Combined Flexural and Axial Loads

- Interaction Diagram
  - Solidly grouted bearing wall
  - Partially grouted bearing wall
- Bearing Walls: Slender Wall Design Procedure
  - Strength
  - Serviceability – Deflections
- Example – Pilaster
- Bearing and Concentrated Loads
- Prestressed Masonry

Key Code Sections

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5.4 Pilasters
9.3.2 Design assumptions
9.3.4.1 Nominal strength
  9.3.4.1.1 Nominal axial and flexural strength
  Section 4.3.3 Radius of gyration
9.3.5 Wall design for out-of-plane loads
  9.3.5.1 Scope
  9.3.5.2 Nominal axial and flexural strength
  9.3.5.3 Nominal shear strength
  9.3.5.4 P-delta effects
  9.3.5.5 Deflections
Concentric Axial Compression

\[ P_n = 0.80 \left[ 0.80 f'_m (A_n - A_{st}) + f_y A_{st} \left( \frac{70r}{h} \right)^2 \right] \quad \frac{h}{r} > 99 \]

\[ P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 EA_p r^2}{h^2} = \frac{\pi^2 (900 f'_m) A_p r}{h^2} = A_n f'_m \left( \frac{94.2 r}{h} \right)^2 \]

- Equation above for CMU; for clay \((E_m = 700f'_m)\), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling

Inclusion of wall weight
Wall weight provides uniform axial load over height of wall. Reasonable approximation is to use half the weight of wall acting at top.

\[ \Phi = 0.9 \]

\[ A_{st} \text{ is area of laterally tied steel} \]

- Equation above for CMU; for clay \((E_m = 700f'_m)\), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling

Inclusion of wall weight
Wall weight provides uniform axial load over height of wall. Reasonable approximation is to use half the weight of wall acting at top.
Buckling Curve for $A_{st} = 0$

\[ P_n = 0.80 \left[ 0.80 f'_{m} A_{n} \left( \frac{70r}{h} \right)^2 \right] \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \]

Questions: Is this a strict average or weighted average? What about different types of units (which changes block area)?

NCMA has tabulated values of average radii of gyration.

I use $r = \sqrt{I_n/A_n}$ in the examples and spreadsheet.
Interaction Diagram

- Assume strain/stress distribution
- Compute forces in masonry and steel
- Sum forces to get axial force
- Sum moment about centerline to get bending moment
- Key points
  - Pure axial load
  - Pure bending
  - Balanced

Example – 8 in. CMU Bearing Wall

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; solid grout
Required: Interaction diagram in terms of capacity per foot

Pure Moment:

\[ M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8b f'_m} \right) \]

\[ \phi M_n = 0.9 \left( 0.934 k - \frac{ft}{ft} \right) = 0.840 k - \frac{ft}{ft} \]

\[ a = \frac{A_s f_y}{0.8b f'_m} = \]

\[ c = \frac{a}{0.8} = \]
Example – 8 in. CMU Bearing Wall

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; solid grout

Required: Interaction diagram in terms of capacity per foot

Pure Moment: 

\[ M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8 b f_m'} \right) \]

\[ = 0.05\text{in}^2 / \text{ft}(60\text{ksi}) \left( 3.8125\text{in} - \frac{1}{2} \frac{0.05\text{in}^2 / \text{ft}(60\text{ksi})}{0.8(12\text{in} / \text{ft})(2.0\text{ksi})} \right) \]

\[ = 11.20k - \text{in} / \text{ft} = 0.934k - \text{ft} / \text{ft} \]

\[ \phi M_n = 0.9(0.934k - \text{ft} / \text{ft}) = 0.840k - \text{ft} / \text{ft} \]

\[ a = \frac{A_s f_y}{0.8 b f_m'} = \frac{0.05\text{in}^2 / \text{ft}(60\text{ksi})}{0.8(12\text{in} / \text{ft})(2.0\text{ksi})} = 0.156\text{in} \]

\[ c = \frac{a}{0.8} = \frac{0.156\text{in}}{0.8} = 0.195\text{in} \]

Pure Axial: 

\[ r = \frac{1}{\sqrt{12}} t = \frac{h}{r} = \frac{144\text{in}}{2.20\text{lin}} = 65.4 \]

\[ P_n = 0.8[0.80 f''_{u}(A_n - A_s) + f_y A_n \left[ 1 - \left( \frac{h}{140r} \right)^2 \right]] \]

\[ \frac{h}{r} \leq 99 \]
Example – 8 in. CMU Bearing Wall

Pure Axial:

\[ r = \frac{\sqrt{\frac{I_n}{A_n}}} = \frac{1}{\sqrt{12}} = 0.289(7.625\text{ in}) = 2.201\text{ in} \]

For solid sections

\[ \frac{h}{r} = \frac{144\text{ in}}{2.201\text{ in}} = 65.4 \]

\[ P_n = 0.8\left[0.80f'(A_n - A_{st}) + f_yA_{st}\left[1 - \left(\frac{h}{140r}\right)^2\right]\right] \leq 99 \]

\[ = 0.8\left[0.80(2.0\text{ ksi})(7.625\text{ in})(12\text{ in} / \text{ ft}) - 0\right] + 0\left[1 - \left(\frac{65.4}{140}\right)^2\right] \]

\[ = 117.2\text{ k} / \text{ ft}(0.7816) = 91.6\text{ k} / \text{ ft} \]

\[ \phi P_n = 0.9(91.6\text{ k} / \text{ ft}) = 82.4\text{ k} / \text{ ft} \]

Choose strain distribution (alternatively \(c\))

Balanced conditions

\[ C_m = \]

\[ T = \]

\[ P_n = C_m - T = \]

\[ \phi P_n = \]

\[ M_n = \]

\[ \phi M_n = \]
Example – 8 in. CMU Bearing Wall

Choose strain distribution (alternatively c)
Balanced conditions

\[
C_m = [0.80(2.0\text{ksi})][0.8][2.09\text{in}][12\text{in} / \text{ft}] = 32.0 \text{kip} / \text{ft}
\]
\[
T = (60\text{ksi})(0.05\text{in}^2 / \text{ft}) = 3.0 \text{kip} / \text{ft}
\]
\[
P_n = C_m - T = (32.0 - 3.0) \text{k/ft} = 29.0 \text{k/ft}
\]
\[
\phi P_n = 0.9(29.0 \text{k/ft}) = 26.1 \text{k/ft}
\]
\[
M_n = 32.0 \text{kip} / \text{ft} \left(3.81\text{in} - \frac{0.8(2.09\text{in})}{2}\right) = 95.4\text{k-in/ft} = 7.95\text{k-ft/ft}
\]
\[
\phi M_n = 0.9(7.95\text{k-ft/ft}) = 7.16\text{k-ft/ft}
\]

Example - Interaction Diagram

<table>
<thead>
<tr>
<th>Point</th>
<th>c (in)</th>
<th>(C_m) (kip/ft)</th>
<th>T (kip/ft)</th>
<th>(\phi P_n) (kip/ft)</th>
<th>(\phi M_n) (kip-ft/ft)</th>
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<tbody>
<tr>
<td>a = d</td>
<td>4.76</td>
<td>73.2</td>
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<td>10.46</td>
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<td>c = d</td>
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<td>45.3</td>
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<td>39.8</td>
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<td>32.0</td>
<td>3.0</td>
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<td>3.0</td>
<td>0</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Example – Moment at Maximum Axial

Maximum Moment at $\varphi P_n = 82.4 \text{ k/ft}, P_n = 91.6 \text{ k/ft}$

Assume steel is not in tension, $P_n = C_m$

$$c = \frac{P_n}{0.8(0.8f_m'\beta)} = \frac{91.6 \text{ k/ft}}{0.8(0.8(2.0 \text{ksi}))12 \text{ in/ft}} = 5.96 \text{ in}$$

$$a = 0.8c = 4.77 \text{ in}$$

$$M_n = C_m\left(\frac{t}{2} - \frac{a}{2}\right) = 91.6 \text{ k/ft} \left(\frac{7.625 \text{ in}}{2} - \frac{4.77 \text{ in}}{2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 10.90 \text{ k-ft/ft}$$

$$\varphi M_n = 0.9(10.90 \text{ k-ft/ft}) = 9.81 \text{ k-ft/ft}$$
Interaction Diagram – Below Balanced

Below the balanced point: \( T = A_s f_y \) \( C = 0.8 f'_m b a \)

\[ P_n = 0.80 f'_m b a - A_s f_y \quad \text{or} \quad a = \frac{A_s f_y + P_n}{0.80 f'_m b} \]

\[ M_n = 0.8 f'_m b a \left( \frac{t_{sp}}{2} - \frac{a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

\[ = \left( P_n + A_s f_y \right) \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

If we could only know one point on the interaction diagram, we would want to know the point corresponding to \( \varphi P_n = P_u \)

\[ a = \frac{A_s f_y + P_u}{0.80 f'_m b} \quad \text{or} \quad M_n = \left( P_u / \phi + A_s f_y \right) \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

These are equations in 9.3.5.2 commentary. They ignore any tension in a possible second layer of steel near the compression face.

For centered bars:

\[ M_n = \left( P_u / \phi + A_s f_y \right) \left( d - \frac{a}{2} \right) \]
Partially Grouted Bearing Wall

- Small _______ forces
  - Partially grouted walls act as _______ walls
  - Compression area is in ______________
- Strength design
  - Higher axial loads act as __________
  - Very high axial loads act as _________
  - Need to calculate $r$ based on grouted cross-section.

Interaction Diagram: Solid vs. Partial Grout
Walls: Slenderness Effects

Three methods to account for slenderness effects:

1. _________________________
   a. Axial capacity (and sometimes moment) reduced
   b. Used to be in TMS 402 Code
   c. deleted because it can be unconservative

2. ___________________________
   a. Second-order moment directly added by P-δ
   b. Usually requires iteration
   c. Difficult for hand calculations for other than simple cases
   d. Basis for second-order analysis in computer programs
   e. Historical method used for masonry design

3. ________________________
   a. Added in 2013 TMS 402 Code
   b. Very general, but a bit conservative.

Walls: Complementary Moment 9.3.5.4.2

• Assumes simple support conditions.
• Assumes midheight moment is approximately maximum moment
• Valid only for the following conditions:
  - \( \frac{P_L}{A_n} \leq 0.05f_m' \) No height limit
  - \( \frac{P_L}{A_g} \leq 0.20f_m' \) height limited by \( \frac{h}{t} \leq 30 \)

**Moment:**

\[
M_u = \frac{w_nh^2}{8} + P_{uf}\frac{e_u}{2} + P_u\delta_u
\]

**Deflection:**

\[
\delta_u = \frac{5M_u h^2}{48E_m I_n} \quad M_u < M_{cr}
\]

\[
P_u = P_{uw} + P_{uf}
\]

\[
P_{uf} = \text{Factored floor load}
\]

\[
P_{uw} = \text{Factored wall load}
\]

\[
\delta_u = \frac{5M_{cr} h^2}{48E_m I_n} + \frac{5(M_u - M_{cr})h^2}{48E_m I_{cr}} \quad M_u > M_{cr}
\]
Walls: Complementary Moment

Solve simultaneous linear equations:

\[
M_u > M_{cr} \quad \quad \quad M_u < M_{cr}
\]

\[
M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + \frac{5M_{cr} P_u h^2}{48E_m I_{cr}} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right)
\]

\[
\delta_u = \frac{5h^2}{48E_m I_{cr}} \left[ \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + M_{cr} \left( \frac{I_{cr}}{I_n} - 1 \right) \right] - \frac{5P_u h^2}{48E_m I_{cr}}
\]

\[
M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} - \frac{5P_u h^2}{48E_m I_{cr}}
\]

\[
\delta_u = \frac{5h^2}{48E_m I_n} \left[ \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} \right] - \frac{5P_u h^2}{48E_m I_n}
\]

Combined Flexural and Axial Loads

Walls: Deflections

\[
I_{cr} = n \left( A_s + \frac{P_u}{f_y} \frac{t_{sp}}{2} \right) (d - c)^2 + \frac{bc^3}{3}
\]

\[
c = \frac{A_s f_y + P_u}{0.64 f_m b}
\]

For centered bars:

\[
I_{cr} = n \left( A_s + \frac{P_u}{f_y} \right) (d - c)^2 + \frac{bc^3}{3}
\]

\[
M_{cr} = \frac{P_u / A_n + f_y}{t_{sp} / 2} I_n
\]

What axial load, \( P_u \), should be used? Suggest using smallest value of \( P_u \).

\[
\text{Deflection Limit} \quad \delta_s \leq 0.007h \quad \text{Calculated using allowable stress load combinations}
\]
Walls: Maximum Reinforcement

- Strain gradient of $\varepsilon_{mu}$ and $\alpha \varepsilon_y$ with $\alpha = 1.5$ for OOP loading
- $P_u$ determined from $D + 0.75L + 0.525Q_E$

Fully grouted with equal tension and compression reinforcement

$$\rho = \frac{A_s}{bd} = \frac{0.64 f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y - \min \left\{ \varepsilon_{mu} - \frac{d'}{d} \left( \frac{\varepsilon_{mu} + \alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \right\} E_s}$$

Fully grouted with concentrated tension reinforcement, or partially grouted with neutral axis in face shell

$$\rho = \frac{A_s}{bd} = \frac{0.64 f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y}$$

Partially grouted walls with concentrated tension reinforcement and neutral axis in web

$$\rho = \frac{0.64 f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left( \frac{b_w}{b} \right) + 0.8 f'' \left( \frac{b - b_w}{bd} \right)}{f_y} - \frac{P_u}{bd}$$

Out-of-Plane Loading: Wind Load

Wind Load on Parapet
- MWFRS (ASCE 7-10 27.4.5, 28.4.2): Used for determining shear wall loads
- Components and Cladding (ASCE 7-10 30.7.1.2, 30.9):
  - Used for designing parapet
  - Used for designing wall-to-diaphragm connection
- Parapet pressure to use for designing wall?
  - Parapet wind load reduces midheight wall moment
  - Very conservative: parapet load is 0
  - Aggressive: full parapet C&C pressure
  - Moderate: extend wall pressure to parapet
Out-of-Plane Loading: Seismic Load

Seismic Load on Parapet
- In first mode, wall and parapet loads are in opposite directions
- Design forces for shear walls and wall-to-diaphragm connections
  - Suggest using wall and parapet loads in same direction
- Seismic parapet force to use for designing wall
  - Conservative: parapet load in opposite direction of wall
  - Aggressive: wall and parapet load in same direction
  - Moderate: no parapet load

Out-of-Plane Loading: Maximum Moment

\[ x = \frac{L}{2} - \frac{M}{wL} \]

\[ M_{\text{max}} = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL} \]

If \( x < 0 \), \( M_{\text{max}} = M \)
Example – Out-of-Plane Wall

Given: 8 in. CMU wall; Grade 60 steel; Type S masonry cement mortar; $f'_m=2000\text{psi}$; roof forces act on 3 in. wide bearing plate at edge of wall.

Required: Reinforcement

Solution: Estimate amount of steel

$$a = d - \sqrt{d^2 - \frac{2P_e(d - t_y/2) + M_u}{\phi(0.8f'_m b)}}$$

$$A_s = \frac{0.8f'_m b a - P_a}{f_y}$$

For centered steel:

$$a = d - \sqrt{d^2 - \frac{2M_u}{\phi(0.8f'_m b)}}$$

Cross-section of top of wall

Determine eccentricity

$$e = \frac{7.625\text{in}}{2} - 1.0\text{in.} = 2.81\text{in.}$$

Example - OOP: Estimate Steel

Use $0.9D+1.0W$ without second-order effects, parapet, and wall weight

- Smaller axial load results in decreased capacity
- Higher axial load results in higher $P-\delta$
- Will need to consider both $0.9D+1.0W$ and $1.2D+0.5L_r+1.0W$

$$P_{uf} = 0.9(500\text{lb/ft}) + 1.0(-360\text{lb/ft}) = 90\text{lb/ft}$$

$$M_{u,app} = \frac{w_d h^2}{8} + P_{uf} \frac{e_u}{2} = \frac{1.0(32\text{psf})(18\text{ft})^2}{8} \left(\frac{12\text{in}}{\text{ft}}\right)^2 + 90\text{lb/ft} \frac{2.81\text{in}}{2} = 15.7k-\text{in/ft}$$

Estimate steel:

$$a = d - \sqrt{d^2 - \frac{2M_u}{\phi(0.8f'_m b)}} = 3.81\text{in} - \sqrt{(3.81\text{in})^2 - \frac{2(15.7k-\text{in/ft})}{0.9(0.8(2.0ksi)(12\text{in/ft}))}} = 0.246\text{in}$$

$$A_s = \frac{0.8f'_m b a - P_a}{f_y} = \frac{0.8(2.0ksi)(12\text{in/ft})(0.246\text{in}) - (0.09k)}{60ksi} = 0.077\text{in}^2/\text{ft}$$

Try #5@48 in., $A_s = 0.0775\text{in}^2/\text{ft}$
Example - OOP

Summary of Strength Design Load Combination Axial Forces
(wall weight is 38 psf for 48 in. grout spacing)

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>( P_{uf} ) (kip/ft)</th>
<th>( P_{uw} ) (kip/ft)</th>
<th>( P_u ) (kip/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D+1.0W</td>
<td>( 0.9(0.5)+1.0(-0.36) ) = 0.090</td>
<td>( 0.9(0.038)(2.67+9) ) = 0.399</td>
<td>0.489</td>
</tr>
<tr>
<td>1.2D+1.0W+0.5Lr</td>
<td>|</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ P_{uf} = \text{Factored floor load (just eccentrically applied load)} \]
\[ P_{uw} = \text{Factored wall load (includes wall and parapet weight; found at midheight of wall between supports (9 ft from bottom))} \]

Example - OOP: \( M_{cr} \)

Find modulus of rupture; use linear interpolation between no grout and full grout

Ungrouped (Type S masonry cement): _____ psi

Fully grouted (Type S masonry cement): 153 psi

\[ f_r = \text{psi} \left( \frac{\text{cells ungrouted}}{\text{cells}} \right) + 153 \text{psi} \left( \frac{\text{cells grouted}}{\text{cells}} \right) = \text{psi} \]

Find \( M_{cr} \), cracking moment:
Commentary allows one to include axial load
Use minimum axial load (once wall has cracked, it has cracked)

\[ M_{cr} = \frac{(P_u / A_n + f_r)I_n}{t_{sp} / 2} \]
Example - OOP: $M_{cr}$

Find modulus of rupture; use linear interpolation between no grout and full grout

Ungrounded (Type S masonry cement): 51 psi
Fully grouted (Type S masonry cement): 153 psi

$$f_r = 51 \text{ psi} \left( \frac{5 \text{ cells ungrouted}}{6 \text{ cells}} \right) + 153 \text{ psi} \left( \frac{1 \text{ cells grouted}}{6 \text{ cells}} \right) = 68 \text{ psi}$$

Find $M_{cr}$, cracking moment:
Commentary allows one to include axial load
Use minimum axial load (once wall has cracked, it has cracked)

$$M_{cr} = \left( \frac{P_u}{A_n} + f_r \right) I_n = \left( \frac{489 \text{ lb}}{\text{ ft}} \right) \left( \frac{40.7 \text{ in}^2}{\text{ ft}} \right) \left( 68 \text{ psi} \right) \frac{332.0 \text{ in}^4}{\text{ ft}}$$

$$M_{cr} = 6.97 \text{ kip} - \text{ in} / \text{ ft} = 0.581 \text{ k} - \text{ ft} / \text{ ft}$$

---

Example - OOP: Moment of Inertia

Find $I_{cr}$, cracked moment of inertia.

$$c = \frac{A_s f_y + P_u}{0.64 f_m b} =$$

$$n = \frac{E_s}{E_m} = \frac{29000 \text{ ksi}}{1800 \text{ ksi}} = 16.1$$

$$I_{cr} = n \left( A_s + \frac{P_u t_{sp}}{f_y} \right) \left( \frac{d - c}{2d} \right)^2 + \frac{bc^3}{3}$$
Example - OOP: Moment of Inertia

Find $I_{cr}$, cracked moment of inertia.

$$c = \frac{A_s f_y + P_u}{0.64 f' m b} = \frac{0.0775in^2 / ft(60ksi) + 0.489k / ft}{0.64(2.0ksi)(12in / ft)} = 0.334in$$

$$n = \frac{E_s}{E_m} = 29000ksi / 1800ksi = 16.1$$

$$I_{cr} = n \left( A_s + \frac{P_u t_{sp}}{f_y 2d} \right) (d - c)^2 + \frac{bc^3}{3}$$

$$= 16.1 \left( 0.0775in^2 / ft + \frac{0.489k / ft}{60ksi} \right) (3.812in - 0.334in)^2 + \frac{12in / ft(0.334in)^3}{3}$$

$$= 16.8in^4 / ft$$

Example - OOP

Find $M_u$.

$P_{ufr} e_u$ is the moment at the top support of the wall. It includes eccentric axial load and wind load from parapet.

$$M_{u, top} = P_{ufr} e_u - \frac{w_{u, parapet} h_{parapet}^2}{2} = 0.090 \frac{ft}{in} (2.81in)^{\frac{1}{2}} - \frac{0.032ksf(2.67ft)^2}{2} = -0.093 \frac{k - ft}{ft}$$

$$M_u = \frac{w_{u} h^2}{8} + P_{ufr} e_u^2 + \frac{5M_{cr} P_{u} h^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right)$$

$$= 1 - \frac{5P_{u} h^2}{48E_m I_{cr}}$$
Example - OOP

Find $M_u$.

$P_{uf} e_u$ is the moment at the top support of the wall. It includes eccentric axial load and wind load from parapet.

$$M_{u,\text{top}} = P_{uf} e_u - \frac{w_{u,\text{parapet}} h_{\text{parapet}}^2}{2} = 0.090 \frac{k}{ft} (2.81 \text{in}) \frac{1}{12 \text{in}} - \frac{0.032 \text{ksf}(2.67 \text{ ft})^2}{2} = -0.093 \frac{k}{ft}$$

$$M_u = \frac{w_u h^2}{8} + P_u e_u + \frac{5M_{cr} P_u h^2}{48E_m} \left( \frac{1}{I_u} - \frac{1}{I_{cr}} \right)$$

$$1 - \frac{5P_u h^2}{48E_m I_{cr}}$$

$$\frac{0.032 \text{ksf}(18 \text{ ft})^2}{8} + \frac{-0.093 \frac{k}{ft}}{2} + \frac{5(0.581 \frac{k}{ft})(0.489 \frac{k}{ft})(18 \text{ ft})^2}{48(1800 \text{ksi})} \left( \frac{1}{332 \text{ in}^2} - \frac{1}{16.8 \text{ in}^2} \right) \left( 144 \text{in}^2 \right)$$

$$= \frac{1.296 - 0.046 - 0.043}{0.9214} \frac{k}{ft} = 1.309 \frac{k}{ft}$$

Combined Flexural and Axial Loads

Example - OOP

Compare to capacity:

$$a = \frac{A_s f_y + P_u / \phi}{0.80 f_m' b} =$$

$$\phi M_n = \phi (P_u / \phi + A_s f_y \left( d - \frac{a}{2} \right))$$

Combined Flexural and Axial Loads
Example - OOP

Compare to capacity:

\[ a = \frac{A_s f_y + P_u \phi}{0.80 f_{m}^b} = \frac{0.0775in^2 / ft(60ksi) + 0.489k / ft / 0.9}{0.80(2.0ksi)(12in / ft)} = 0.270 in \]

\[ \phi M_n = \phi \left( P_u / \phi + A_s f_y \right) \left( d - \frac{a}{2} \right) \]

\[ = 0.9 \left( 0.489k / ft / 0.9 + 0.0775in^2 / ft(60ksi) \right) \left( 3.812in - \frac{0.270in}{2} \right) \]

\[ = 17.19 kip - in / ft = 1.432k - ft / ft \]

\[ M_u = 1.309kip - ft / ft \leq 1.432kip - ft / ft = \phi M_n \quad \text{OK} \]

Example – OOP Strength

Summary of Strength Design Load Combinations

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>( M_u ) (kip-ft/ft)</th>
<th>( \phi M_n ) (kip-ft/ft)</th>
<th>( M_u/\phi M_n )</th>
<th>Second Order ( M ) / First Order ( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2D+1.6L_r+0.5W</td>
<td>0.799</td>
<td>1.755</td>
<td>0.455</td>
<td>1.074</td>
</tr>
<tr>
<td>1.2D+1.0W+0.5L_r</td>
<td>1.416</td>
<td>1.574</td>
<td>0.900</td>
<td>1.097</td>
</tr>
<tr>
<td>0.9D+1.0W</td>
<td>1.309</td>
<td>1.432</td>
<td>0.914</td>
<td>1.047</td>
</tr>
</tbody>
</table>

Try #5 @ 56 in.

What changes?

Overstressed by 5.4%
Example – OOP Deflections

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>D+0.6W</th>
<th>0.6D+0.6W</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (k/ft)</td>
<td>0.5+0.6(-0.36)  = 0.284</td>
<td>0.6(0.5)+0.6(-0.36) = 0.084</td>
</tr>
<tr>
<td>P_w (k/ft)</td>
<td>38(2.67+9) = 0.443</td>
<td>0.6(0.443) = 0.266</td>
</tr>
<tr>
<td>P (k/ft)</td>
<td>0.727</td>
<td>0.350</td>
</tr>
<tr>
<td>c (in)</td>
<td>0.350</td>
<td>0.325</td>
</tr>
<tr>
<td>I_cr (in^4/ft)</td>
<td>17.48</td>
<td>16.46</td>
</tr>
<tr>
<td>M_top (k-ft/ft)</td>
<td>-0.002</td>
<td>-0.049</td>
</tr>
<tr>
<td>M (k-ft/ft)</td>
<td>0.805</td>
<td>0.767</td>
</tr>
<tr>
<td>δ (in)</td>
<td>0.474</td>
<td>0.426</td>
</tr>
</tbody>
</table>

Deflection Limit \( \delta_s \leq 0.007h = 0.007(18 \text{ ft})^{\frac{12\text{ in}}{\text{ ft}}} = 1.51\text{ in} \quad \text{OK} \)

Deflections, Sample Calculations: \( \mathbf{D + 0.6W} \)
Replace factored loads with service loads

\[
\delta = \frac{5h^2}{48E_mI_{cr}} \left[ \frac{wh^2}{8} + P_f \frac{e}{2} + M_{cr} \left( \frac{I_{cr}}{I_n} - 1 \right) \right] \left[ 1 - \frac{5Ph^2}{48E_mI_{cr}} \right]
\]

\[
\begin{align*}
&= \frac{5(18 \text{ ft})^2}{48(1800\text{ksi})(17.5 \frac{\text{ in}}{\text{ ft}})^2} \left( \frac{0.6(0.032\text{kfsf})(18 \text{ ft})^2}{8} + \frac{-0.002k^{\text{ ft}}}{2} + 0.581k^{\text{ ft}} \left( \frac{17.5 \frac{\text{ in}}{\text{ ft}}}{332 \frac{\text{ in}}{\text{ ft}}} - 1 \right) \right) \frac{144\text{ in}^2}{1\text{ ft}^2} \\
&\quad \times 1 - \frac{5(0.727k^{\text{ ft}})(18 \text{ ft})^2}{48(1800\text{ksi})(17.5 \frac{\text{ in}}{\text{ ft}})} \frac{144\text{ in}^2}{1\text{ ft}^2}
\end{align*}
\]

\( = 0.0393 \text{ ft} = 0.474\text{ in.} \)
Example – Chase a rabbit

Chase a rabbit, what to use for $I_{cr}$? $c$ or $kd$ or something else??
For #5 @ 48 in., $P = 0.727$ k/ft, $c = 0.350$ in., $I_{cr} = 17.5$ in$^4$/ft
$kd = 0.8845$ in., $I_{cr} = 17.9$ in$^4$/ft

$$(n\rho)_{ef} = n\rho + \frac{nP}{F_{bd}} = 16.1\left(\frac{0.0775in^2/ft}{12in/ft(3.812in.)}\right) + \frac{16.1(0.711kip/ft)}{32ksi(12in/ft)(3.812in.)} = 0.0351$$

$$k = \sqrt{(n\rho)^2_{ef} + 2(n\rho)_{ef} - (n\rho)_{ef} = \sqrt{(0.0351)^2 + 2(0.0351) - (0.0351)} = 0.232}$$

$$I_{cr} = n\left(A_s + \frac{P}{F_s}\frac{t_{sp}}{2d}\right)(d - kd)^2 + \frac{b(kd)^3}{3} = 16.5in^4/ft$$

And both are wrong: Need to find neutral axis that corresponds to $P$, $M$ combination. This would require iteration, and would be quite complicated.

Example – Check Maximum Reinforcement

Check Maximum Reinforcement:
- neutral axis is in face shell
- $P_u$ is just dead load = $0.5 + 0.038(2.67 + 9) = 0.943$k/ft

$$\rho_{max} = \frac{A_s}{bd} = \frac{0.64 f'_m \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y}\right) - P_u}{b f_y}$$

$$= \frac{0.0025}{0.0025 + 1.5(0.00207)} - \frac{0.943 k}{60ksi} = \frac{0.00917}{60ksi} = 0.00917$$

$$\rho = \frac{A_s}{bd} = \frac{0.0775 \frac{ft^2}{ft}}{12 \frac{in}{ft}(3.81in)} = 0.00169 \text{ OK}$$
Example: Seismic Loads

Given: 8 in. normal weight (125 pcf) CMU wall; Grade 60 steel; Type S PCL mortar (special reinforced wall); $f'_m = 2000\text{psi}$; roof forces act at 7.32 in. eccentricity; $S_{DS} = 1.43$, $I = 1.0$.

Required: Reinforcement

Solution: Estimate amount of steel

- Lateral load directly proportional to wall weight
- Need to know grout spacing to determine wall weight
- Wall is primarily in flexure
- Check different grout spacings in flexure and determine reasonable amount of steel

After 2009 NEHRP Recommended Seismic Provisions: Design Examples *FEMA P-751 / September 2012*

<table>
<thead>
<tr>
<th>Grout spacing</th>
<th>$w_w$ (psf)</th>
<th>$w_u$ (psf)</th>
<th>$M_u$ (k-ft/ft)</th>
<th>$a$ (in.)</th>
<th>$A_s$ (in²/ft)</th>
<th>Bar size</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in.</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 in.</td>
<td>51*</td>
<td>29</td>
<td>2.86</td>
<td>0.562</td>
<td>0.18</td>
<td>#6@24 (0.22in²/ft)</td>
</tr>
<tr>
<td>48 in.</td>
<td>44</td>
<td>25</td>
<td>2.47</td>
<td>0.479</td>
<td>0.15</td>
<td>#7@48 (0.15in²/ft)</td>
</tr>
</tbody>
</table>

* FEMA P-751 uses 65 psf to account for bond beams and additional grouted cells (27% more)

$$a = d - \sqrt{\frac{d^2 - \frac{2M_u}{f'_m h}}{\phi (0.8f'_m b)}}$$

$$A_s = \frac{0.8f'_m b a - P_u / \phi}{f_y}$$

Try #6 @ 24 in., use 56 psf for wall weight (10% increase)

$w_u = 32.0$ psf
Example: Seismic Loads

Summary of Strength Design Load Combination Axial Forces
- \( 0.9D + 1.0E = (0.9 - 0.2S_{DS})D = (0.9 - 0.2 \times 1.43)D = 0.614D \)
- \( 1.2D + 1.0E = (0.9 + 0.2S_{DS})D = (1.2 + 0.2 \times 1.43)D = 1.486D \)

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>( P_{uf} ) (kip/ft)</th>
<th>( P_{uw} ) (kip/ft)</th>
<th>( P_u ) (kip/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D + 1.0E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2D + 1.0E</td>
<td>1.486(0.2) = 0.297</td>
<td>1.486(0.056)(2+14)</td>
<td>= 1.331</td>
</tr>
</tbody>
</table>

Example: Seismic Loads

Find modulus of rupture; use linear interpolation between no grout and full grout

Ungrouted (Type S PCL): 84 psi
Fully grouted (Type S PCL): 163 psi

\[ f_r = 84 \text{ psi} \left( \frac{2 \text{ ungrouted cells}}{3 \text{ cells}} \right) + (163 \text{ psi}) \left( \frac{1 \text{ grouted cell}}{3 \text{ cells}} \right) = 110 \text{ psi} \]

Find \( M_{cr} \), cracking moment:

\[ M_{cr} = \frac{\left( \frac{P_u}{A_n} + f_r \right)I_n}{t_{sp}/2} = \frac{\left( \frac{673 \text{ lb}}{51.3 \text{ in}^2/\text{ft}} + 110 \text{ psi} \right) \left( 355.3 \text{ in}^3/\text{ft} \right)}{3.81 \text{ in}} = 11.48 \frac{k-\text{in}}{\text{ft}} = 0.957 \frac{k-\text{ft}}{\text{in}} \]

Wall properties determined from NCMA TEK 14-1B Section
Properties of Concrete Masonry Walls
Example: Seismic Loads

### Strength

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>0.9D+1.0E</th>
<th>1.2D+1.0E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ (in.)</td>
<td>0.903</td>
<td>0.965</td>
</tr>
<tr>
<td>$I_{cr}$ (in$^4$/ft)</td>
<td>34.5</td>
<td>35.9</td>
</tr>
<tr>
<td>$M_u$ (k-ft/ft)</td>
<td>3.53</td>
<td>4.20</td>
</tr>
<tr>
<td>Second Order / First Order</td>
<td>1.11</td>
<td>1.31</td>
</tr>
<tr>
<td>$a$ (in.)</td>
<td>0.726</td>
<td>0.782</td>
</tr>
<tr>
<td>$\phi M_n$ (k-ft/ft)</td>
<td>3.61</td>
<td>3.85</td>
</tr>
<tr>
<td>$M_u/\phi M_n$</td>
<td>0.98</td>
<td>1.09</td>
</tr>
</tbody>
</table>

### Deflections

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>0.6D+0.7E</th>
<th>D+0.7E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (in.)</td>
<td>3.44</td>
<td>4.07</td>
</tr>
<tr>
<td>$\delta/\delta_{allow}$ ($\delta_{allow} = 2.35$in.$)$</td>
<td>1.46</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Example: Seismic Loads

- Load combinations with higher axial forces control due to P-\(\delta\) effects
- FEMA P-751 does not check deflections; reasoning is standard does not have mid-height deflection limit for walls; TMS 402 does have a deflection limit

<table>
<thead>
<tr>
<th>Option</th>
<th>$M_u/\phi M_n$</th>
<th>$\delta/\delta_{allow}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#7@ 24 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7@ 24 in. LW units (w$_{wall} = 51$psf; w$_u = 29$ psf)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7@ 24 in. $f'_m=2500$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#5@ 8 in. Fully grout; $f'_m=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6@ 8 in. Fully grout; $f'_m=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-#5@ 24 in, 2 in. cover; $f'_m=2000$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 in. CMU, #6@ 32 in. w$_{wall} = 75$ psf; w$_u = 43$ psf</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Would need non-traditional rebar positioner
Example: Seismic Loads

Check Maximum Reinforcement: NA in face shell; \( P_u \) is just dead load

\[
\rho_{\text{max}} = \frac{0.64 f'_{\text{m}} \left( \frac{\varepsilon_{\text{mu}}}{\varepsilon_{\text{mu}} + \alpha \varepsilon_{\text{y}}} \right) - \frac{P_u}{bd}}{f_y} = \frac{0.64(2.0\text{ksi}) \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) - \frac{1.10}{60\text{ksi}}}{12 \text{in/ft}} \left( \frac{3.81\text{in}}{\text{ft}} \right) = 0.00911
\]

\[
\frac{A_s}{b} = \rho_{\text{max}} d = 0.00911(3.81\text{in}) \left( \frac{12\text{in}}{\text{ft}} \right) = 0.42\text{in}^2/\text{ft}
\]

<table>
<thead>
<tr>
<th>Grout spacing</th>
<th>( P_u ) (kip/ft)</th>
<th>( \rho_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in. CMU; 24 in.</td>
<td>0.2+0.056(2+14) = 1.10 k/ft</td>
<td>0.00911 = 0.42 in²/ft</td>
</tr>
<tr>
<td>#5 @ 24 in. = 0.16 in²/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 in. CMU; full grout</td>
<td>0.2+0.081(2+14) = 1.50k/ft</td>
<td>0.00897 = 0.42 in²/ft</td>
</tr>
<tr>
<td>#6 @ 8 in. = 0.66 in²/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 in. CMU; 32 in.</td>
<td>0.2+0.075(2+14) = 1.40k/ft</td>
<td>0.00918 = 0.64 in²/ft</td>
</tr>
<tr>
<td>#6 @ 32 in. = 0.16 in²/ft</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Moment Magnification Method

Complementary Moment

\[
M_u = \psi M_{u,0}
\]

\[
\psi = \begin{cases} 
M_u < M_{cr} & : I_{\text{eff}} = 0.75I_n \\
M_u \geq M_{cr} & : I_{\text{eff}} = I_{cr} 
\end{cases}
\]

\[
P_e = 
\]
Moment Magnification Method

First Order Moment

\[ M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + 5M_{cr} P_u \frac{h^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right) \left( 1 - \frac{5P_u h^2}{48E_m I_{cr}} \right) \]

Complementary Moment

\[ M_u = \psi M_{u,0} \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} \]

\[ M_u < M_{cr}: \ I_{eff} = 0.75 I_n \]

\[ M_u \geq M_{cr}: \ I_{eff} = I_{cr} \]

\[ P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \]

First-order deflection (simply supported wall):

\[ \delta_0 = \frac{5wh^4}{384E_m I_e} + \frac{P_f eh^2}{16E_m I_e} \]

Moment Magnification, Deflections

Rewriting TMS 402 OOP equations:

Same as proposed by Bischoff, P. (2005).
Example – Moment Magnification

Given: CMU wall; Grade 60 steel; Type S PCL mortar (special reinforced wall); $f_m'=2000$ psi; roof dead load of 400 lb/ft at 3.5 in. outside face of wall; $S_{DS}=1.43$, $I=1.0$. 12 ft high x 12 ft wide door openings. See elevation on next page.

Required: Reinforcement

Solution:

After 2009 NEHRP Recommended Seismic Provisions:
Design Examples FEMA P-751 / September 2012
Example - Moment Magnification

- Load has approximately doubled
- Use 12 inch block, two layers of bars, 2 inch cover, \(d \sim 9.2\) in.
- Normal weight units, 24 in. bar spacing, \(w_w = 75\) psf, say 85 psf
- \(w_u = 0.4S_{DS}lw_w = 0.4(1.43)(1)(2*85\text{psf}) = 97\) psf
- \(M_u = 0.097\text{kfsf}(28\text{ft})^2/8 = 9.51\text{k-ft/ft}\)

\[
a = 9.2in - \sqrt{(9.2in)^2 - \frac{2(9.51\frac{k}{ksi})(12\frac{ft}{ksi})}{0.9(0.8)(2ksi)(12\frac{ksi}{ft})}} = 0.748in.
\]

\[
A_s = \frac{0.8f_y'(ba - P_u / \phi)}{f_y} = \frac{0.8(2\text{ksi})(12\frac{ksi}{ft})(0.748in)}{60\text{ksi}} = 0.24\text{ in}^2/\text{ft}
\]

• Over 8 ft pier width, \(A_s = 8\text{ft}(0.24\text{in}^2/\text{ft}) = 1.92\text{in}^2\)
• 1.92\text{in}^2 = 6.2 \#5 bars or 4.4 \#6 bars.
• Try 5 - \#6 bars; two at each jamb, and one in center.
• \(d = 11.62 - 2 - 0.31 = 9.31\) in.
• \(e = 11.62/2 + 3.5 = 9.31\) in.
• 85 psf is reasonable wall weight
Example – Moment Magnification

Shear and Moment Diagrams for Pier: 0.9D + 1.0E

Shear

Moment

Combined Flexural and Axial Loads

59

Example – Moment Magnification

Shear and Moment Diagrams for Pier: 0.9D + 1.0E

Shear

Moment

Combined Flexural and Axial Loads

60
Example – Moment Magnification

Analyze at point of maximum moment:

- $M_{u,0} = 77.6 \text{ k-ft}$
- $P_u = 0.614(8k+0.085\text{ksf(20ft)(12.3ft)}) = 17.8 \text{ kips}$
- Find $I_{cr}$:

\[
c = \frac{A_s f_y + P_u}{0.64 f_m' b} = \frac{5(0.44in^2)(60ksi) + 17.8k}{0.64(2.0ksi)(96in)} = 1.22in\quad \text{c is in face shell}
\]

\[
I_{cr} = n \left( A_s + \frac{P_u t_{sp}}{f_y} \right) \left( d - c \right)^2 + \frac{bc^3}{3}
\]

\[
= 16.1 \left( 2.2in^2 + \frac{17.8k}{60ksi} \frac{11.62in}{2(9.31in)} \right) \left( 9.31in - 1.22in \right)^2 + \frac{96in(1.22in)^3}{3}
\]

\[
= 257in^4
\]
Example – Moment Magnification

Use Moment Magnifier Method:

\[ P_e = \frac{\pi^2 E \cdot I_{eff}}{h^2} = \frac{\pi^2 (1800 \text{ksi})(2571 \text{in}^4)}{(28 \text{ ft})(12 \text{ in/ft})^2} = 404.6k \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} = \frac{1}{1 - \frac{17.8k}{404.6k}} = 1.046 \]

\[ M_u = \psi M_{u,0} = 1.046(77.6k - \text{ft}) = 81.1k - \text{ft} \]

Check capacity:

\[ a = \frac{A_s f_y + P_u / \phi}{0.80 f_m' b} = \frac{2.2\text{in}^2(60\text{ksi}) + 17.8k / 0.9}{0.80(2\text{ksi})(96\text{in})} = 0.988\text{in.} \]

\[ M_n = \left( P_u / \phi + A_s f_y \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \right) \]

\[ = \left( 17.8k / 0.9 + 2.2\text{in}^2(60\text{ksi}) \left( \frac{11.62\text{in} - 0.99\text{in}}{2} \right) + 2.2\text{in}^2(60\text{ksi}) \left( 9.3\text{in} - \frac{11.62\text{in}}{2} \right) \right) \]

\[ = 1269k - \text{in} = 105.8k - \text{ft} \]

\[ \phi M_n = 0.9(105.8k - \text{ft}) = 95.2k - \text{ft} > M_u = 81.1k - \text{ft} \quad \text{OK} \]

For \(1.2D+1.0E\), \( M_u = 89.4k-\text{ft} \quad \phi M_n = 104.9k-\text{ft} \quad \text{OK} \)
Example – Moment Magnification

Check Maximum Reinforcement:
• neutral axis is in face shell
• $P_u$ is just dead load = 8.0k + 0.085ksf(20ft)(12.3ft) = 28.9k

$$\rho = \frac{A_s}{bd} = \frac{0.64 f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha e_y} \right) - \frac{P_u}{bd}}{f_y - \min \left\{ \varepsilon_{mu} - \frac{d'}{d} (\varepsilon_{mu} + \alpha e_y) \cdot e_y \right\} E_s}$$

$$= \frac{0.64(2ksi) \left( \frac{0.0025}{0.0025 + 1.5(0.00207)} \right) - \frac{28.9k}{96in(9.31in)}}{60ksi - \min \left\{ \varepsilon_{mu} - \frac{2.3in}{9.3in} (0.0025 + 1.5(0.00207)), 0.00207 \right\} 29000ksi} = 0.0194$$

$$\rho = \frac{A_s}{bd} = \frac{2.2in^2}{96in(9.31in)} = 0.0025 \quad \text{OK}$$

Check deflections:
\[0.6D + 0.6W\]

$$A_n = 2.5in(96in) + 5(8in)(11.62in - 2.5in) = 605in^2$$

$$I_n = \frac{1}{12} (96in)(11.62in)^3 - \frac{1}{12} (96in - 5(8in))(11.62in - 2.5in)^3 = 9022in^4$$

$$f_r = 84psi \left( \frac{7 \text{ ungrouted cells}}{12 \text{ cells}} \right) + (163psi) \left( \frac{5 \text{ grouted cells}}{12 \text{ cells}} \right) = 117psi$$

$$M_{cr} = \frac{\left( \frac{17.8k}{605in^4} + 0.117ksi \right) \left(9022in^4 \right)}{5.81in} = 227.4kip - in = 18.9k - ft$$
Example – Moment Magnification

\[ c = \frac{A_y f_y + P}{0.64 f_y^* b} = \frac{2.2in^2(60ksi) + 11.6k}{0.64(2.0ksi)96in} = 1.17in \]

\[ I_{cr} = n \left( A_y + \frac{P_u t_{sp}}{f_y 2d} \right) (d - c)^2 + \frac{bc^3}{3} \]

\[ = 16.1 \left( 2.20in^2 + \frac{11.6k}{60ksi} \frac{11.62in}{2(9.31in)} \right) (9.31in - 1.17in)^2 + \frac{96in(1.17in)^3}{3} = 2528in^4 \]

\[ I_e = \frac{I_{cr}}{1 - \frac{M_{cr}}{M} \left( 1 - \frac{I_{cr}}{I_n} \right)} = \frac{2528in^4}{1 - \frac{18.9k}{54.2k - \frac{1 - 2528in^4}{9022in^4}}} = 3375in^4 \]
Example – Moment Magnification

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>0.6D+0.7E</th>
<th>D+0.7E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{max}$ (k-ft)</td>
<td>54.2</td>
<td>57.0</td>
</tr>
<tr>
<td>$P$ (kip)</td>
<td>11.6</td>
<td>34.2</td>
</tr>
<tr>
<td>$I_{cr}$ (in$^4$)</td>
<td>2528</td>
<td>2687</td>
</tr>
<tr>
<td>$I_e$ (in$^4$)</td>
<td>3375</td>
<td>3502</td>
</tr>
<tr>
<td>$\delta_0$ (in)</td>
<td>1.22</td>
<td>1.25</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>1.030</td>
<td>1.088</td>
</tr>
<tr>
<td>$\delta$ (in)</td>
<td>1.26</td>
<td>1.36</td>
</tr>
</tbody>
</table>

$\delta_{all} = 0.007h = 0.007(28\text{ft})(12\text{in}/\text{ft}) = 2.35\text{in.}$

Example - Pilaster Design

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; $f_m=2000$ psi; Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Wind load of 26 psf (pressure and suction) and uplift of 8.1 kips (e=5.8 in.); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Reinforcement

Solution:

\[ d = 15.625 - 7.625/2 = 11.8 \text{ in} \]
Example - Pilaster Design

Weight of pilaster:
Weight of fully grouted 8 in wall (lightweight units) is 75 psf. Pilaster is like a double thick wall. Weight is \(2(75\text{psf})(16\text{in})(1\text{ft/12in}) = 200 \text{lb/ft}\)

1.2D + 1.6S

Critical location is top of pilaster. \(P_u = 26.9 \text{kips} \quad M_u = 156.0 \text{kip-in}\)

\[
a = d - \sqrt{d^2 - \frac{2[P_u (d - h/2) + M_u]}{f_a(b/2)}}
\]

\[
A_y = \frac{0.8 f_y b - P_u / \phi}{f_y} = \]

\[
= 11.8\text{in} - \sqrt{11.8\text{in}}^2 - \frac{2[26.9k (11.8\text{in} - 15.6\text{in} / 2) + 156k - in]}{0.9(0.8)(2.0ksi)(15.6\text{in})} = 1.04\text{in}
\]

\[
A_y = \frac{0.8 f_y b - P_u / \phi}{f_y} = \frac{0.8(2.0ksi)(15.62\text{in})(1.04\text{in}) - 26.9k / 0.9}{60ksi} = -0.066\text{in}^2
\]
Example - Pilaster Design

Why the negative area of steel?
Sufficient area from just masonry to resist applied forces.
Determine \( a \) from just compression.

\[
a = \frac{P_u}{0.8f'_m b} = \frac{26.9 \text{kip}}{0.8(2.0 \text{ksi})15.6 \text{in}} = 1.08 \text{in}
\]

Find the moment

\[
M = P_u \left( \frac{t}{2} - \frac{a}{2} \right) = 26.9 \text{kip} \left( \frac{15.6 \text{in}}{2} - \frac{1.08 \text{in}}{2} \right) = 195 \text{kip-in} \quad M_u = 156 \text{ kip-in}
\]

Sufficient capacity from just masonry. No steel needed.

---

Example - Pilaster Design

0.9D + 1.0W  Check wind suction

At top of pilaster.  \( P_u = 0.9(9.6) - 1.0(8.1) = 0.54 \text{ kips} \)
\( M_u = 0.54 \text{ kips}(5.8 \text{ in}) = 3.1 \text{ kip-in} \)

\[
M_{\text{max}} = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2} \quad x = \frac{L}{2} - \frac{M}{wL} \quad \text{If } x < 0 \text{ or } x > L, M_{\text{max}} = M
\]

Find axial force at this point. Include weight of pilaster.
Example - Pilaster Design

**0.9D + 1.0W**  
Check wind suction  
At top of pilaster.  \( P_u = 0.9(9.6) - 1.0(8.1) = 0.54 \) kips  
\( M_u = 0.54\text{kips}(5.8\text{in}) = 3.1 \text{ kip-in} \)

\[
M_{\text{max}} = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2} \quad x = \frac{L}{2} - \frac{M}{wL} \quad \text{If x<0 or x>L, } M_{\text{max}} = M
\]

\[
x = \frac{L}{2} - \frac{M}{wL} = \frac{288\text{in}}{2} - \frac{3.1\text{kip-in}}{0.416\text{kip/ft}} = 143.7\text{in}
\]

\[
M_u = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2} = \frac{3.1\text{ k-ip}}{2} + \frac{0.0347k/\text{in}(288\text{in})^2}{8} + \frac{(3.1\text{ k-ip})^2}{2(0.0347k/\text{in})(288\text{in})^2} = 361.0\text{ k-ip}
\]

Find axial force at this point. Include weight of pilaster.

\( P_u = 0.54k + 0.9\left(0.20k/\text{ft}\right)(143.7\text{in})\text{ ft}/12\text{in} = 2.69k \)

Design for \( P_u = 2.7 \) kips, \( M_u = 361 \text{ k-ip} \)

---

**Example - Pilaster Design**

**0.9D + 1.0W**  
At top: \( P_u = 0.5 \) k \( M_u = 3 \text{ k-ip} \)  
\( x = 144 \text{ in} \)

\( a = 1.49 \text{ in} \)  
\( A_s = 0.57 \text{ in}^2 \)

**1.2D + 1.0W + 0.5S**  
At top: \( P_u = 8.2 \) k \( M_u = 48 \text{ k-ip} \)  
\( x = 139 \text{ in} \)

\( a = 1.74 \text{ in} \)  
\( A_s = 0.52 \text{ in}^2 \)

**1.2D + 1.6S + 0.5W**  
At top: \( P_u = 22.8 \) k \( M_u = 132 \text{ k-ip} \)  
\( x = 117 \text{ in} \)

\( a = 1.41 \text{ in} \)  
\( A_s = 0.12 \text{ in}^2 \)

Required steel = 0.57 \text{ in}^2  
Use 2-#5 each face, \( A_s = 0.62 \text{ in}^2 \)  
Total bars, 4-#5, one in each cell

---

Combined Flexural and Axial Loads
Columns

- Structural member, not built integrally into a wall, designed primarily to resist compressive loads parallel to its longitudinal axis.
- Minimum side dimension is 8 in. (5.3.1.1(b))
- Distance between lateral supports ≤ 99r \( \{h/r \leq 99\} \) (5.3.1.1(a))
- Minimum reinforcement is 0.0025\( A_n \) (5.3.1.3)
- Maximum reinforcement is 0.04\( A_n \) (5.3.1.3)
  - Additional maximum reinforcement requirements in strength design
- Minimum of 4 bars (5.3.1.3)
- Fully grouted (5.3.1.2)

Ties: (5.3.1.4)
- \( \geq 1/4 \) in. diameter; located in mortar joint or grout
- spacing \( \leq 16 \) longitudinal bar diameter, 48 tie diameter, or least cross-sectional dimension
**Bearing Walls**

**Location of Reaction:**
Wall section

Members that rotate will cause reaction to shift towards edge

Members that experience little rotation (deep truss)

**Bearing area (4.3.4):**

\[ A_1 \sqrt{\frac{A_2}{A_1}} \leq 2A_1 \]

Wall section

A2 ends at edge of member or head joint in stack bond

**Strength Design:**

\[ \varphi = 0.6 \text{ (9.1.4.2)} \]

\[ B_n = 0.8f'_{m}A_{br} \text{ (9.1.8)} \]

**Distribution of Concentrated Loads Along Wall:** (5.1.3)
Load is dispersed along a 2 vertical: 1 horizontal line.

(a) Distribution of concentrated load through bond beam

**Bearing Walls**

**Distribution of Concentrated Loads Along Wall:** (5.1.3)
Load is dispersed along a 2 vertical: 1 horizontal line.

Load

Load is dispersed at 2:1 slope

Bond Beam

Check bearing on hollow wall

Effective Length

(a) Distribution of concentrated load through bond beam
Bearing Walls

(b) Distribution of concentrated load in wall

Combined Flexural and Axial Loads
Load indicating washer (LIW)

Combined Flexural and Axial Loads