Shear Walls

- Load Distribution to Shear Walls
  - Shear wall stiffness
  - Shear walls with openings
  - Diaphragm types
- Types of Masonry Shear Walls
- Maximum Reinforcement Requirements
- Shear Strength
- Example: simple building

Shear Walls: Stiffness

- **h/d < 0.25**: stiffness predominates
- **0.25 < h/d < 4.0**: Both shear and bending stiffness are important
- **h/d > 4.0**: stiffness predominates
9.1.5.2 Deflection calculations shall be based on cracked section properties. Assumed properties shall not exceed half of gross section properties, unless a cracked-section analysis is performed.

Cantilever wall
\[ \Delta_{\text{cant}} = \frac{E_m t}{h} \left( \frac{h}{L} \right) \left[ 4 \left( \frac{h}{L} \right)^2 + 3 \right] \]

Fixed wall (fixed against rotation at top)
\[ \Delta_{\text{fixed}} = \frac{E_m t}{h} \left( \frac{h}{L} \right) \left[ \frac{h}{L} \right] \left[ 4 \left( \frac{h}{L} \right)^2 + 3 \right] \]

Real wall is probably between two cases; diaphragm provides some rotational restraint, but not full fixity.

T- or L- Shaped Shear Walls
Section 5.1.1 Wall intersections designed either to:

a) ________________:

b) ________________:

Connection that transfers shear: (must be in running bond)

a) Fifty percent of masonry units interlock
b) Steel connectors at max 4ft.
c) Intersecting bond beams at max 4 ft. Reinforcing of at least 0.1in\(^2\) per foot of wall

Metal lath or wire screen to support grout

1/8 in. x 1 1/2 in. x 28 in. with 2 in. long 90 deg bends at each end to form U or Z shape
Effective Flange Width (5.1.1.2.3)

Effective flange width on either side of web shall be smaller of actual flange width, distance to a movement joint, or:

- Flange in compression: $6t$
- Flange in tension:
  - Unreinforced masonry: $6t$
  - Reinforced masonry: 0.75 times floor-to-floor wall height

Example: Flanged Shear Wall

Given: Fully grouted shear wall
Required: Stiffness of wall
Solution: Determine stiffness from basic principles.

\[
A = \text{Area of wall}
\]

Find centroid from outer flange surface

\[
\bar{y} = \frac{7.62 \text{in}(48 \text{in})(3.8 \text{in}) + 7.62 \text{in}(40 \text{in})(20 \text{in})}{671 \text{in}^2} = 11.16 \text{in}
\]

\[
I = \text{Moment of inertia}
\]

\[
A_c \approx \text{Corrected area}
\]
Example: Flanged Shear Wall

\[ k = \frac{P}{\Delta} = \frac{P}{\frac{Ph^3}{3E_m I_n} + \frac{Ph}{A_v E_v} + \frac{1}{(112in)^3} + \frac{112in}{3E_m (86000in^4) + \frac{305in^2}{0.4E_m}}} = (0.157in)E_m \]

Shear Walls with Openings

1. Divide wall into piers.
2. Find flexibility of each pier
3. Stiffness is reciprocal of flexibility
4. Distribute load according to stiffness

\[ f = \frac{h^3}{6EI} \left( \frac{2k_i + 2k_b + 2k_i + 3}{k_i + 2k_i k_b + k_b} \right) + \frac{12h}{5EA} (1 + \nu) \]

\[ M_t = Vh \left( \frac{k_i (1 + k_i)}{k_i + 2k_i k_b + k_b} \right) \]

\[ M_b = Vh \left( \frac{k_b (1 + k_i)}{k_i + 2k_i k_b + k_b} \right) \]

As the top spandrel decreases in height, the top approaches a fixed condition against rotation.

If \( h_b = 0 \)

\[ f = \frac{h^3}{6EI} \left( \frac{2 + k_i}{2k_i + 1} \right) + \frac{12h}{5EA} (1 + \nu) \]

\[ M_t = Vh \left( \frac{k_i}{2k_i + 1} \right) \]

\[ M_b = Vh \left( \frac{1 + k_i}{2k_i + 1} \right) \]

Example - Shear Walls with Openings

Given:

Required:
A. Stiffness of wall
B. Forces in each pier under 10 kip horizontal load

Sample calculations: Pier B

\[ h = 4\text{ft} \quad h_i = 4\text{ft} \quad h_b = 8\text{ft} \quad k_i = h/h_i = 4\text{ft}/4\text{ft} = 0.0 \quad k_b = h/h_b = 4\text{ft}/8\text{ft} = 0.5 \]

\[ I = \frac{tL^2}{12} = \]
\[ A = tL = (2.0\text{ft})t \]

\[ f = \frac{h^3}{6EI} \left( \frac{2k_i + k_b + 2k_{ib} + 3}{k_i + 2k_b k_{ib} + k_{ib}} \right) + \frac{12h}{5EA}(1+v) \]

\[ k = \frac{1}{f} = \frac{1}{47.6/Et} = 0.0210Et \]
### Example - Shear Walls with Openings

**Pier** | h (ft) | h₀ (ft) | h₁ (ft) | kₗ | k₀ | I (ft³) | A (ft) | Δᵢ | Δₛ | f | k |
---|---|---|---|---|---|---|---|---|---|---|---|
A | 12 | 0 | 3 | inf | 0.667t | 2t | 308.4 | 18 | 326.4/Eₜ | 0.0031Et |
B | 4 | 8 | 4 | 1 | 0.667t | 2t | 41.6 | 6 | 47.6/Eₜ | 0.0210Et |
C | 4 | 8 | 4 | 1 | 0.667t | 2t | 41.6 | 6 | 47.6/Eₜ | 0.0210Et |

Δᵢ is flexural deformation; Δₛ is shear deformation

Total stiffness is 0.0451Et
Average stiffness from finite element analysis 0.0440Et
Solid wall stiffness 0.1428Et (free at top); 0.25Et (fixed at top)

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### Example - Shear Walls with Openings

**Sample calculations:** Pier B

\[ V_b = \frac{k_b}{\sum k} \]

\[ V = \frac{0.0210 Et}{0.0451 Et} \]

\[ 10k = 4.66 kips \]

\[ M_i = V h \left( \frac{k_i (1 + k_h)}{k_i + 2k_i k_h + k_h} \right) = \]

\[ M_b = V h \left( \frac{k_b (1 + k_h)}{k_i + 2k_i k_b + k_h} \right) = \]

<table>
<thead>
<tr>
<th>Pier</th>
<th>V (kips)</th>
<th>Mᵢ (k-ft)</th>
<th>Mₜ (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.68</td>
<td>3.50</td>
<td>4.66</td>
</tr>
<tr>
<td>B</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
<tr>
<td>C</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
</tbody>
</table>
Example - Shear Walls with Openings

To find axial force in piers, look at FBD of top 4ft of wall.

This 65.9k-ft moment is carried by axial forces in the piers. Axial forces are determined from $M_{ci}/I$, where each pier is considered a concentrated area. Find centroid from left end.

\[
y = \frac{\sum y_i A_i}{\sum A_i} = \]

Example - Shear Walls with Openings

The axial force in pier A is:

FBD's of piers:
Example - Shear Walls with Openings

FBD’s of entire wall to obtain forces in bottom right spandrel:

\[
\sum F_x = 10k - 0.68k - F_x = 0 \\
F_x = 9.32k
\]

\[
\sum F_y = -4.45k + F_y = 0 \\
F_y = 4.45k
\]

\[
\sum M_z = -10k(16\, ft) + (4.66k - f_t) \\
- 4.45k(1\, ft) + 4.45k(11\, ft) + M_z = 0 \\
M_z = 110.8k - f_t
\]

Shear Walls with Openings

Shear walls with openings will be composed of solid wall portions, and portions with piers between openings.

Piers:
- Requirements only in strength design
- Length ≥ 3 x thickness; width ≤ 6 x thickness
- Height < 5 x length
- Factored axial compression ≤ 0.3A_{nf}f'_m (9.3.4.3.1)
- One bar in each end cell (9.3.4.3.2)
- Minimum reinforcement is 0.0007bd (9.3.4.3.2)
Shear Walls: Building Layout

1. All elements either need to be isolated, or will participate in carrying the load
2. Elements that participate in carrying the load need to be properly detailed for seismic requirements
3. Most shear walls will have openings
4. Can design only a portion to carry shear load, but need to detail rest of structure

Coupled Shear Walls
Diaphragms

- Diaphragm: ________________ system that transmits ____________ forces to the vertical elements of the lateral load resisting system.
- Diaphragm classification:
  - ________________: distribution of shear force is based on tributary ________ (wind) or tributary ________ (earthquake)
  - ________________: distribution of shear force is based on relative ________________.

Lateral Force Resisting System

Typical classifications:
- ___________: Precast planks without topping, metal deck without concrete, plywood sheathing
- ___________: Cast-in-place concrete, precast concrete with concrete topping, metal deck with concrete

Rigid Diaphragms

Direct Shear: \[ F_v = V \sum \frac{R_{R_i}}{R_{R_i}} \]

Torsional Shear: \[ F_t = Ve \sum \frac{R_{R_i}d_i}{R_{R_i}d_i^2} \]

\( V \) = total shear force
\( R_{R_i} \) = relative rigidity of lateral force resisting element i
\( d_i \) = distance from center of stiffness
\( e \) = eccentricity of load from center of stiffness
Diaphragms: Example

Given: The structure shown is subjected to a 0.2 kip/ft horizontal force. Relative rigidities are given, where the relative rigidity is a normalized stiffness.

Required: Distribution of force assuming:
- flexible diaphragm
- rigid diaphragm.

Solution: Flexible diaphragm – wind
Distribute based on tributary area

For seismic, the diaphragm load would be distributed the same (assuming a uniform mass distribution), but when wall weights were added in, the forces could be different.
Rigid Diaphragms: Example

Solution: Rigid diaphragm

<table>
<thead>
<tr>
<th>Wall</th>
<th>x</th>
<th>RR</th>
<th>x( RR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>5</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>350</td>
<td></td>
</tr>
</tbody>
</table>

Center of stiffness = 350/10 = 35 ft

<table>
<thead>
<tr>
<th>Wall</th>
<th>RR</th>
<th>d (ft)</th>
<th>RR(d)</th>
<th>RR(d^2)</th>
<th>F_v</th>
<th>F_t</th>
<th>F_total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>-35</td>
<td>-140</td>
<td>4900</td>
<td>8</td>
<td>-4.1</td>
<td>3.9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>15</td>
<td>75</td>
<td>1125</td>
<td>10</td>
<td>2.2</td>
<td>12.2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>65</td>
<td>65</td>
<td>4225</td>
<td>2</td>
<td>1.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>350</td>
<td></td>
<td>10250</td>
<td>20</td>
<td>0.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Diaphragms Design Forces

Solution: Forces are shown for a rigid diaphragm.

The moment is generally taken through chord forces, which are simply the moment divided by the width of the diaphragm. In masonry structures, the chord forces are often take by bond beams.
Drag Struts and Collectors

- Shear forces: generally considered to be uniformly distributed across the width of the diaphragm.
- Drag struts and collectors: transfer load from the diaphragm to the lateral force resisting system.

Diaphragm Behavior

Three lateral force resisting systems: Length=20 ft; Height=14 ft

- Moment Resisting Frame
  - W24x68
  - W14x68
  - k = 83.2 kip/in

- Braced Frame
  - W16x40
  - L 4x4x5/16
  - k = 296 kip/in

- Masonry Shear Wall
  - E = 1800 ksi
  - Face shell bedding
  - End cells fully grouted
  - k = 1470 kip/in

Lateral force resisting systems at 24 ft o.c. Diaphragm assumed to be concrete slab, E=3120ksi, v=0.17, variable thickness, load of 1 kip/ft.
Diaphragm Behavior

Shear Walls

(unreinforced) shear wall (7.3.2.2): Unreinforced wall

(unreinforced) shear wall (7.3.2.3): Unreinforced wall with prescriptive reinforcement.

40db or 24 in. Joint reinforcement at 16 in. o.c. or bond beams at 10 ft.

Reinforcement not required at openings smaller than 16 in. in either vertical or horizontal direction

Reinforcement of at least 0.2 in²
Shear Walls

__________ reinforced shear wall (7.3.2.4): Reinforced wall with prescriptive reinforcement of detailed plain shear wall.

__________ reinforced shear wall (7.3.2.5): Reinforced wall with prescriptive reinforcement of detailed plain shear wall. Spacing of vertical reinforcement reduced to 48 inches.

__________ reinforced shear wall (7.3.2.6):
1. Maximum spacing of vertical and horizontal reinforcement is \( \min\{1/3 \text{ length of wall}, 1/3 \text{ height of wall}, 48 \text{ in.} \} \). [24 in. for masonry in other than running bond].
2. Minimum area of vertical reinforcement is 1/3 area of shear reinforcement
3. Shear reinforcement anchored around vertical reinforcing with standard hook
4. Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
5. Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall [0.0015 for horizontal reinforcement for masonry in other than running bond].

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Minimum Reinforcement of Special Shear Walls

<table>
<thead>
<tr>
<th>Reinforcement Ratio</th>
<th>8 in. CMU wall</th>
<th>12 in. CMU wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_s ) (in^2/ft)</td>
<td>Possibilities</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.064</td>
<td>#4@32, #5@56</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.092</td>
<td>#4@24, #5@32, #6@40</td>
</tr>
<tr>
<td>0.0013</td>
<td>0.119</td>
<td>#4@16, #5@32, #6@40</td>
</tr>
</tbody>
</table>

Use specified dimensions, e.g. 7.625 in. for 8 in. CMU walls.
Special Shear Walls: Shear Capacity

Minimum shear strength (7.3.2.6.1.1):

- Design shear strength, $\phi V_n$, greater than shear corresponding to 1.25 times nominal flexural strength, $M_n$.
- Except $V_n$ need not be greater than $2.5V_u$.

Normal design: $\phi V_n$ has to be greater than $V_u$. Thus, $V_n$ has to be greater than $V_u/\phi = V_u/0.8 = 1.25V_u$. Thus, the requirement approximately doubles the shear.

Seismic Design Category

<table>
<thead>
<tr>
<th>Seismic Design Category</th>
<th>Allowed Shear Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>A or B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D and higher</td>
<td></td>
</tr>
</tbody>
</table>

Response modification factor:
Seismic design force divided by response modification factor, which accounts for ductility and energy absorption.

Knoxville, Tennessee
Category C for Use Group I and II
Category D for Use Group III (essential facilities)

<table>
<thead>
<tr>
<th>Shear Wall</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary plain</td>
<td>1.5</td>
</tr>
<tr>
<td>Detailed plain</td>
<td>2</td>
</tr>
<tr>
<td>Ordinary reinforced</td>
<td>2</td>
</tr>
<tr>
<td>Intermediate reinforced</td>
<td>3.5</td>
</tr>
<tr>
<td>Special reinforced</td>
<td>5</td>
</tr>
</tbody>
</table>
Maximum reinforcing (9.3.3.5)

No limits on maximum reinforcing for following case (9.3.3.5.4):
\[
\frac{M_u}{V_u d_v} \leq 1 \quad \text{and} \quad R \leq 1.5
\]
Squat walls, not designed for ductility

In other cases, can design by either providing boundary elements or limiting reinforcement.

Boundary element design (9.3.6.5):
More difficult with masonry than concrete
Boundary elements not required if:
\[
P_u \leq 0.1 f_m' A_g \quad \text{geometrically symmetrical sections}
\]
\[
P_u \leq 0.05 f_m' A_g \quad \text{geometrically unsymmetrical sections}
\]

AND
\[
\frac{M_u}{V_u d_v} \leq 1 \quad \text{OR} \quad V_u \leq 3 A_n \sqrt{f_m'} \quad \text{AND} \quad \frac{M_u}{V_u d_v} \leq 3
\]

Maximum reinforcing (9.3.3.5)

Reinforcement limits: Calculated using
Maximum stress in steel of \( f_y \)
Axial forces taken from load combination \( D+0.75L+0.525Q_e \)
Compression reinforcement, with or without lateral ties, permitted to be included for calculation of maximum flexural tensile reinforcement

Uniformly distributed reinforcement
\[
\alpha = 1.5 \quad \text{ordinary walls}
\]
\[
\alpha = 3 \quad \text{intermediate walls}
\]
\[
\alpha = 4 \quad \text{special walls}
\]

Compression steel with area equal to tension steel
\[
\rho = \frac{A_t}{bd} = \frac{0.64 f_m' b \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P}{bd}}{f_y - \min \left\{ \frac{d}{d'} \left( \varepsilon_{mu} + \alpha \varepsilon_y \right) \right\} E_x}
\]
Maximum reinforcing

Consider a wall with uniformly distributed steel:

\[ C_m + C_s - T_s = P \]

\[ C_m = 0.8 f_m' \left( 0.8 \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} + d_y \right) b \]

\[ T_s = f_y A_s \left( \frac{\alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \frac{\varepsilon_{mu} - \varepsilon_y}{\alpha \varepsilon_y} + \frac{1}{2} \frac{\varepsilon_y}{\alpha \varepsilon_y} \]

\[ C_s = f_y A_s \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left[ \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu}} \right] \]

\[ = f_y A_s \left( \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \]

\[ A_s \text{ taken as total steel} \]
\[ d_y \text{ is actual depth of masonry} \]
Maximum reinforcing, $\varepsilon_s = 4\varepsilon_y$

Maximum reinforcing, $\varepsilon_s = 3\varepsilon_y$
Maximum reinforcing, $\varepsilon_s = 1.5\varepsilon_y$

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Shear Strength (9.3.4.1.2)

$$V_n = (V_{um} + V_{ns})\gamma_g$$ $\phi = 0.8$ $\gamma_g = 0.75$ for partially grouted shear walls and 1.0 otherwise

$$V_m = \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_{nv} \sqrt{f_m'} + 0.25P_u$$ $M_u/V_u d_v$ need not be taken > 1.0 $P_u$ = axial load

$$V_s = 0.5 \left( \frac{A_v}{s} \right) f_y d_v$$ Vertical reinforcement shall not be less than one-third horizontal reinforcement; reinforcement shall be uniformly distributed, max spacing of 8 ft (9.3.6.2)

Maximum $V_n$ is:

$$V_n \leq \left( 6 A_{nv} \sqrt{f_m'} \right) \gamma_g$$ $M_u/V_u d_v \leq 0.25$

$$V_n \leq \left( 4 A_{nv} \sqrt{f_m'} \right) \gamma_g$$ $M_u/V_u d_v \geq 1.0$

Interpolate for values of $M_u/V_u d_v$ between 0.25 and 1.0

$$V_n = \left( \frac{4}{3} \left( 5 - 2 \frac{M_u}{V_u d_v} \right) A_{nv} \sqrt{f_m'} \right) \gamma_g$$

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## Partially Grouted Walls

<table>
<thead>
<tr>
<th>Method</th>
<th>$V_{exp}/V_n$</th>
<th>$V_{exp}/V_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. Dev.</td>
</tr>
<tr>
<td>Partially Grouted Walls (Minaie et al, 2010; 60 tests)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 Provisions</td>
<td>0.90</td>
<td>0.26</td>
</tr>
<tr>
<td>Multiply shear strengths by $A_e/A_n$</td>
<td>1.53</td>
<td>0.43</td>
</tr>
<tr>
<td>Using just face shells</td>
<td>1.77</td>
<td>0.78</td>
</tr>
<tr>
<td>Fully Grouted Walls (Davis et al, 2010; 56 tests)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 Provisions</td>
<td>1.16</td>
<td>0.17</td>
</tr>
</tbody>
</table>

$$0.90/1.16 = 0.776; \text{ rounded to } 0.75$$

---

## Example

Given: 10ft. high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, $f'_m=2000$psi; 2-#5 each end; #5 at 48in. (one space will actually be 40 in.); superimposed dead load of 1 kip/ft. $S_{DS} = 0.4$

**Required:** Maximum seismic load based on flexural capacity; shear steel required to achieve this capacity.

**Solution:** Check capacity in flexure using 0.9D+1.0E load combination.

Weight of wall: 38 psf(10ft) = 0.38 k/ft  
(Wall weight from lightweight units, grout at 48 in. o.c.)

$$P_u = (0.9-0.2S_{DS})D = (0.9-0.2*0.4)(1k/ft+0.38k/ft)$$

$$= 0.82(1.38k/ft) = 1.13 \text{ k/ft}$$

Total = 1.13k/ft(16ft) = 18.1 kips
Example

Solve for stresses, forces and moment in terms of \( c \), depth to neutral axis.

\[
\begin{align*}
T_1 & \quad T_2 & \quad T_3 & \quad T_4 & \quad C_2 & \quad C_1 \\
\downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow \\
188 \text{ in} & \quad 140 \text{ in} & \quad 92 \text{ in} & \quad 52 \text{ in} & \quad c & \quad 8 \text{ in}
\end{align*}
\]

Example

• Use trial and error to find \( c \) such that \( P_n = 18.1 \text{ kips}/0.9 = 20.1 \text{ kips} \)
  
  \[ P_n = C_1 + C_2 - T_1 - T_2 - T_3 - T_4 \]

• \( c = 14.85 \text{ inches} \)
  
  \[ C_1 = 97.6 \text{ kips}; \quad C_2 = 15.5 \text{ kips} \]
  
  \[ T_1 = 37.2 \text{ kips}; \quad T_2 = 18.6 \text{ kips}; \quad T_3 = 18.6 \text{ kips}; \quad T_4 = 18.6 \text{ kips (all yielded)} \]

• Sum moments about middle of wall (8 ft from end) to find \( M \)

\[ M_n = 37.2(188-96) + 18.6(140-96) + 18.6(92-96) + 18.6(52-96) + 97.6[96-8/2] + 15.5[96-8-(0.8*14.85-8)/2] \]

\[ M_n = 13660 \text{ kip-in} = 1140 \text{ kip-ft}; \quad M_u = 0.9(1140 \text{ kip-ft}) = 1025 \text{ kip-ft} \]

• Seismic = \( M_u/h = (1025 \text{ kip-ft})/(10\text{ft}) = 102.5 \text{ kips} \)
Example

Calculate net shear area, \( A_{nv} \), including grouted cells.

Net area: \( A_{nv} = 2.5in(192in) + 5(8in)(7.62in - 2.5in) = 685in^2 \)

\[
M_u/\left(V_u d_v\right) = \left(V_u h\right)/\left(V_u d_v\right) = 10 \text{ ft} / 16 \text{ ft} = 0.625
\]

Maximum \( V_n \)

\[
V_u \leq 0.8\left(V_n\right) = 0.8\left(114.9\text{ kips}\right) = 91.9\text{ kips}
\]

Maximum shear controls; seismic = 91.9 kips

Example

Top of wall (critical location for shear):
\( P_u = (0.9-0.2S_{DS})D = 0.82(1k/ft) = 0.82 \text{ k/ft} \)
Total: 0.82k/ft(16ft) = 13.1 kips

\[
V_{nm} = \left[4.0 - 1.75(0.625)\right]\left(685in^2\right)\sqrt{2000 \text{ psi}} + 0.25(13.1\text{ kips}) = 89.0\text{ kips}
\]

\[
V_{ns} = \frac{V_n - \gamma_g V_{nm}}{\gamma_g} = \frac{114.9 - 0.75(89.0k)}{0.75} = 64.2k
\]

Use #5 bars in bond beams. Determine spacing.

\[
V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v \quad \Rightarrow \quad s = \frac{0.5A_v f_y d_v}{V_{ns}}
\]

Use ___ bond beams, spaced at ____ in. o.c. vertically.
Example: Maximum Reinforcing

Check with boundary elements:

- Geometrically symmetrical section:
  - $0.1f'_{m}A_{n} = 0.1(2.0\text{ksi})(685\text{in}^2) = 137\text{ kips}$
  - Check with $P_u = (1.2+0.2S_{DS})D = 1.28(1.38\text{k/ft}) = 1.77\text{ k/ft}$
    - Total = $1.77\text{k/ft}(16\text{ft}) = 28.3\text{ kips}$  **OK**
  - Since $M_u(V_u d_v) = 0.625 \leq 1$ No boundary elements required
  - Maximum reinforcement provisions satisfied

If we needed to check maximum reinforcing, there are at least two ways. These are shown on the following slides.
A live load of 1 k/ft is assumed for maximum reinforcement calculations.

**Example: Maximum Reinforcing**

Check on axial load:

a) Set steel strain to limit and find $c$

b) Find axial load for this $c$

c) If applied axial load is less than axial load, OK

Steel strain = $1.5\varepsilon_y = 0.00310$
$c = 0.0025/(0.0025+0.00310)*192 = 85.7\text{ in.}$
From spreadsheet, $P_u = 258\text{ kips}$

$$P_u = D+0.75L+0.525Q_E = (1.38\text{k/ft} + 0.75(1\text{k/ft}))16\text{ft} = 34.1\text{ kips} \quad \text{OK}$$

Note: Did not include compression steel which is allowed when checking maximum reinforcement.
If compression steel had been included, $P_u = 299\text{ kips}$
**Example: Maximum Reinforcing**

Find steel strain for given axial load:

a) Through trial and error (or solver), find $c$ for given axial load

b) Determine steel strain

c) If steel strain greater than limit, OK

\[ P = 34.1 \text{ kips} \quad \text{(from D+0.75L+0.525QE)} \]
\[ c = 20.4 \text{ in.} \quad \text{(from spreadsheet and solver)} \]
\[ \varepsilon_s = \frac{(192-20.4)}{20.4} \times 0.0025 = 0.0210 = 10.2\varepsilon_y \quad \text{OK} \]

Note: Did not include compression steel which is allowed when checking maximum reinforcing.
If compression steel had been included: $c = 11.3$ in. $\varepsilon_s = 0.0400 = 19.3\varepsilon_y$

---

**Example: Special Shear Wall**

If this were a special shear wall:

- $\phi V_n \geq$ shear corresponding to $1.25 M_n$ (7.3.2.6.1.1)
- $1.25 M_n = 1.25(1140 \text{ k-ft}) = 1425 \text{ k-ft}$
- $\phi V_n = M_n/h = 1425 \text{k-ft/10ft} = 142.5 \text{kips}$
- $V_n = 142.5k/0.8 = 178.1 \text{kips}$
- But, maximum $V_n$ is 114.9 kips (64% of required).

Cannot build wall. Need to fully grout to get greater shear capacity.
Example: Special Shear Wall

**Fully grouted:**
- Use spreadsheet to find $M_n$ corresponding to $P_n = 20.1$ kips
- $M_n = 1156$ kip-ft; $M_u = 1041$ kip-ft a 1.4% increase
- Seismic = 104.1 kip-ft
- $1.25M_n = 1.25(1156 \text{ k-ft}) = 1445 \text{ k-ft}$
- $\phi V_n = M_n/h = 1445\text{k-ft}/10\text{ft} = 144.5 \text{kips}$
- $V_n = 144.5\text{k}/0.8 = 180.6 \text{kips}$

$$A_{nv} = 7.625 \text{in.}(196 \text{in.}) = 1494 \text{in}^2$$

$$V_{nm} = \left[4.0 - 1.75(0.625)\right]\sqrt{2000 \text{ psi} + 0.25(13.1 \text{kips})} = 194.2 \text{kips}$$

No shear steel required; choose horizontal reinforcement to satisfy prescriptive requirements.

---

Example: Special Shear Wall

Prescriptive reinforcement:
- Minimum reinforcement in either direction: 0.0007$A$
  - Vertical: $0.0007(1494 \text{ in}^2) = 1.05 \text{ in}^2$
  - 7-#5’s provided: $7(0.31 \text{ in}^2) = 2.17 \text{ in}^2$ OK
- Spacing is min{1/3 length of wall, 1/3 height of wall, 48 in.} = {192 in./3, 120 in./3, 48 in.} = min{64, 40, 48 in.}
- Need to decrease spacing of vertical reinforcement to 40 inches
  - Use spacings of 40, 32, 40, 32, 40 inch to match wall coursing.
  - This will have a small impact on moment capacity, increasing $M_u$ from 1041 kip-ft to 1166 kip-ft, a 12% increase
  - This increases the required nominal shear strength to 202.4 kips. This is slightly greater than the nominal shear strength due to the masonry (194.2 kips), but with the prescriptive reinforcement, everything will be OK.
Example: Special Shear Wall

Prescriptive reinforcement:
• Minimum reinforcement in either direction: 0.0007A
  • Horizontal: 0.0007(120 in)(7.625 in) = 0.64 in²
  • Use #5’s at 40 in (required spacing); total is 3 bars
  • 3(0.31 in²) = 0.92 in² OK
• Total reinforcement: 0.002A
  • Vertical: 2.79in²/1494in² = 0.00187A
  • Horizontal: 0.92in²/915in² = 0.00100A
  • Total = 0.00187A + 0.00100A = 0.00287A OK

Example

Given: Wall system constructed with 8 in. grouted CMU (Type S mortar).
Required: Determine the shear distribution to each wall

Solution: Use gross properties (will not make any difference for shear distribution).
Example

Pier b

\[ k_b = \frac{Et}{(h/L)^2 + \frac{h}{L} + 3} = \frac{E(7.62in)}{(\frac{112in}{64in})^2 + \frac{112in}{64in} + 3} = 0.286inE \]

Piers a and c  Previously determined stiffness from basic principles.

\[ k_{a,c} = 0.157inE \]

Portion of shear load = stiffness of pier over the sum of the stiffnesses.

\[ V_a = \frac{k_{a,c}V}{\sum k} = \frac{0.157inE}{0.157inE + 0.286inE + 0.157inE} = 0.26V \quad V_b = 0.48V \]

Deflection calculations:
\[ k = (0.600in.)E = (0.600in.)(1800ksi) = 1080 \text{ kip/in} \]
Reduce by a factor of 2 to account for cracking: \( k = 540 \text{ kip/in} \)

Example

Required: Design the system for a horizontal earthquake load of 35 kips, a dead load of 1200 lb/ft length of building, a live load of 1000 lb/ft length of building, and a vertical earthquake force of 0.2 times the dead load. Use a special reinforced shear wall (R>1.5).

Solution: Use strength design. Check deflection.

\[ \Delta = \frac{P}{k} = \frac{35k}{540k/\text{in}} = 0.065\text{in} \]

\[ \Delta = \frac{0.065\text{in}}{112\text{in}} = 0.000579 \quad \text{or} \quad \Delta = \frac{h}{1728} \]
Example

Wall b: \[ V_{ub} = 0.48V_u = 0.48(35k) = 16.8k \]
\[ D = 1.2k/ft(1ft/12in) + 0.075ksf(112in)(64in)(1ft^2/144in^2) = 14.1kips \]

Use load combination 0.9D+E
\[ M_u = 16.8k(112in)(1ft/12in) = 156.8k-ft \quad P_u = 0.9(14.1k) - 0.2(14.1k) = 9.87k \]

Minimum prescriptive reinforcement for special shear walls:
Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall

Use prescriptive reinforcement of \((0.002-0.0007)A_g = 0.0013A_g\) in vertical direction
\[ A_s = 0.0013(64in)(7.625in) = 0.63in^2 \]

Example: Flexural Steel

Use load combination 0.9D+E \[ M_u = 156.8k-ft \quad P_u = 9.87k \]

Try 2-#4 each end; 1-#4 middle: \[ A_s = 1.00in^2 \]
Middle bar is at 28 in. (8 cells, cannot place bar right in center)
From spreadsheet: \( \phi M_n = 152.7k-ft \quad \text{NG} \)
Note: used solver to find value

Try 1-#6 each end; 1-#6 middle: \[ A_s = 1.32in^2 \quad \phi M_n = 194.1k-ft \quad \text{OK} \]
Try 2-#5 each end; 1-#5 middle: \[ A_s = 1.55in^2 \quad \phi M_n = 219.1k-ft \quad \text{OK} \]

Use 2-#5 each end; 1-#5 middle: \[ A_s = 1.59in^2 \]
Example: Maximum Reinforcing

Check maximum reinforcing:
Axial force from load combination $D + 0.75L + 0.525Q_E$

$L = 1\text{k}/\text{ft}(104\text{in})(1\text{ft}/12\text{in}) = 8.67\text{kip}$

$D + 0.75L + 0.525Q_E = 14.1\text{k} + 0.75(8.67\text{k}) + 0.525(0) = 20.60\text{kip}$

For maximum reinforcing calculations, compression steel can be included even if not laterally supported

For #5 bars and $P = 20.60\text{kip}$, $c = 6.35\text{in}$. (from solver on spreadsheet)

$$\varepsilon_s = \frac{d - kd}{kd} = \frac{60\text{in} - 6.35\text{in}}{6.35\text{in}} = 0.0025 = 0.0211 = 10.21\varepsilon_y \geq 4\varepsilon_y \quad \text{OK}$$

Example: Shear

$$\frac{M_u}{V_u d_v} = \frac{V_u(112\text{in})}{V_u(64\text{in})} = 1.75 > 1$$
Use $M_u/V_u d_v = 1.0$

Find $P_u$ neglecting wall weight

$P_u = 0.9(10.4\text{k}) - 0.2(10.4\text{k}) = 7.28\text{k}$

$$V_{nm} = \left[4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right]A_n\sqrt{f_m'} + 0.25P_u$$

$$= \left[2.25\right](7.625\text{in})(64\text{in})\sqrt{2000\text{psi}(1k/1000\text{lb})} + 0.25(7.28\text{k}) = 50.9\text{kips}$$

Ignore any shear steel: $V_n = 50.9\text{kips}$

$$\phi V_n = 0.8(50.9\text{kips}) = 40.7\text{kips} > 16.8\text{kips} \quad \text{OK}$$
Example: Shear

\[ M_n = \phi M_n / \phi = 219.1 \text{k-ft}/0.9 = 243.4 \text{k-ft} \]

\[ V \text{ corresponding to } M_n: [243.3 \text{k-ft}/(112 \text{in})](12 \text{in}/\text{ft}) = 26.1 \text{kips} \]

1.25 times \( V \): \( 1.25(26.1) = 32.6 \text{kips} < \phi V_n = 40.7 \text{kips} \) OK

Minimum prescriptive horizontal reinforcement: \( 0.0007 A_g \)

\[ A_s = 0.0007(64 \text{in})(7.625 \text{in}) = 0.34 \text{in}^2 \]

Maximum spacing is \( \{64 \text{in.}/3, 48 \text{in.}\} = 21.3 \text{ in.} \)

Use bond beams at 16 in. o.c. (I would be fine with 24 in.)

Wall b: Vertical bars: 2-#5 each end; 1-#5 middle

Horizontal bars: #4 at 24 in. o.c. (Total 5-#4)

Example

Wall a: \( V_{ua} = 0.26 V_u = 0.26(35k) = 9.1k \)

\[ D = 1.2k/\text{ft}(20\text{in+40in})(1\text{ft}/12\text{in}) + 0.075\text{ksf}(112\text{in})(40\text{in+48in})(1\text{ft}^2/144\text{in}^2) = 11.1 \text{kips} \]

Use load combination 0.9D+E

\[ M_u = 9.1k(112\text{in})(1\text{ft}/12\text{in}) = 84.9\text{k-ft} \]

\[ P_u = 0.9(11.1k) - 0.2(11.1k) = 7.77k \]

Minimum prescriptive reinforcement: \( 0.0013 A_g \)

\[ A_s = 0.0013(88\text{in})(7.625\text{in}) = 0.87\text{in}^2 \]

Design issues:

Flange in tension: easy to get steel; problems with \( \rho_{\text{max}} \)

Flange in compression: hard to get steel; \( \rho_{\text{max}} \) is easy

Steel in both flange and web affect \( \rho_{\text{max}} \)

Need to also consider loads in perpendicular direction

Need to check shear at interface of flange and web
Example

Use load combination 0.9D+E \( M_u = 84.9\text{k-ft} \) \( P_u = 7.77\text{k} \)

Axial force for \( \rho_{\text{max}} \) from load combination \( D+0.75L+0.525QE \)
\( L=1\text{k/ft}(60\text{in})(1\text{ft/12in}) = 5.00\text{kip} \)
\( D+0.75L+0.525QE = 11.1k+0.75(5.00k)+0.525(0) = 14.85\text{kip} \)

To be consistent with other wall, try #5 bars
Try 3 in the flange (either distributed as shown, or one in the corner, and two in the jamb
Try 2 in the web jamb

Flange in tension: \( \phi M_n = 150.8\text{k-ft} \) \( \text{OK} \)
Max reinforcing check: \( \epsilon_s = 0.0127 = 6.14\epsilon_y \)
Note: with 2-#5 in flange, \( \phi M_n = 106.9\text{k-ft} \), but might need 3 bars for perpendicular direction or out-of-plane

Flange in compression: \( \phi M_n = 125.8\text{k-ft} \) \( \text{OK} \)
Max reinforcing check: \( \epsilon_s = 0.0564 = 27.3\epsilon_y \)

Example: Shear

\[
\frac{M_u}{V_u d_v} = \frac{V_u (112\text{in})}{V_u (40\text{in})} = 2.8 > 1 \quad \text{Use } M_u/V_u d_v = 1.0
\]

\[
V_u = [2.25](7.625\text{in})(40\text{in})\sqrt{2000 \text{ psi}}(1k / 1000\text{ lb}) + 0.25(4.2k) = 31.7\text{kips}
\]

Ignore any shear steel: \( V_u = 31.7\text{kips} \quad \phi V_u = 0.8(31.7\text{kips}) = 25.4\text{kips} \quad \text{OK} \)

\[
M_n = \phi M_n/\phi = 150.8\text{k-ft/0.9} = 167.6\text{k-ft}
\]
\( V \) corresponding to \( M_n: \) [167.6-k-ft/(112in)](12in/ft) = 18.0kips
1.25 times \( V: \) 1.25(18.0k) = 22.4kips < \( \phi V_u = 25.4\text{kips} \quad \text{OK} \)

Minimum prescriptive horizontal reinforcement: 0.0007\( A_g \)
Use 5 bond beams, 24in. o.c. (top spacing is only 32in.)

Walls a,c: Vertical bars: 5-#5 as shown in previous slide
Horizontal bars: #4 at 24 in. o.c. (Total 5-#4)
Example

Section 5.1.1.2.4 requires shear to be checked at interface. Check intersection of flange and web in walls a and c.

Approximate shear force is tension force in flexural steel.

\[ V_u = A_v f_y = 3(0.31\text{in}^2)(60\text{ksi}) = 55.8\text{kips} \]

Conservatively take \( M_d/V_u d_v = 1.0 \), and \( P_u = 0 \).

\[ V_m = [4.0 - 1.75(M/V_d v)]A_v \sqrt{f_y} + 0.25P_v \\
= [4.0 - 1.75(1.0)](7.625\text{in})(112\text{in})\sqrt{2000\text{psi}}(1k/1000\text{lb}) = 85.9\text{kips} \]

\[ \phi V_m = 0.8(85.9\text{kips}) = 68.7\text{kips} \quad \text{OK} \]

Example: Drag Struts

Distributed shear force: \( \frac{V_u}{L} = \frac{35k}{12\text{in}} = \frac{12in}{224\text{in}} = 1.875\text{k/ft} \)

Look at top of wall:

\[
\begin{align*}
1.88k/ft & \quad 2.8k & \quad 3.4k & \quad 16.8k & \quad 3.4k & \quad 2.8k & \quad 1.88k/ft \\
9.1k & \quad \text{Drag strut} & \quad 3.4k & \quad \text{Drag strut} & \quad 3.33f & \quad 3.33f & \quad 5.33f & \quad 3.33f & \quad 3.33f & \quad 3.33f & \quad 9.1k
\end{align*}
\]