**Assumptions: (9.3.2)**

1. Plane sections remain plane
2. All masonry in tension is neglected
3. Perfect bond between steel and grout
4. Member is straight prismatic section
5. \( \varepsilon_m = \) ______ clay masonry
6. \( \varepsilon_m = \) ______ concrete masonry
7. Masonry stress = \( f_m' \)
8. Masonry stress acts over \( a = \) _____

\[ \rho = \frac{A_s}{bd} \]

\[ M_n = \phi M_{cr} \]

\( \phi = 0.9 \) (9.1.4.4)
Beams - Strength Design (9.3.4.2)

Beam: Factored axial compressive force $\leq 0.05A_nf'_m$

Minimum reinforcement: $M_n \geq 1.3 \times$ cracking strength; or $A_s \geq (4/3)A_{s,req'd}$

Modulus or rupture, $f_r = \text{Table 9.1.9.2}$

Maximum reinforcement: (9.3.3.5): $\varepsilon_s = 1.5\varepsilon_y$

$$\rho_{\text{max}} = \frac{0.8(0.8)f'_m}{f_y} \left(\frac{\varepsilon_m}{\varepsilon_m + \varepsilon_s}\right)$$

<table>
<thead>
<tr>
<th>Steel Ratio</th>
<th>Grade 60 steel</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Clay</td>
</tr>
<tr>
<td>Max reinf. ($f'_m/f_y$)</td>
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</tr>
<tr>
<td>$f_y = 60$ ksi; $f'_m = 2.00$ ksi</td>
<td>0.01131</td>
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Modulus of Rupture, psi  

<table>
<thead>
<tr>
<th>Masonry Type</th>
<th>Mortar Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portland cement/lime or mortar cement</td>
</tr>
<tr>
<td></td>
<td>M or S</td>
</tr>
<tr>
<td>Normal to Bed Joints</td>
<td></td>
</tr>
<tr>
<td>Solid Units</td>
<td>133</td>
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<tr>
<td>Hollow Units</td>
<td></td>
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<tr>
<td>Ungrounded</td>
<td>84</td>
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<tr>
<td>Fully Grouted</td>
<td>163</td>
</tr>
<tr>
<td>Parallel to bed joints in running bond</td>
<td></td>
</tr>
<tr>
<td>Solid Units</td>
<td>267</td>
</tr>
<tr>
<td>Hollow Units</td>
<td></td>
</tr>
<tr>
<td>Ungrounded and partially grouted</td>
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</tr>
<tr>
<td>Fully grouted</td>
<td>267</td>
</tr>
<tr>
<td>Parallel to bed joints in stack bond</td>
<td></td>
</tr>
<tr>
<td>Continuous grout section parallel to bed joints</td>
<td>335</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

* Use linear interpolation for partially grouted masonry.
Maximum reinforcement: (9.3.3.5): $\varepsilon_s = 1.5 \varepsilon_y$

Strength Design Procedure

1. Determine $a$, depth of compressive stress block

   $$a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m'b}}$$

2. Solve for $A_s$

   $$A_s = \frac{0.8f_m'b a}{f_y}$$
Example - Masonry Beam

Given: M = 150 k-in. dead load; M = 150 k-in. live load; Grade 60 steel, f’m = 2000 psi; 8 in. CMU; depth of section limited to three courses; Type S masonry cement mortar

Required: Design section

Solution: For three units, d = 2(8 in.) + 4 in. = 20 in.

\[(M_n)_{reqd} = \]

\[a = d - \sqrt{d^2 - \frac{2M_n}{0.8f'_mb}} = \]

Solve for \(A_s\)

\[A_s = \frac{0.8f'_mab}{f_y} = \]

Use \((A_s = \quad)\)

Example - Masonry Beam, Check Steel

Check minimum steel

\[f_r = \]

Cracking moment: \(M_{cr} = f_r \frac{bh^2}{6} = 160 \text{ psi} \left(\frac{7.625\text{ in}}{24\text{ in}}\right)^2 = 117 \text{ k-in.} \]

Required \(M_n = 1.3M_{cr} = 1.3(117 \text{ k-in}) = 152 \text{ k-in.} < 467 \text{ k-in.} \quad \text{OK}\)
Example - Masonry Beam, Check Steel

Check maximum steel

\[ \rho = \frac{A_s}{bd} = \frac{0.44\text{in}^2}{(7.625\text{in})(20\text{in})} = 0.00288 \]

TMS 402 code \((1.5\varepsilon_y)\)

\[ \rho_{\text{max}} = \frac{0.8(0.8)f'_{m}'}{f_y} \left( \frac{\varepsilon_m}{\varepsilon_y + \varepsilon_s} \right) \]

\[ \rho_{\text{max}} = \frac{0.8(0.8)(2000\text{psi})}{60000\text{psi}} \left( \frac{0.0025}{0.0025 + 1.5 \left( \frac{60\text{ksi}}{29000\text{ksi}} \right)} \right) = 0.00952 \quad \text{OK} \]

Partially Grouted Walls

\[ b = \text{width of effective flange} = \min\{s, 6t, 72 \text{ in}\} \quad (5.1.2) \]

A. Neutral axis in flange:
   a. Almost always the case
   b. Design and analysis for solid section

B. Neutral axis in web
   a. Design as a T-beam section

C. Often design based on a 1 ft width

<table>
<thead>
<tr>
<th>Spacing (inches)</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
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<tbody>
<tr>
<td>8</td>
<td>0.16</td>
<td>0.30</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>16</td>
<td>0.082</td>
<td>0.15</td>
<td>0.23</td>
<td>0.33</td>
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<tr>
<td>24</td>
<td>0.055</td>
<td>0.10</td>
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<td>0.22</td>
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<td>32</td>
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<td>0.075</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>40</td>
<td>0.033</td>
<td>0.060</td>
<td>0.093</td>
<td>0.13</td>
</tr>
<tr>
<td>48</td>
<td>0.028</td>
<td>0.050</td>
<td>0.078</td>
<td>0.11</td>
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<tr>
<td>56</td>
<td>0.024</td>
<td>0.043</td>
<td>0.066</td>
<td>0.094</td>
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<td>64</td>
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<td>0.058</td>
<td>0.082</td>
</tr>
<tr>
<td>72</td>
<td>0.018</td>
<td>0.033</td>
<td>0.052</td>
<td>0.073</td>
</tr>
</tbody>
</table>
Partially Grouted Walls - Example

Given: 8 in. CMU wall; 12 ft high; Grade 60 steel, \( f_m \)=2000 psi; Wind load of \( w_u \)= 30 psf

Required: Reinforcing (place in center of wall)

Solution:

\[
M_u = \frac{w_u h^2}{8} = \frac{30 \text{lb/ft}^2 (12 \text{in/ft})(12 \text{ft})^2}{8} = 6480 \text{lb-in/ft} = 0.54k \text{ ft-lb/ft}
\]

Use # __ @ __ inches \( (A_s=____\text{in}^2/\text{ft}) \)

Partially Grouted Walls – Example, cont

Minimum Reinforcement: No requirements for walls

Maximum Reinforcement: Requirements apply, although can be difficult to meet for heavily reinforced shear walls

\[
\rho_{\text{max}} = 0.00952 \quad \rho = \frac{0.05/(12*3.81)}{12} = 0.00109 \quad \text{OK}
\]

Horizontal spanning masonry between bars:

- Some treat as unreinforced masonry, although debate as to whether you can mix unreinforced and reinforced masonry.
- There is arching occurring, so not truly a simply supported flexural member between vertical bars.
Partially Grouted Walls – Example, cont

Try #4 @ 72 in. (0.033 in²/ft)

Problem: 72 inch is greater than maximum spacing of 6t = 48 in.
Solution:
• Design as 48 in. wide section; load is 30psf(72in.)(1ft/12in.)
  = 180 lb/ft
• Design partially grouted wall on basis of 1 ft unit section; increase
  load by factor of 72/48=1.50

From a practical standpoint, there is little difference.
#4, b = 48 in., φMₙ = 3361 lb-ft
#4, b = 72 in., φMₙ = 3384 lb-ft (0.7% greater)
Mᵤ applied [wᵤ = 30psf(72in/(12in/ft)=180lb/ft] = 3240 lb-ft  OK

Out-of-Plane: Maximum Reinforcement

For an 8 in. CMU wall, fully grouted, bars in the center, Grade 60
steel, and f'ₚ =2000 psi, the following table lists the maximum
reinforcement for various axial loads.

<table>
<thead>
<tr>
<th>P/(bdfₚ)</th>
<th>Aₛ (in² per ft)</th>
<th>Spacing (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>#4</td>
</tr>
<tr>
<td>0</td>
<td>0.436</td>
<td>8 (5.5)</td>
</tr>
<tr>
<td>0.05</td>
<td>0.359</td>
<td>8 (6.7)</td>
</tr>
<tr>
<td>0.10</td>
<td>0.283</td>
<td>16 (8.5)</td>
</tr>
<tr>
<td>0.15</td>
<td>0.207</td>
<td>16 (11.6)</td>
</tr>
<tr>
<td>0.20</td>
<td>0.130</td>
<td>24 (18.4)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.054</td>
<td>48 (44.3)</td>
</tr>
</tbody>
</table>
Partially Grouted Walls - Tolerances

Placement tolerances: (3.4.B.11)

d ≤ 8 in. ± 1/2 in.
8 in. < d ≤ 24 in. ± 1 in.
d > 24 in. ± 1 1/4 in.

Along wall: ± 2 in.

8 in. CMU; f'_m=2000 psi; Grade 60

Shear - Strength Design (9.3.4.1.2)

\[ V_n = V_{nm} + V_{ns} \]
\[ V_{nm} = 4 - 1.75 \left( \frac{M_u}{V_u d_v} \right) A_{nv} \sqrt{f'_m} \]
\[ V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_v d_v \]

\( \phi = 0.8 \)

\( d_v = \) actual depth of masonry

\[ M_u / (V_u d_v) \] need not be taken > 1.0

\[ s_{max} = \min \{d / 2, 48in.\} \]

\[ V_n \leq 6 A_{nv} \sqrt{f'_m} \] where \( M_u / (V_u d_v) \leq 0.25 \)

\[ V_n \leq 4 A_{nv} \sqrt{f'_m} \] where \( M_u / (V_u d_v) \geq 1.00 \)

Linearly interpolate between 0.25 and 1.0

Conservative approximation, \( M_u / (V_u d_v) = 1.0 \)

\[ V_{nm} = 2.25 A_{nv} \sqrt{f'_m} \]
Shear - Strength Design

Detailing of shear reinforcement for beams (9.3.4.2.3):
A. Single bar with 180-degree hook at each end
B. Hook shear reinforcement around flexural reinforcement
C. Minimum area of shear reinforcement is 0.0007bdv
D. First bar within dv/4
E. Maximum spacing is dv/2 or 48 in.

Sections within dv/2 from face of support can be designed for shear at dv/2 (8.3.5.4):
A. Noncantilever beam
B. Reaction introduces compression into end region of member
C. No concentrated load between dv/2 and face of support

Deflections

Deflection of beam or lintels supporting unreinforced masonry is limited to L/600 (5.2.1.4.1)
Deflections of approximately L/300 needed to be visible.

\[ I_{eff} = I_n \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \leq I_n \]  
(Equation 5-1)

\[ I_{cr} = \frac{b k^3 d^3}{3} + nA_s(d-kd)^2 \]  
f:\ Table 9.1.9.2

Deflections do not need to be checked when \( L \leq 8d \) (5.2.1.4.3). End restraint of masonry beams reduces deflections from 20 to 45% of those of simply supported beams.

Doubly reinforced beam

\[ I_{cr} = \frac{b k^3 d^3}{3} + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2 \]
**Flexural Members – Determination of k**

\[ \rho = \frac{A_s}{bd} \]

\[ n = \frac{E_s}{E_m} \]

\[ k = \sqrt{(n\rho)^2 + 2n\rho - n\rho} \]

Doubly reinforced beam

\[ k = \sqrt{(n\rho)^2(1+r)^2 + 2n\rho \left( \frac{1+rd'}{d} \right)} - n\rho(1+r) \]

\[ r = \frac{(n-1)A_s'}{nA_s} \]

**Example**

Given: 10 ft. opening; dead load (including beam) of 2.5 kip/ft; live load of 1.0 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; \( f_m = 2000 \text{ psi} \)

**Required:** Design beam

**Solution:**

5.2.1.3: Length of bearing of beams shall be a minimum of 4 in.; typically assumed to be 8 in.

5.2.1.1.1: Span length of members not built integrally with supports shall be taken as the clear span plus depth of member, but need not exceed distance between center of supports.

- Span = 10 ft +2(4 in.) =10.67 ft

5.2.1.2: Compression face of beams shall be laterally supported at a maximum spacing of

- 32 multiplied by the beam thickness. 32(7.625 in.) = 244 in. = 20.3 ft
- \( 120b'^2/d \). 120(7.625 in.)² / (20 in.) = 349 in. = 29.1 ft
Example – Flexure

Flexural Design:

\[ w_u = 1.2D + 1.6L = 1.2(2.5k/ft) + 1.6(1.0k/ft) = 4.6k/ft \]

\[ M_u = \frac{w_uL^2}{8} = \frac{(4.6k/ft)(10.67ft)^2}{8} = 65.42k-ft \]

\[ a = d - \sqrt{d^2 - \frac{2M_n}{0.8f''_n b}} = 20in - \sqrt{(20in)^2 - \frac{2\left(\frac{65.42k-ft}{0.9}\right)\left(\frac{12in}{ft}\right)}{0.8(2.0ksi)(7.625in)}} = 3.97in \]

\[ A_s = \frac{0.8f''_n b a}{f_y} = \frac{0.8(2.0ksi)(7.625in)(3.97in)}{60ksi} = 0.807in^2 \]

Use 2 - #6 (A_s = 0.88in^2)  \hspace{1cm} M_n = 78.48k-ft \hspace{1cm} \varphi M_n = 70.63k-ft

Minimum Reinforcement Check:

Maximum Reinforcement Check:
Example - Development

\[ l_{de} = \left( \frac{0.13d_{b}^{2}f_{y} \gamma}{K \sqrt{f_{m}^{'}}} \right) \]

Cover = 1.25 (face shell thickness) + 0.5 (coarse grout) + 0.375 (assumed #3 stirrup) = 2.125 in.

\[ \gamma = 1.3 \text{ (#6, #7)} \]

\[ K = \min\{\text{masonry cover, clear spacing between adjacent bars, } 9d_{b}\} \]

\[ K = \min\{2.125, 9(0.75)\} = 2.125 \text{ in} \]

\[ l_{de} = \left( \frac{0.13(0.75\text{in})^{2}(60000 \text{ psi})(1.3)}{2.125\text{in} \sqrt{2000 \text{ psi}}} \right) = 60 \text{in} = 5.00 \text{ft} \]
Example - Shear

Conservative approximation, $M_r/V_u d_v = 1.0$

Bennett design: Use $d$ instead $d_v$ for beams

$A_{nv} = b d_v = (7.625 \text{ in})(20 \text{ in}) = 152.5 \text{ in}^2$

$V_{nm} =$

$\phi V_{nm} =$

$V_n = 4 A_{nv} \sqrt{f_m} = 4(152.5 \text{ in}^2) \sqrt{2000 \text{ psi}} = 27.28 \text{kips}$

$\phi V_n \leq 0.8(27.28 \text{kips}) = 21.82 \text{kips}$

$V_u$ at $d/2$ from face of supports is 19.17 kips; solve for $A_v$

---

Example - Shear

$\phi V_{ns} = V_u - \phi V_{nm} = 19.17 - 12.27 = 6.90 \text{kips}$

$V_{ns} = \frac{6.90 \text{kips}}{0.8} = 8.62 \text{kips}$

$V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_y d_v$

Space bars at every 8 in. (every cell)

$A_v =$
9.3.4.2.3 Transverse reinforcement

When transverse reinforcement is required, the following provisions shall apply:
(a) Transverse reinforcement shall be a single bar with a 180° hook at each end.
(b) Transverse reinforcement shall be hooked around longitudinal reinforcement.
(c) The minimum area of transverse reinforcement shall be 0.0007bd_\gamma_.
(d) The first transverse bar shall not be located more than one-fourth of the beam depth, dv, from the end of the beam.
(e) The maximum spacing shall not exceed 1/2 the depth of the beam nor 48 in.

Intent of provision (c) is area over a length of dv.

\[
0.0007bd_\gamma_ = 0.0007(7.625in)(24in) = 0.128in^2
\]

\[
l_{dc} = \frac{0.13d_\gamma_2f_{\gamma} \sqrt{f''_m}}{K \sqrt{f''_m}} = \frac{0.13(0.375)^2(60000)(1.0)}{\min\left\{9(0.375), 1.25 + 0.5\right\}\sqrt{2000}} = 14.0in
\]

With hook: \( l_{dc} = 14.0 - 13(0.375) = 9.1in \)

---

Example - Details

Check width:
2(1.25) Face shells
2(0.75) #6 bars
2(0.5) coarse grout space
2(0.375) #3 stirrups
1.0 space between bars
Total 6.75 in. OK

Need stirrups over first
2.67 ft = 32 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use 2-#3 stirrups at 8 in.
Example – Alternate Details

8.1.6.6.1.5 Pairs of U-stirrups or ties placed to form a closed unit shall be considered properly spliced when length of laps are $1.7l_k$. In grout at least 18 in. deep, such splices with $A_y f_y$ not more than 9,000 lb per leg shall be permitted to be considered adequate if legs extend the full available depth of grout.

$$A_y f_y = \left(0.11 \text{in}^2 \right) \left(60000 \text{ psi} \right) = 6600 \text{ lb}$$

---

Example - Deflections

$L/d = 10.67 \text{ ft} / 20 \text{ in} (12 \text{ in} / \text{ ft}) = 6.4$ Although deflections are not required to be checked, we will check to illustrate process.

$$w = D + L = 2.5k / \text{ ft} + 1.0k / \text{ ft} = 3.5k / \text{ ft}$$

$$M_a = \frac{wL^2}{8} = \frac{(3.5k / \text{ ft})(10.67 \text{ ft})^2}{8} = 49.78k - \text{ ft}$$

$$\rho = 0.00577 \quad n = \frac{E_s}{E_m} = \frac{29000 \text{ ksi}}{900f'_{m}} = \frac{29000 \text{ ksi}}{900(2.0 \text{ ksi})} = \frac{29000 \text{ ksi}}{1800 \text{ ksi}} = 16.11$$

$$n \rho = 0.09296$$

$$k = \sqrt{(n \rho)^2 + 2n \rho - n \rho} = \sqrt{(0.0930)^2 + 2(0.0930) - 0.0930} = 0.348$$

$$kd = 6.96 \text{ in}$$

$$I_n = \frac{bh^3}{12} = \frac{7.625 \text{ in}(24 \text{ in})^3}{12} = 8784 \text{ in}^4$$

$$I_{cr} = \frac{bk^3d^3}{3} + nA_y(d-kd)^2 = \frac{7.625 \text{ in}(6.96 \text{ in})^3}{3} + (16.11)(0.88 \text{ in}^2)(20 \text{ in} - 6.96 \text{ in})^2 = 3268 \text{ in}^4$$
Example - Deflections

\[ I_{\text{eff}} = I_e \left( \frac{M_{\text{cr}}}{M_a} \right)^3 + I_o \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^3 \right] \]

\[= 8784in^4 \left( \frac{9.76k - ft}{49.78k - ft} \right)^3 + 4020in^4 \left[ 1 - \left( \frac{9.76k - ft}{49.78k - ft} \right)^3 \right] = 3309in^4 \]

\[ \delta = \frac{5wL^4}{384EI} = \frac{5(3500lb\ ft)(10.67ft)^4}{384(180000psi)(3309in^4)} \frac{1728in^3}{1ft^3} = 0.171in \]

\[ \frac{L}{600} = \frac{(10.67ft)}{600} \frac{12in}{1ft} = 0.213in \]

Deflection OK

Example – Shear Revisited

\[ V_{nm} = \left[ 4 - 1.7S \left( \frac{M_a}{V_a d_v} \right) A_{nm} \sqrt{f_m'} \right] \]

Simply supported beam; uniform load

\[ V_u = w_u \left[ \frac{L}{2} - x \right] \]

\[ M_u = w_u \left[ \frac{L}{2} x - \frac{x^2}{2} \right] \]

\((\phi V_{ns})_{\text{max}} = 4.73\ kips\ at\ x = 1.64\ ft\]

Previous was 6.90 kips
Lintels - Arching

Arching creates thrust force. Shoring must remain in place long enough for masonry to develop sufficient strength to resist thrust.

Assumptions:
- Height of arch is half the height above the lintel
- \( w \) is conservatively taken as entire wall weight above lintel plus any superimposed load

\[ w' = \frac{P}{1.15h'} \]
Lintels - Arching, Shear strength

Equate shear force (horizontal reaction) and shear strength. Solve for x.

Allowable shear stress: \( 1.5 \sqrt{f'_m} \quad V = 1.5 \sqrt{f'_m} b x \sqrt{2} = \sqrt{f'_m} b x \)

2/3 from parabolic shear stress distribution; \( b \) is 2x face shell thickness for ungrouted, wall thickness for grouted.

\[
\frac{w(L+x)^2}{4h} = \sqrt{f'_m} bx \quad \frac{w}{4h} x^2 + \left( \frac{wL}{2h} - b \sqrt{f'_m} \right) x + \frac{wL^2}{4h} = 0 \quad \text{(A)}
\]

Allowable shear stress: \( \nu + 0.45 \frac{N_v}{A_n} \quad V = \frac{2}{3} \left[ \nu b x + 0.45 \frac{w(L+x)}{b} \right] \)

\[
\frac{w}{4h} x^2 + \left( \frac{wL}{2h} - \frac{2}{3} \frac{v b}{3} - 0.45 w \right) x + \left[ \frac{wL^2}{4h} - \frac{2}{3} \frac{0.45 w L}{2} \right] = 0 \quad \text{(B)}
\]

Solve for x using quadratic formula; use maximum value of x.

Lintels - Arching, Example

Given: 6 ft. opening in 8 in. CMU wall; \( f'_m = 1350 \) psi; face shell bedding; normal weight units.

Required: Distance x vs. height above opening

Solution: \( L = 6 \text{ft.} + 4 \text{in.} + 4 \text{in.} = 6.67 \text{ft.} \quad b = 2.5 \text{in.} \quad \nu = 37 \text{psi} \)

If load is just from wall, then \( w/h = \) wall weight (psf)

Equation (A) is independent of wall height. \( x = 5.4 \) in.

There is also a maximum value of x, which is 99.2 ft. This is a real flat arch with a large thrust.

For practical heights, Equation (B) does not control.

Negative values are from friction theory that friction force (0.45\( N_v \)) is independent of area. This is not true - some area is needed.

Not much masonry is needed to take thrust from an arch.
**Deep Beams, 5.2.2**

- Lintels in which there is a large height of masonry above the opening
- Walls not continuously supported (supported on pier foundations)

- Plane sections do not remain plane
- Internal level arm smaller than computed from linear strain distribution

Effective span length, $l_{eff}$, smaller of:
- Center-to-center distance between supports
- $1.15$ multiplied by the clear span

<table>
<thead>
<tr>
<th>$z$ – internal lever arm</th>
<th>Simple spans</th>
<th>Continuous spans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \leq \frac{l_{eff}}{d_v} &lt; 2$</td>
<td>$z = 0.2\left(l_{eff} + 2d_v\right)$</td>
<td>$1 \leq \frac{l_{eff}}{d_v} &lt; 3$</td>
</tr>
<tr>
<td>$\frac{l_{eff}}{d_v} &lt; 1$</td>
<td>$z = 0.6l_{eff}$</td>
<td>$\frac{l_{eff}}{d_v} &lt; 1$</td>
</tr>
</tbody>
</table>

**Deep Beams**

- Flexural reinforcement
  - Distributed flexural reinforcement for half beam depth
  - Maximum spacing of one-fifth $d_v$ or 16 in.
  - Joint reinforcement can be used as flexural reinforcement
  - Horizontal reinforcement anchored to develop yield strength at face of supports
- Shear reinforcement (when required)
  - Minimum area of vertical reinforcement is $0.0007bd_v$
  - Horizontal shear reinforcement shall have area $\geq$ half vertical shear reinforcement
  - Maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.
- Total reinforcement: sum of horizontal and vertical reinforcement shall be at least $0.001bd_v$. 

Reinforced Masonry - Flexural Members 43
Example 1: Deep Beams

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 3.0 kip/ft; live load of 2.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000 \text{ psi}$

Required: Design beam

Solution:

Center-to-center between supports = 10 ft + 2(4 in.) = 10.67 ft
1.15(clear span) = 1.15(10 ft) = 11.5 ft

Effective span length, $l_{eff} = \min(10.67, 11.5) = 10.67$ ft

$$M_u = \frac{1.2(3.0 \text{kip/ft}) + 1.6(2.0 \text{kip/ft})((10.67 \text{ft})^2)}{8} = 96.7 \text{kip-ft}$$

$$\frac{l_{eff}}{d_v} = \frac{10.67 \text{ft}}{6 \text{ft}} = 1.78$$

$$z = 0.2(l_{eff} + 2d_v) = 0.2(10.67 + 2(6 \text{ ft})) = 4.53 \text{ ft} = 54.4 \text{ in}$$

Example 1: Deep Beams, cont.

$$A_s = \frac{(M_u / \phi)}{zf_y} = \frac{(96.7 \text{kip-ft}/0.9)}{4.53 \text{ft}(60 \text{ksi})} = 0.395 \text{in}^2$$

Use 2-#4 bars

Using standard beam theory, $A_{s,req} = 0.320 \text{ in}^2$ (19% less)

Flexural reinforcement requirements:

- distributed flexural reinforcement for half beam depth
  - need flexural reinforcement over bottom 36 inches.
- maximum spacing of one-fifth $d_v$ or 16 in.
  - $(1/5)(72) = 14.4$ in.
- Use W1.7 (9 gage) joint reinforcement every 8 in. in bottom 5 bed joints (as a practical matter, use in every bed joint in beam)
Example 1: Deep Beams, cont.

\[ V_u = \frac{1.2(3.0 \text{ kip/ft}) + 1.6(2.0 \text{ kip/ft})}{2} (10.67 \text{ ft}) = 36.3 \text{ kip} \]

\[ V_{nm} = \left[ 4 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_{nv} \sqrt{f'_m} = 2.25 \left[ (7.625 \text{ in})(68 \text{ in}) \right] \sqrt{2000 \text{ psi}} \frac{1 \text{ kip}}{1000 \text{ lb}} = 52.2 \text{ kips} \]

\[ \phi V_{nm} = 0.8(52.2 \text{ kip}) = 41.7 \text{ kips} \]

No shear reinforcement required

Total Reinforcement: 0.001 \( bd_v = 0.001(7.625 \text{ in})(72 \text{ in}) = 0.55 \text{ in}^2 \)

2-\#4 (0.40 in$^2$) + 5(2)(0.017 in$^2$) = 0.57 in$^2$ \( \checkmark \) joint reinforcement

Development:
Clear cover = 1.25 (face shell) + 0.5 (coarse grout) = 1.75 in.

\[ l_{de} = \frac{0.13d^2 b f_y \gamma}{\min\{9d_b, \text{ clear cover}\} \sqrt{f'_m}} \]

\[ = \frac{0.13(0.5 \text{ in})^2(60000 \text{ psi})1.0}{\min\{9(0.5 \text{ in}), 1.75 \text{ in}\} \sqrt{2000 \text{ psi}}} = 24.9 \text{ in.} \]

Extend bars 24 in. beyond face of support
(close enough; cover of 1.82 in., 1/16 in. more than 1.75 in., results in \( l_{de} = 24 \text{ in.} \))
Example 2: Deep Beams

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 6.0 kip/ft; live load of 3.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f_m' = 2000$ psi

Required: Design beam

Solution:

$$M_u = \frac{[1.2(6.0kip/ft)+1.6(3.0kip/ft)][10.67ft]^2}{8} = 170.7kip-ft$$

$$z = 4.53\text{ft} \quad A_s = \left(\frac{M_u}{\phi}\right) = \frac{170.7kip-ft/0.9}{4.53\text{ft}(60ksi)} = 0.697\text{in}^2$$

Use 2-#4 bars in each of bottom 2 courses (4 total bars)

Using standard beam theory, $A_{s,req} = 0.650\text{ in}^2$ (7% less)

Use joint reinforcement in every course for distributed reinforcement

Example 2: Deep Beams, cont.

$$V_u = \frac{[1.2(6.0kip/ft)+1.6(3.0kip/ft)][10.67ft]}{2} = 64.0kip$$

$$\phi V_{n,max} = 0.8\left[4A_{nv}\sqrt{f_m'}\right] = 0.8\left[4\left(518\text{in}^2\right)\sqrt{2000\text{psi} \frac{1\text{kip}}{1000\text{lb}}}ight] = 74.1kips$$

$$\phi V_{nm} = 41.7kips \quad \phi V_{ns,req} = 64.0 - 41.7 = 22.3kips$$

$$A_v = \frac{V_{ns}}{0.5 f'_{dv} d_v} = \frac{(22.3kips/0.8)(8in)}{0.5(60ksi)(72in)} = 0.103\text{in}^2$$

Use 1-#3 at 8 in. vertical shear reinforcement
Example 2: Deep Beams, cont.

- Shear reinforcement details
  - minimum area of vertical reinforcement is $0.0007bd_v$
    - $0.0007bd_v = 0.0007(7.625)(72) = 0.384$ in$^2$
    - Total vertical is $0.11in^2(15) = 1.65$ in$^2$
  - horizontal shear reinforcement shall have area ≥ half vertical shear reinforcement
    - Use 1-#4 each course
    - Do not need joint reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.
    - Maximum spacing is 14.4 in.