Example: Flanged Shear Wall

**Given:** Fully grouted shear wall

**Required:** Stiffness of wall

**Solution:** Determine stiffness from basic principles.

\[
A = \text{Find centroid from outer flange surface}
\]

\[
\bar{y} = \frac{7.62in(48in)(3.81in) + 7.62in(40in)(20in)}{671in^2} = 11.16in
\]

\[
I = \text{Elevation}
\]

\[
A_c \approx \text{Plan}
\]
**Example: Flanged Shear Wall**

\[ k = \frac{P}{\Delta} = \frac{P}{\frac{Ph^3}{3E_mI_n} + \frac{Ph}{A_vE_v}} = \frac{1}{\frac{(112in)^3}{3E_m(86000in^4)} + \frac{112in}{305in^2(0.4E_m)}} = (0.157in)E_m \]

**Shear Walls with Openings**

1. Divide wall into piers.
2. Find flexibility of each pier
3. Stiffness is reciprocal of flexibility
4. Distribute load according to stiffness

\[ f = \frac{h^3}{6EI} \left( \frac{2k_i + k_b + 2k_b + 3}{k_i + 2k_b + k_b} \right) + \frac{12h}{5EA} (1 + \nu) \]

\[ k_i = \frac{h}{h_i} \quad k_b = \frac{h}{h_b} \]

\[ M_i = Vh \left( \frac{k_i(1 + k_i)}{k_i + 2k_b} \right) \]

\[ M_b = Vh \left( \frac{k_b(1 + k_i)}{k_i + 2k_b} \right) \]

As the top spandrel decreases in height, the top approaches a fixed condition against rotation.

If \( h_b = 0 \)

\[ f = \frac{h^3}{6EI} \left( \frac{2 + k_i}{2k_i + 1} \right) + \frac{12h}{5EA} (1 + \nu) \]

\[ M_i = Vh \left( \frac{k_i}{2k_i + 1} \right) \quad M_b = Vh \left( \frac{1 + k_i}{2k_i + 1} \right) \]

**Example - Shear Walls with Openings**

**Given:**
- Shear Walls

**Required:**
A. Stiffness of wall
B. Forces in each pier under 10 kip horizontal load

---

**Sample calculations:** Pier B

- \( h = 4 \text{ft} \)
- \( h_i = 4 \text{ft} \)
- \( h_b = 8 \text{ft} \)
- \( k_i = h/h_i = 4\text{ft}/4\text{ft} = 0.0 \)
- \( k_b = h/h_b = 4\text{ft}/8\text{ft} = 0.5 \)

\[
I = \frac{tL^2}{12} \quad A = tL = (2 \text{ft})t
\]

\[
f = \frac{h^3}{6EI} \left( \frac{2k_i + k_b + 2k_b + 3}{k_i + 2k_b} \right) + \frac{12h}{5EA} (1 + \nu)
\]

\[
k = \frac{1}{f} = \frac{1}{47.6 \times Et} = 0.0210 Et
\]
### Example - Shear Walls with Openings

<table>
<thead>
<tr>
<th>Pier</th>
<th>h (ft)</th>
<th>h₁ (ft)</th>
<th>h₀ (ft)</th>
<th>kᵢ</th>
<th>k₀</th>
<th>l (ft³)</th>
<th>A (ft)</th>
<th>Δᵢ</th>
<th>Δₛ</th>
<th>f</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>4</td>
<td>0</td>
<td>3</td>
<td>inf</td>
<td>0.667t</td>
<td>2t</td>
<td>308.4</td>
<td>18</td>
<td>326.4/Et</td>
<td>0.0031Et</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>0.5</td>
<td>0.667t</td>
<td>2t</td>
<td>41.6</td>
<td>6</td>
<td>47.6/Et</td>
<td>0.0210Et</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>0.5</td>
<td>0.667t</td>
<td>2t</td>
<td>41.6</td>
<td>6</td>
<td>47.6/Et</td>
<td>0.0210Et</td>
</tr>
</tbody>
</table>

Δᵢ is flexural deformation; Δₛ is shear deformation

Total stiffness is 0.0451Et
Average stiffness from finite element analysis 0.0440Et
Solid wall stiffness 0.1428Et (free at top); 0.25Et (fixed at top)

---

### Sample calculations: Pier B

\[
V_b = \frac{k_b}{\sum k} = \frac{0.667t}{2t} = 0.3337\text{Et} \approx 0.0210Et
\]

\[
10k = 4.66\text{kips}
\]

\[
M_i = Vh\left(\frac{k_i(1+k_b)}{k_i + 2k_i k_b + k_b}\right) = 3.50\text{ (k-ft)}
\]

\[
M_b = Vh\left(\frac{k_b(1+k_i)}{k_i + 2k_i k_b + k_b}\right) = 4.66\text{ (k-ft)}
\]

---

<table>
<thead>
<tr>
<th>Pier</th>
<th>V (kips)</th>
<th>Mᵢ (k-ft)</th>
<th>Mₜ (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.68</td>
<td>3.50</td>
<td>4.66</td>
</tr>
<tr>
<td>B</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
<tr>
<td>C</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
</tbody>
</table>
Example - Shear Walls with Openings

To find axial force in piers, look at FBD of top 4ft of wall.

This 65.9k-ft moment is carried by axial forces in the piers. Axial forces are determined from \( \frac{Mcl}{I} \), where each pier is considered a concentrated area. Find centroid from left end.

\[
\bar{y} = \frac{\sum y_i A_i}{\sum A_i}
\]

The axial force in pier A is:

FBD's of piers:
Example - Shear Walls with Openings

FBD’s of entire wall to obtain forces in bottom right spandrel:

\[ \sum F_x = 10k - 0.68k - F_x = 0 \]
\[ F_x = 9.32k \]

\[ \sum F_y = -4.45k + F_y = 0 \]
\[ F_y = 4.45k \]

\[ \sum M_z = -10k(16\text{ ft}) + (4.66k - ft) - 4.45k(1\text{ ft}) + 4.45k(11\text{ ft}) + M_z = 0 \]
\[ M_z = 110.8k - ft \]

Shear Walls: Building Layout

1. All elements either need to be isolated, or will participate in carrying the load
2. Elements that participate in carrying the load need to be properly detailed for seismic requirements
3. Most shear walls will have openings
4. Can design only a portion to carry shear load, but need to detail rest of structure
Diaphragms

- **Diaphragm**: __________ system that transmits ______________ forces to the vertical elements of the lateral load resisting system.
- **Diaphragm classification**:
  - ______________: distribution of shear force is based on tributary ______ (wind) or tributary ______ (earthquake)
  - ______________: distribution of shear force is based on relative ______________.

Typical classifications:
- __________: Precast planks without topping, metal deck without concrete, plywood sheathing
- __________: Cast-in-place concrete, precast concrete with concrete topping, metal deck with concrete
Rigid Diaphragms

Direct Shear:

\[ F_v = V \sum \frac{R R_i}{R R_i} \]

Torsional Shear:

\[ F_{tt} = V e \sum \frac{R R_i d_i}{R R_i d_i^2} \]

\( V = \) total shear force  
\( R R_i = \) relative rigidity of lateral force resisting element \( i \)  
\( d_i = \) distance from center of stiffness  
\( e = \) eccentricity of load from center of stiffness

Diaphragms: Example

Given: The structure shown is subjected to a 0.2 kip/ft horizontal force. Relative rigidities are given, where the relative rigidity is a normalized stiffness. 
Required: Distribution of force assuming:  
- flexible diaphragm  
- rigid diaphragm.
Flexible Diaphragms: Example

Solution: Flexible diaphragm – wind
Distribute based on tributary area

For seismic, the diaphragm load would be distributed the same (assuming a uniform mass distribution), but when wall weights were added in, the forces could be different.

Rigid Diaphragms: Example

Solution: Rigid diaphragm

Center of stiffness = 350/10 = 35 ft

<table>
<thead>
<tr>
<th>Wall</th>
<th>RR</th>
<th>d (ft)</th>
<th>RR(d)</th>
<th>RR(d^2)</th>
<th>F_v</th>
<th>F_t</th>
<th>F_total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>-35</td>
<td>-140</td>
<td>4900</td>
<td>8</td>
<td>-4.1</td>
<td>3.9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>15</td>
<td>75</td>
<td>1125</td>
<td>10</td>
<td>2.2</td>
<td>12.2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>65</td>
<td>65</td>
<td>4225</td>
<td>2</td>
<td>1.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>350</td>
<td>10250</td>
<td>20</td>
<td>0.0</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>
Diaphragms Design Forces

Solution: Forces are shown for a rigid diaphragm.

The moment is generally taken through chord forces, which are simply the moment divided by the width of the diaphragm. In masonry structures, the chord forces are often taken by bond beams.

Drag Struts and Collectors

- Shear forces: generally considered to be uniformly distributed across the width of the diaphragm.
- Drag struts and collectors: transfer load from the diaphragm to the lateral force resisting system.
Diaphragm Behavior

Three lateral force resisting systems: Length=20 ft; Height=14 ft

- Moment Resisting Frame
  - $k = 83.2 \text{ kip/in}$

- Braced Frame
  - $k = 296 \text{ kip/in}$

- Masonry Shear Wall
  - $E = 1800 \text{ ksi}$
  - Face shell bedding
  - End cells fully grouted
  - $k = 1470 \text{ kip/in}$

Lateral force resisting systems at 24 ft o.c. Diaphragm assumed to be concrete slab, $E=3120\text{ksi}$, $v=0.17$, variable thickness, load of 1 kip/ft.

Diaphragm

24 ft

Lateral Force Resisting System

Center Reaction (kips) vs. Diaphragm Thickness (inch) graph

- Moment Frame
- Braced Frame
- Shear Wall
- Masonry Shear Wall
- Flexible Diaphragm
- Rigid Diaphragm
Diaphragm Behavior

Is any of the following true?

1. 1- & 2-family dwelling
   Light-frame construction where:
   1. Topping of concrete or similar materials is not placed over wood structural panel diaphragms except for nonstructural topping no greater than 1 ½” and
   2. Each line of vertical elements of the seismic force-resisting system complies with the allowable story drift of Table 12.12-1.

Vertical elements one of the following:
* Steel braced frames
* Composite steel and concrete braced frames
* Concrete, masonry, steel or composite shear walls

START

Is diaphragm wood structural panels or untopped steel decking?
N
Y

See Figure 2

Assume Flexible
Assume Rigid

Is diaphragm:
* Concrete slab?
* Concrete filled metal deck?
N
Y

Is span-to-depth ratio ≤ 3 and no horizontal irregularities?
N
Y

Figure 2


Shear Walls 34

Diaphragm Behavior

Is MDD > 2 (ADVE)?

MAXIMUM DIAPHRAGM DEFLECTION (MDD)
AVERAGE DRIFT OF VERTICAL ELEMENT (ADVE)

Assume Flexible Per ASCE 7-10 Section 12.3.1.3
Assume Rigid per 2012 IBC Section 202

Shear Walls 35
Shear Walls

___________ (unreinforced) shear wall (7.3.2.2): Unreinforced wall

___________ (unreinforced) shear wall (7.3.2.3): Unreinforced wall with prescriptive reinforcement.

Reinforcement not required at openings smaller than 16 in. in either vertical or horizontal direction

Reinforcement of at least 0.2 in$^2$

Shear Walls

___________ reinforced shear wall (7.3.2.4): Reinforced wall with prescriptive reinforcement of detailed plain shear wall.

___________ reinforced shear wall (7.3.2.5): Reinforced wall with prescriptive reinforcement of detailed plain shear wall. Spacing of vertical reinforcement reduced to 48 inches.

___________ reinforced shear wall (7.3.2.6):
1. Maximum spacing of vertical and horizontal reinforcement is min{1/3 length of wall, 1/3 height of wall, 48 in. [24 in. for masonry in other than running bond]}. 
2. Minimum area of vertical reinforcement is 1/3 area of shear reinforcement
3. Shear reinforcement anchored around vertical reinforcing with standard hook
4. Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
5. Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall [0.0015 for horizontal reinforcement for masonry in other than running bond].

Shear Walls
## Minimum Reinforcement of Special Shear Walls

<table>
<thead>
<tr>
<th>Reinforcement Ratio</th>
<th>( A_s ) (in²/ft)</th>
<th>Possibilities</th>
<th>( A_s ) (in²/ft)</th>
<th>Possibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0007</td>
<td>0.064</td>
<td>#4@32, #5@56</td>
<td>0.098</td>
<td>#4@24, #5@32, #6@48</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.092</td>
<td>#4@24, #5@32, #6@40</td>
<td>0.140</td>
<td>#4@16, #5@24, #6@32</td>
</tr>
<tr>
<td>0.0013</td>
<td>0.119</td>
<td>#4@16, #5@32, #6@40</td>
<td>0.181</td>
<td>#5@16, #6@24</td>
</tr>
</tbody>
</table>

Use specified dimensions, e.g. 7.625 in. for 8 in. CMU walls.

## Special Shear Walls: Shear Capacity

Minimum shear strength (7.3.2.6.1.1):

- Design shear strength, \( \phi V_n \), greater than shear corresponding to 1.25 times nominal flexural strength, \( M_n \)
- Except \( V_n \) need not be greater than 2.5\( V_u \).

Normal design: \( \phi V_n \) has to be greater than \( V_u \). Thus, \( V_n \) has to be greater than \( V_u / \phi = V_u / 0.8 = 1.25 V_u \). Thus, the requirement approximately doubles the shear.
Seismic Design Category

<table>
<thead>
<tr>
<th>Seismic Design Category</th>
<th>Allowed Shear Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>A or B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D and higher</td>
<td></td>
</tr>
</tbody>
</table>

Response modification factor:
Seismic design force divided by response modification factor, which accounts for ductility and energy absorption.

Knoxville, Tennessee
Category C for Use Group I and II
Category D for Use Group III (essential facilities

<table>
<thead>
<tr>
<th>Shear Wall</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary plain</td>
<td>1.5</td>
</tr>
<tr>
<td>Detailed plain</td>
<td>2</td>
</tr>
<tr>
<td>Ordinary reinforced</td>
<td>2</td>
</tr>
<tr>
<td>Intermediate reinforced</td>
<td>3.5</td>
</tr>
<tr>
<td>Special reinforced</td>
<td>5</td>
</tr>
</tbody>
</table>

Maximum reinforcing (9.3.3.5)

No limits on maximum reinforcing for following case (9.3.3.5.4):

\[ \frac{M_u}{V_u d_v} \leq 1 \quad \text{and} \quad R \leq 1.5 \]

Squat walls, not designed for ductility

In other cases, can design by either providing boundary elements or limiting reinforcement.

Boundary element design (9.3.6.5):
More difficult with masonry than concrete

Boundary elements not required if:

\[ P_u \leq 0.1 f'_m A_g \quad \text{geometrically symmetrical sections} \]

\[ P_u \leq 0.05 f'_m A_g \quad \text{geometrically unsymmetrical sections} \]

AND

\[ \frac{M_u}{V_u l_w} \leq 1 \quad \text{OR} \quad V_u \leq 3 A_n \sqrt{f'_m} \quad \text{AND} \quad \frac{M_u}{V_u l_w} \leq 3 \]
Maximum reinforcing (9.3.3.5)

Reinforcement limits: Calculated using

Maximum stress in steel of \( f_y \)
Axial forces taken from load combination \( D + 0.75L + 0.525Q_E \)
Compression reinforcement, with or without lateral ties, permitted to be included for calculation of maximum flexural tensile reinforcement

Uniformly distributed reinforcement

\[
A_s = \frac{0.64 f'_m b}{d_v} \left( \frac{\varepsilon_{mu} - \alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P}{d_v}
\]

\( \alpha = 1.5 \) ordinary walls
\( \alpha = 3 \) intermediate walls
\( \alpha = 4 \) special walls

Compression steel with area equal to tension steel

\[
\rho = \frac{A_s}{bd} = \frac{0.64 f'_m b}{f_y - \min \left\{ \frac{d}{d'} \left( \frac{\varepsilon_{mu} + \alpha \varepsilon_y}{\varepsilon_{mu}} \right) \right\}} - \frac{P}{bd}
\]

Shear Walls

Maximum reinforcing

Consider a wall with uniformly distributed steel:

\[
C_m + C_s - T_s = P
\]

\[
C_m = 0.8 f'_m \left( 0.8 - \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) b
\]

\[
T_s = f_y A_s \left( \frac{\alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \frac{\varepsilon_{mu} - \varepsilon_y}{\varepsilon_{mu}} + \frac{1}{2 \alpha \varepsilon_y}
\]

\[
C_s = f_y A_s \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left[ \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu}} \right]
\]

\[
= f_y A_s \left( \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)
\]

\( A_s \) taken as total steel
\( d_s \) is actual depth of masonry
Maximum reinforcing

\[ T_s - C_s = C_m - P \]

\[ T_s - C_s = f_y A \left( \frac{\alpha \varepsilon_y - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - f_y A \left( \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu} + \varepsilon_y} \right) = f_y A \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \]

\[ T_s - C_s = f_y A \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) = 0.8 f_m' \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} d_y \right) b - P = C_m - P \]

\[ A_s = \frac{0.64 b f_m' \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)}{d_y} \frac{P}{d_y} \]

Maximum reinforcing, \( \varepsilon_s = 4 \varepsilon_y \)
Maximum reinforcing, $\varepsilon_s = 3\varepsilon_y$

**Graph 1:**
- Reinforcement ratio, $\rho$ vs. $[P/(bd)]/f_m$
- Graphs for Clay $f_m=2\text{ksi}$, CMU $f_m=2\text{ksi}$, CMU $f_m=2.5\text{ksi}$, and Clay $f_m=4\text{ksi}$.

Maximum reinforcing, $\varepsilon_s = 1.5\varepsilon_y$

**Graph 2:**
- Reinforcement ratio, $\rho$ vs. $[P/(bd)]/f_m$
- Graphs for CMU $f_m=2\text{ksi}$ and CMU $f_m=2.5\text{ksi}$.
**Shear Strength (9.3.4.1.2)**

\[ V_n = (V_{im} + V_{ns})\gamma_g \]
\[ \phi = 0.8 \]
\[ \gamma_g = 0.75 \text{ for partially grouted shear walls and 1.0 otherwise} \]

\[ V_m = \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_{nv} \sqrt{f_m'} + 0.25P_u \]
\[ \text{M}_u/V_u d_v \text{ need not be taken > 1.0} \]
\[ P_u = \text{axial load} \]

\[ V_s = 0.5 \left( \frac{A_v}{s} \right) f_y d_v \]
Vertical reinforcement shall not be less than one-third horizontal reinforcement; reinforcement shall be uniformly distributed, max spacing of 8 ft (9.3.6.2)

Maximum \( V_n \) is:
\[ V_n \leq \left( 6A_{nv} \sqrt{f_m'} \right) \gamma_g \]
\[ (M_u/V_v d_v) \leq 0.25 \]
\[ V_n \leq \left( 4A_{nv} \sqrt{f_m'} \right) \gamma_g \]
\[ (M_u/V_v d_v) \geq 1.0 \]

Interpolate for values of \( M_u/V_v d_v \) between 0.25 and 1.0
\[ V_n = \left( \frac{4}{3} \left( 5 - 2 \frac{M_u}{V_v d_v} \right) A_{nv} \sqrt{f_m'} \right) \gamma_g \]

**Partially Grouted Walls**

<table>
<thead>
<tr>
<th>Method</th>
<th>( V_{exp} / V_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Partially Grouted Walls (Minaie et al, 2010; 60 tests)</td>
<td></td>
</tr>
<tr>
<td>2008 Provisions</td>
<td>0.90</td>
</tr>
<tr>
<td>Multiply shear strengths by ( A_g/A_n )</td>
<td>1.53</td>
</tr>
<tr>
<td>Using just face shells</td>
<td>1.77</td>
</tr>
<tr>
<td>Fully Grouted Walls (Davis et al, 2010; 56 tests)</td>
<td></td>
</tr>
<tr>
<td>2008 Provisions</td>
<td>1.16</td>
</tr>
</tbody>
</table>

0.90/1.16 = 0.776; rounded to 0.75
Design of Shear Walls with Single Layer of Reinforcement

1. Determine $a$, depth of compressive stress block

\[
a = d - \sqrt{d^2 - \frac{2P_u(d - l_w/2) + M_u}{\phi(0.8 f'_m t_{sp})}}
\]

2. Solve for $A_s$

\[
A_s = \frac{0.8 f'_m t_{sp} a - P_u / \phi}{f_y}
\]

3. Check axial capacity
4. Check maximum reinforcement
5. Check shear

Example

Given: 2 ft long, 8 ft high CMU pier; Type S masonry cement mortar; Grade 60 steel; fully grouted. $P_u = 11$ kips, $V_u = 7$ kips, $M_u = 28$ k-ft

Required: Required amount of steel

Solution: Choose/determine material properties.

$f'_m = 2000$ psi; $f_y = 60,000$ psi

\[
a = d - \sqrt{d^2 - \frac{2P_u(d - l_w/2) + M_u}{\phi(0.8 f'_m t_{sp})}}
\]

\[
A_s = \frac{0.8 f'_m t_{sp} a - P_u / \phi}{f_y}
\]
Example

Try #4 bars
Consider second layer of steel:

\[ P_n = C_m - T_1 - T_2 \]

Solve for \( c \) such that \( P_n = 11 \text{kips}/0.9 = 12.2 \text{kips} \)
\( c = 2.986 \text{ inches} ; \ C_m = 29.14 \text{kips} ; \ T = 12 \text{kips} ; \ T_1 = 4.92 \text{kips} \)

\[ \phi M_n = \]

\[ = 334 \text{k} - \text{in} = 27.9 \text{k} - \text{ft} \]

3. Check axial load

\[ r = \sqrt{\frac{L}{A}} = \sqrt{\frac{11 \cdot 24^3}{20^2}} = 0.289t = 0.289(7.625\text{in.}) = 2.20\text{in} \]
\[ \frac{h}{r} = \frac{8\text{ft}(12\text{in})}{2.20\text{in.}} = 43.6 < 99 \]

\[ P_n = 0.80\left[0.80f_m'(A_n - A_{st}) + f_yA_{st}\left[1 - \left(\frac{h}{140r}\right)^2\right]\right] \]

\[ = 0.80\left[0.80(2.0\text{ksi})(7.62\text{in.}(24\text{in.}) - 0) + 60\text{ksi}(0)\left[1 - \left(\frac{43.6}{140}\right)^2\right]\right] \]

\[ = 234.2\text{k}(0.903) = 211.5\text{k} \]

\[ \phi P_n = 0.9(211.5\text{k}) = 190.3\text{k} \]

11k < 190k \hspace{1cm} \text{OK}

Applied load is 6% of axial capacity
Example

4. Check maximum reinforcement:

\[ \rho = \frac{A_s}{bd} = \frac{0.64 f'_m}{\left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}} \]

Fully grouted with equal tension and compression reinforcement

Assume axial force is from 0.9D, so \( P_u \) for maximum reinforcement is 11k/0.9 = 12.2 kips. For an ordinary wall:

\[ \rho = \frac{0.64 (2.0 \text{ksi}) \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) - \frac{12.2k}{7.62\text{in.}(20\text{in.})}}{60 \text{ksi} - \min \left\{ \frac{4\text{in.}}{20\text{in.}} \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) \right\} 29000 \text{ksi} } = 0.0245 \]

\[ A_{s,\text{max}} = \rho bd = 0.0245(7.62\text{in.})(20\text{in.}) = 3.74\text{in}^2 \]

<table>
<thead>
<tr>
<th>Axial Force, ( P_u )</th>
<th>( A_{s,\text{reqd}} )</th>
<th>Maximum Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 kips</td>
<td>0.32 in(^2)</td>
<td>4.35 in(^2), 1.47 in(^2), 0.91 in(^2)</td>
</tr>
<tr>
<td>11 kips (6% of axial capacity)</td>
<td>0.21 in(^2)</td>
<td>3.74 in(^2), 1.15 in(^2), 0.66 in(^2)</td>
</tr>
<tr>
<td>19 kips (10% of axial capacity)</td>
<td>0.13 in(^2)</td>
<td>3.30 in(^2), 0.92 in(^2), 0.48 in(^2)</td>
</tr>
<tr>
<td>38 kips (20% of axial capacity)</td>
<td>0</td>
<td>2.24 in(^2), 0.36 in(^2), 0.06 in(^2)</td>
</tr>
</tbody>
</table>
Example

5. Check Shear:

\[
\frac{M_u}{V_u d_y} = \frac{28k - ft}{7k(2 ft)} = 2 \quad \text{Use } M_d/(V_u d_y) = 1.0
\]

\[
V_{nm} = \left(4.0 - 1.75 \left(\frac{M}{V u d_y}\right)\right) A_{nv} \sqrt{f'_m} + 0.25P_u
\]

\[
= (4.0 - 1.75(1.0))(7.62\text{in.}(24\text{in.}))\sqrt{2000 \text{psi}} \frac{1\text{ksi}}{1000 \text{psi}} + 0.25(11k)
\]

\[
= 18.4k + 2.8k = 21.2k
\]

\[
\phi V_{nm} = 0.8(21.2k) = 16.9k \quad > \quad V_u = 7k \quad \text{OK}
\]

Example

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, \( f'_m = 2000 \text{psi} \); 2-#5 each end; #5 at 48 in. (one space will actually be 40 in.)

Required: Interaction diagram

Solution: Illustrate with one point, \( c = 54 \text{ inches} \), \( a = 0.8(54) = 43.2 \text{ in.} \)
Example

\[ \phi P_n = \]

Sum moments about middle of wall (8 ft from end) to find \( \phi M_n \)

\[ \phi M_n = \]
Multiple Layers of Reinforcement:

- Design equations for single layer of reinforcement can be used for preliminary steel estimates
- Spacing of intermediate reinforcing bars often controlled by out-of-plane loading
- Shear area for a partially grouted shear wall is area of face shells and grouted cells

Example

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; $f'_{m} = 2000$psi; superimposed dead load of 1 kip/ft. In-plane seismic load (from ASCE 7-10) of 90 kips. $S_{DS} = 0.4$

Required: Design the shear wall; ordinary reinforced shear wall

Solution: Check using 0.9D+1.0E load combination.
- Need to know weight of wall to determine $P_u$.
- Need to know reinforcement spacing to determine wall weight
- Estimate $P_u$ by ignoring wall weight and seismic vertical force
  - $P_u = 0.9(1\text{kip/ft})(16\text{ft}) = 14.4\text{k}$
  - $M_u = (90\text{k})(10\text{ft}) = 900\text{k-ft}$
Example

Estimate required reinforcement:

1. Use equations for single layer of reinforcement (d ~ d_v - 4 = 188 in.):
   - a = 6.0 in., A_s = 0.95 in^2 This is about 3.1 #5 bars; try 2-#5 at end and #5 @ 64 in.

2. Use spreadsheet/computer program and interaction diagram:

   ![Interaction Diagram]

   Overstressed by 3.3%; may work

Example

Weight of wall: 36psf(10ft)(16ft) = 5760 lb
Lightweight units, grout at 64 in. o.c. 36 psf (estimated)

\[ P_u = [0.9 - 0.2(S_{DS})]D = [0.9 - 0.2(0.4)](1k/ft(16ft)+5.8k) = 17.8 \text{ kips} \]

With slightly higher axial force

\[ M_n = 995 \text{ k-ft} \]

\[ \phi M_n \] has increased to 896 k-ft

Wall weight may be slightly greater due to bond beams
Example

Calculate net area, $A_{nv}$, including grouted cells.

$$A_{nv} = \frac{M_u}{(V_u d_v)}$$

Maximum $V_n$ \( \phi V_n = \)

---

Example

Top of wall (critical location): \( P_u = (0.9 - 0.2S_{DS})D = 0.82(1\text{k/ft})(16\text{ft}) = 13.1\text{kips}\)

$$\phi V_{nm} = \phi \left[ \left( 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right) A_{nv} \sqrt{f_m} + 0.25 P_u \right]$$

$$V_n = \frac{V_u}{\phi} = (V_{nm} + V_{ns})/\gamma_g \quad \Rightarrow \quad V_{ns} = \$$

Use #5 bars in bond beams. Determine spacing.

$$V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_y d_v \quad \Rightarrow \quad s =$$

ASD: \( s \leq \min(d/2, 48\text{ in.}) = \min(94\text{ in.}, 48\text{ in.}) = 48\text{ in.} \) Code 8.3.5.2.1

In strength design, this provision only applies to beams (9.3.4.2.3 (e))

Suggest that minimum spacing also be applied to shear walls.

Use #5 bars at 16 inches
Joint Reinforcement

- **9.1.9.3.2** Maximum specified yield strength of 85,000 psi
- **9.3.3.1** (b) Minimum diameter of 3/16 in. diameter.
- **9.3.3.2.3** Anchor around edge reinforcing bar in the edge cell
  - by bar placement between adjacent crosswires, or
  - $90^\circ$ bend in longitudinal wires with 3-in. bend extensions in mortar or grout.
- **9.3.3.7** Seismic Requirements
  - Seismic Design Categories (SDC) A and B
    - At least two 3/16 in. wires; Maximum spacing of 16 in.
  - SDC C, D, E, and F; partially grouted walls
    - At least two 3/16 in. wires; Maximum spacing of 8 in.
  - SDC C, D, E, and F; fully grouted walls
    - At least four 3/16 in. wires; Maximum spacing of 8 in.

---

**Joint Reinforcement**

Equivalent Joint Reinforcement Options:

<table>
<thead>
<tr>
<th>Joint Reinforcement</th>
<th>Equivalent Bar Reinforcement</th>
<th>Replaces this Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 3/16 in. wires at 16 in.</td>
<td>0.0472 in$^2$/ft</td>
<td>#4 @ 56 in.; #5 @ 80 in.</td>
</tr>
<tr>
<td>2 – 3/16 in. wires at 8 in.</td>
<td>0.0945 in$^2$/ft</td>
<td>#4 @ 32 in.; #5 @ 40 in.</td>
</tr>
<tr>
<td>4 – 3/16 in. wires at 8 in.</td>
<td>0.189 in$^2$/ft</td>
<td>#4 @ 16 in.; #5 @ 24 in.</td>
</tr>
</tbody>
</table>

Bar reinforcement yield stress = 60 ksi
Joint reinforcement yield stress = 70 ksi

- Instead of bond beams in example, could use 4 – 3/16 in. wires at 8 in.
  - but no manufacturer makes this
- Smaller distributed reinforcement generally results in better behavior.
Example: Special Wall

**Given:** 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; $f'_m=2000$psi; superimposed dead load of 1 kip/ft. In-plane seismic load (from ASCE 7-10) of 90 kips. $S_{DS} = 0.4$

**Required:** Design the shear wall; special reinforced shear wall

**Solution:** Check using 0.9D+1.0E load combination.

- Shear capacity design provisions (Section 7.3.2.6.1.1)
  - $\phi V_n \geq$ shear corresponding to $1.25M_n$.
    - Minimum increase is $1.25/0.9 = 1.39$
    - $M_n = 995$ k-ft; $1.25M_n = 1243$ k-ft; design for shear of 124 kips
  - $V_n$ need not exceed $2.5V_u$
    - Normal design $V_n \geq V_d/\phi = V_d/0.8 = 1.25V_u$
    - Increases shear by a factor of 2
  - **Fully grout wall** (Max $\phi V_n$ was 91.9 kips)
Example: Special Wall

- Design for shear of 124 kips
- But wait, spacing of #5’s was decreased to 48 inches
  - \( M_n = 1124 \text{ k-ft}, 1.25M_n = 1405 \text{ k-ft}, \) Design for 140 kips
- But wait, wall is fully grouted. Wall weight has increased to 75 psf
  - For \( P_u = 23.0k, \) fully grouted, \( M_n = 1167 \text{ k-ft}, 1.25M_n = 1459 \text{ k-ft} \)
- Design for 146 kips
- But wait, Section 7.3.2.6 has maximum spacing requirements:
  - \( \frac{1}{3} \) length of wall = (192 in.)/3 = 64 in.
  - \( \frac{1}{3} \) height of wall = (120 in.)/3 = 40 in.
  - 48 in. for running bond; 24 in. for not laid in running bond
- Decrease spacing of vertical reinforcement from 48 in. to 40 in.
- For 2-#5 at end, and #5 @ 40in., \( M_n = 1269 \text{ k-ft} \) \( (P_u = 23.0k) \)
  - \( 1.25M_n = 1586 \text{ k-ft}; \) Design for 159 kips
- Bottom line: any change in wall will change \( M_n, \) which will change design requirement; often easier to just use \( 2.5V_u. \)

\[
\phi V_{nm} = \phi \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) A_{nv} \sqrt{f_m} + 0.25P_u \right] \\
= 0.8 \left[ 4.0 - 1.75(0.625)(1464\text{in.}^2)\sqrt{2000\text{psi}} \frac{1\text{kip}}{1000\text{lb}} + 0.25(13.1k) \right] = 154.8\text{ kips}
\]

\[
V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{158.6k - 154.8k}{0.8} = 4.7k \\
\text{Use #5 bars in bond beams.} \\
\text{Determine spacing.}
\]

\[
V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_y d_v \quad \Rightarrow \quad s = \frac{0.5 A_v f_y d_v}{V_{ns}} = \frac{0.5(0.31\text{in})(60\text{ksi})(192\text{in})}{4.7k} = 383\text{in}
\]

Use maximum spacing of \( \frac{1}{3}\) (height) = 40 in.

Section 7.3.2.6(d): Shear reinforcement shall be anchored around vertical reinforcing bars with a standard hook.
Example: Special Wall

- Prescriptive Reinforcement Requirements (7.3.2.6)
  - 0.0007 in each direction
  - 0.002 total

- Vertical: \(8(0.31\text{in}^2)/1464\text{in}^2 = 0.00169\) (spacing of 40 in.)
- Horizontal: \(3(0.31\text{in}^2)/[120\text{in}(7.625\text{in})] = 0.00102\)
- Total = 0.00169 + 0.00102 = 0.00271 OK

Section 9.3.3.5 Maximum Reinforcement
Since \(M_u/(V_{ud}) < 1\), strain gradient is based on 1.5\(\varepsilon_y\).

<table>
<thead>
<tr>
<th>Strain</th>
<th>(c/d), CMU</th>
<th>(c/d), Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5(\varepsilon_y)</td>
<td>0.446</td>
<td>0.530</td>
</tr>
<tr>
<td>3(\varepsilon_y)</td>
<td>0.287</td>
<td>0.360</td>
</tr>
<tr>
<td>4(\varepsilon_y)</td>
<td>0.232</td>
<td>0.297</td>
</tr>
</tbody>
</table>

\(c = 0.446(188\text{in.}) = 83.8\text{ in.}\)

- Calculate axial force based on \(c = 83.8\text{ in.}\).
- Include compression reinforcement
- \(\phi P_n = 732\text{ kips}\)
- Assume a live load of 1 k/ft
- \(D + 0.75L + 0.525Q_E = (1\text{k/ft} + 0.75(1\text{k/ft})16\text{ft} = 28\text{ kips}\) OK
Example: Special Wall

- Section 9.3.6.5: Maximum reinforcement provisions of 9.3.3.5 do not apply if designed by this section (boundary elements)
- Special boundary elements not required if:
  
  \[
  \begin{align*}
  P_u & \leq 0.1 f'_m A_g & \text{geometrically symmetrical sections} \\
  P'_u & \leq 0.05 f'_m A_g & \text{geometrically unsymmetrical sections}
  \end{align*}
  \]

  AND

  \[\frac{M_u}{V_u d_v} \leq 1 \quad \text{OR} \quad V'_u \leq 3 A_n \sqrt{f'_m} \quad \text{AND} \quad \frac{M_u}{V_u d_v} \leq 3\]

  For our wall, \( M_u / V_u d_v < 1 \)

  \( P_u < 0.1 f'_m A_g = 0.1(2.0\text{ksi})(1464\text{in}^2) = 293 \text{kips} \)

Special Walls: Summary

- Prescriptive Reinforcement Requirements (7.3.2.6)
  - 0.0007 in each direction
  - 0.002 total
- Spacing Requirements (7.3.2.6)
- Shear Capacity Design (Section 7.3.2.6.1.1)
  - \( \phi V_n \geq \) shear corresponding to \( 1.25 M_n \)
  - \( V_n \) need not exceed \( 2.5 V_u \)
- Maximum Reinforcement Requirements (9.3.3.5; 9.3.6.5)
Example: T-Wall

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; $f_m' = 2000$ psi; superimposed dead load of 1 kip/ft. In-plane seismic load (from ASCE 7-10) of 90 kips. $S_{DS} = 0.4$; intersecting wall on one side.

Required: Design the shear wall; ordinary reinforced shear wall

Solution: Check using 0.9D+1.0E load combination.

Flange in tension: Approximate as a single layer of reinforcement. Use design procedure for single layer of reinforcement with $P_u = 18.1$ k, $M_u = 900$ k-ft., $b = 7.625$ in., $l_w = 200$ in. (16 ft + 8 in. flange); $d = 196$ in.

Required reinforcement: $a = 5.9$ in., $A_s = 0.87$ in$^2$ or 3 - #5.

Flange in compression: Reinforcement will be approximately the same as for a non-flanged wall. Increase in compression area will only slightly reduce required steel.

Axial load on just the web creates a moment with a small tension in the flange and compression in the web.
**Example: T-Wall**

**Trial Design**

Tension: 3 bars total; assume spacing of 48 in. for OOP
90 in. flange
Compression: 3 grouted cells within flange

Flange compression area:
\[(48+48+7.62)(2.5) + 3(8)(7.625-2.5) = 382\text{in}^2\]
Equivalent thickness = \(382\text{in}^2/7.625\text{in.} = 50.1\text{in.}\)

Check maximum reinforcement with flange in tension:
\[c/d = 0.446 \ (\alpha = 1.5)\]
\[c = 0.446(196\text{in.}) = 87.4\text{ in.}\]
\[\phi P_n = 285.3\text{ kips}\]
\[P_u = D+0.75L\]
\[= (6.1+16) +0.75(16)\]
\[= 34.1\text{ kips}\]

Perhaps should include steel just outside effective tension flange:
\[\phi P_n = 251.8\text{ kips}\]
Example: T-Wall

Shear strength is based on the flange, and similar to previous example. Use #5 @ 40 in.

Check shear at interface. Check using intersecting bond beams.

Reinforcement from #5@40in.  
0.093in²/ft close to 0.1in²/ft  
\[
\frac{0.31\text{in}^2}{40\text{in.}} \times \frac{12\text{in.}}{1\text{ft}} = 0.093\text{in}^2/\text{ft}
\]

Shear at interface: Set equal to tension force in reinforcement in flange.  
\[
V = T = f_y A_{sf} = 60000\text{psi}(3)(0.31\text{in}^2) = 55800 \text{ lb}
\]

\[
A_{nv} = 2.5in(120in) + 3(8in)(7.62in - 2.5in) = 423\text{in}^2
\]

\[
\phi V_{nm} = \phi \left[4.0 - 1.75 \left(\frac{M_u}{V_u d_v}\right)\right] A_{nv} \sqrt{f_m'} + 0.25P_u
\]

\[
= 0.8(2.25)(423\text{in}^2) \sqrt{2000\text{psi}} = 34050\text{lb}
\]

If used γ = 0.75, \(\phi V_{nm} = 25,500\text{ lbs}\)
Example: T-Wall

Need to fully grout interface: $A_{nv} = 915\text{in}^2$; $\phi V_n = 73600\text{ lb}$

- Suggest adding shear straps between each bond beam (16 in. o.c.)
- Suggest using bond beam units and cutting out half of face-shell in flange blocks to get good grout at interface.

Additional cells to be grouted