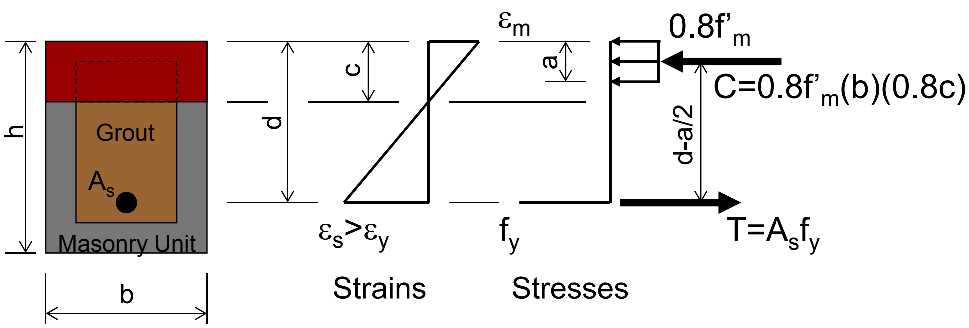
Flexural Members - Strength Design



$$\rho = \frac{A_s}{bd}$$

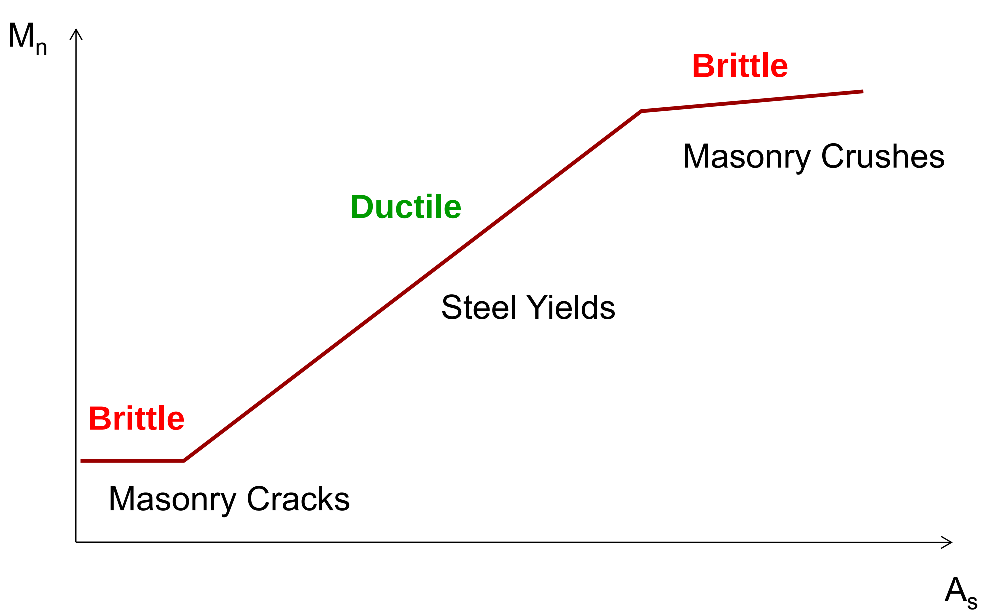
<u>Assumptions:</u> (9.3.2)

1. Plane sections remain plane
2. All masonry in tension is neglected
3. Perfect bond between steel and grout
4. Member is straight prismatic section
5. ε_m=__________ clay masonry
6. ε_m=__________ concrete masonry
7. Masonry stress = _____f'$_m$
8. Masonry stress acts over a=_____c

$$M_n = \left(\quad - - \underline{\qquad\qquad} \right)$$

ϕ=0.9 (9.1.4.4)

Beams – Behavior



Beams - Strength Design (9.3.4.2)

Beam: Factored axial compressive force ≤ $0.05A_n f'_m$

Minimum reinforcement: $M_n \geq 1.3$ x cracking strength; or $A_s \geq (4/3)A_{s,req'd}$

Modulus or rupture, f_r = Table 9.1.9.2

Maximum reinforcement: (9.3.3.5): $\varepsilon_s = 1.5\varepsilon_y$

$$\rho_{\max} = \frac{0.8(0.8)f'_m}{f_y}\left(\frac{\varepsilon_m}{\varepsilon_m + \varepsilon_s}\right)$$

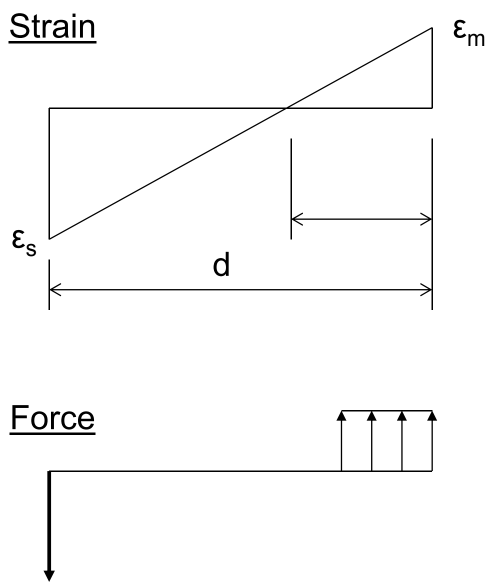
Steel Ratio	Grade 60 steel	
	Clay	CMU
Max reinf. (f'_m/f_y)	0.339	0.285
f_y = 60 ksi; f'_m = 2.00 ksi	0.01131	0.00952

Modulus of Rupture, psi Table 9.1.9.2

Masonry Type	Mortar Type			
	Portland cement/lime or mortar cement		Masonry Cement	
	M or S	N	M or S	N
Normal to Bed Joints (Vertical span)				
Solid Units	133	100	80	51
Hollow Units*				
Ungrouted	84	64	51	31
Fully Grouted	163	158	153	145
Parallel to bed joints in running bond (Horizontal span)				
Solid Units	267	200	160	100
Hollow Units				
Ungrouted and partially grouted	167	127	100	64
Fully grouted	267	200	160	1005
Parallel to bed joints in stack bond				
Continuous grout section parallel to bed joints	335	335	335	335
Other	0	0	0	0

* Use linear interpolation for partially grouted masonry.

Beams – Maximum Reinforcement



Maximum reinforcement: (9.3.3.5): $\varepsilon_s = 1.5\varepsilon_y$

Strength Design Procedure

1. Determine *a*, depth of compressive stress block

$$a = d - \sqrt{d^2 - \frac{2M_n}{0.8 f'_m b}}$$

2. Solve for A_s

$$A_s = \frac{0.8 f'_m b a}{f_y}$$

Example - Masonry Beam

Given: M = 150 k-in. dead load; M = 150 k-in. live load; Grade 60 steel, f'_m = 2000 psi; 8 in. CMU; depth of section limited to three courses; Type S masonry cement mortar

Required: Design section

Solution: For three units, d = 2(8 in.) + 4 in. = 20 in.

$(M_n)_{reqd}$=

$$a = d - \sqrt{d^2 - \frac{2M_n}{0.8 f'_m b}} =$$

Solve for A_s $$A_s = \frac{0.8 f'_m b a}{f_y} =$$

Use (A_s=)

Example - Masonry Beam, Check Steel

Check minimum steel

f_r =

Cracking moment: $$M_{cr} = f_r \frac{bh^2}{6} = 160\,psi \frac{(7.625in)(24in)^2}{6} = 117k - in$$

Required M_n = 1.3M_{cr} = 1.3(117 k-in) = 152 k-in. < 467 k-in. **OK**

Example - Masonry Beam, Check Steel

Check maximum steel

$$\rho = \frac{A_s}{bd} = \frac{0.44in^2}{(7.625in)(20in)} = 0.00288$$

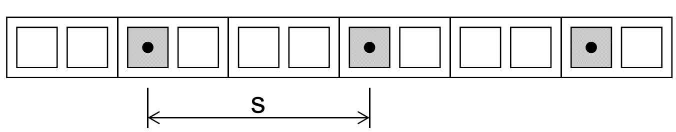
TMS 402 code ($1.5\varepsilon_y$)

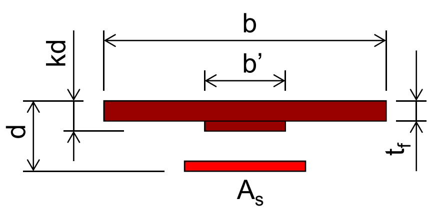
$$\rho_{\max} = \frac{0.8(0.8)f'_m}{f_y}\left(\frac{\varepsilon_m}{\varepsilon_m + \varepsilon_s}\right)$$

$$\rho_{\max} = \frac{0.8(0.8)(2000\,psi)}{60000\,psi}\left(\frac{0.0025}{0.0025 + 1.5\left(\frac{60ksi}{29000ksi}\right)}\right) = 0.00952$$

OK

Partially Grouted Walls





b = width of effective flange = min{s, 6t, 72 in}
(5.1.2)

A. Neutral axis in flange:
   a. Almost always the case
   b. Design and analysis for solid section
B. Neutral axis in web
   a. Design as a T-beam section
C. Often design based on a 1 ft width

Spacing (inches)	Steel Area in^2/ft			
	#3	#4	#5	#6
8	0.16	0.30	0.46	0.66
16	0.082	0.15	0.23	0.33
24	0.055	0.10	0.16	0.22
32	0.041	0.075	0.12	0.16
40	0.033	0.060	0.093	0.13
48	0.028	0.050	0.078	0.11
56	0.024	0.043	0.066	0.094
64	0.021	0.038	0.058	0.082
72	0.018	0.033	0.052	0.073

Partially Grouted Walls - Example

Given: 8 in. CMU wall; 12 ft high; Grade 60 steel, f'_m=2000 psi; Wind load of w_u = 30 psf

Required: Reinforcing (place in center of wall)

Solution:

$$M_u = \frac{w_u h^2}{8} = \frac{30lb/ft^2(12in/ft)(12\,ft)^2}{8} = 6480lb-in/ft = 0.54k-ft/ft$$

Use # __ @ __ inches (A_s=____in^2/ft)

Partially Grouted Walls – Example, cont

Minimum Reinforcement: No requirements for walls

Maximum Reinforcement: Requirements apply, although can be difficult to meet for heavily reinforced shear walls

ρ_{max} = 0.00952 ρ = 0.05/(12*3.81) = 0.00109 **OK**

Horizontal spanning masonry between bars:

- Some treat as unreinforced masonry, although debate as to whether you can mix unreinforced and reinforced masonry.
- There is arching occurring, so not truly a simply supported flexural member between vertical bars.

Partially Grouted Walls – Example, cont

Try #4 @72 in. (0.033 in^2/ft)

Problem: 72 inch is greater than maximum spacing of 6t = 48 in.
Solution:
- Design as 48 in. wide section; load is 30psf(72in.)(1ft/12in.) = 180 lb/ft
- Design partially grouted wall on basis of 1 ft unit section; increase load by factor of 72/48=1.50

From a practical standpoint, there is little difference.
#4, b = 48 in., φM_n = 3361 lb-ft
#4, b = 72 in., φM_n = 3384 lb-ft (0.7% greater)
M_u applied [w_u = 30psf(72in/(12in/ft)=180lb/ft] = 3240 lb-ft **OK**

Out-of-Plane: Maximum Reinforcement

For an 8 in. CMU wall, fully grouted, bars in the center, Grade 60 steel, and f'_m=*2000 psi,* the following table lists the maximum reinforcement for various axial loads.

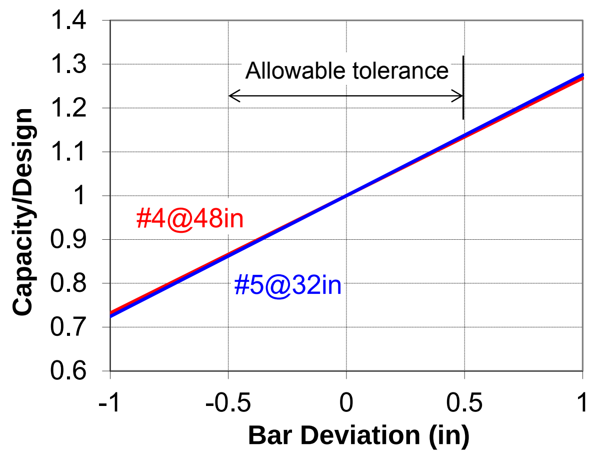
$P/(bdf'_m)$	A_s (in^2 per ft)	Spacing (inches)		
		#4	#5	#6
0	0.436	8 (5.5)	16 (8.5)	16 (12.1)
0.05	0.359	8 (6.7)	16 (10.4)	16 (14.7)
0.10	0.283	16 (8.5)	16 (13.1)	24 (18.7)
0.15	0.207	16 (11.6)	24 (18.0)	32 (25.5)
0.20	0.130	24 (18.4)	32 (28.5)	48 (40.5)
0.25	0.054	48 (44.3)	72 (68.6)	104 (97.4)

Partially Grouted Walls - Tolerances

Placement tolerances: (3.4.B.11)

d ≤ 8 in.	± 1/2 in.
8 in. < d ≤ 24 in.	± 1 in.
d > 24 in.	± 1 1/4 in.

Along wall: ± 2 in.



8 in. CMU; f'_m=2000 psi; Grade 60

Shear - Strength Design (9.3.4.1.2)

$$V_n = V_{nm} + V_{ns} \qquad V_{nm} = \left[4 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_{nv}\sqrt{f'_m} \qquad V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$

$$\phi = 0.8$$

d_v = actual depth of masonry
$M_u/(V_u d_v)$ need not be taken > 1.0

$$s_{max} = \min\{d/2, 48in.\}$$

$V_n \le 6A_{nv}\sqrt{f'_m}$ where $M_u/(V_u d_v) \le 0.25$

$V_n \le 4A_{nv}\sqrt{f'_m}$ where $M_u/(V_u d_v) \ge 1.00$

Linearly interpolate between 0.25 and 1.0

Conservative approximation, $M_u/(V_u d_v) = 1.0$

$$V_{nm} = 2.25 A_{nv}\sqrt{f'_m}$$

Shear - Strength Design

Detailing of shear reinforcement for beams (9.3.4.2.3):

A. Single bar with 180-degree hook at each end
B. Hook shear reinforcement around flexural reinforcement
C. Minimum area of shear reinforcement is $0.0007bd_v$
D. First bar within $d_v/4$
E. Maximum spacing is $d_v/2$ or 48 in.

Sections within d/2 from face of support can be designed for shear at d/2 (8.3.5.4):

A. Noncantilever beam
B. Reaction introduces compression into end region of member
C. No concentrated load between d/2 and face of support

Deflections

Deflection of beam or lintels supporting unreinforced masonry is limited to L/600 (5.2.1.4.1)
Deflections of approximately L/300 needed to be visible.

$$I_{eff} = I_n \left(\frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] \le I_n$$

(Equation 5-1)

$$I_{cr} = \frac{bk^3 d^3}{3} + nA_s (d - kd)^2$$

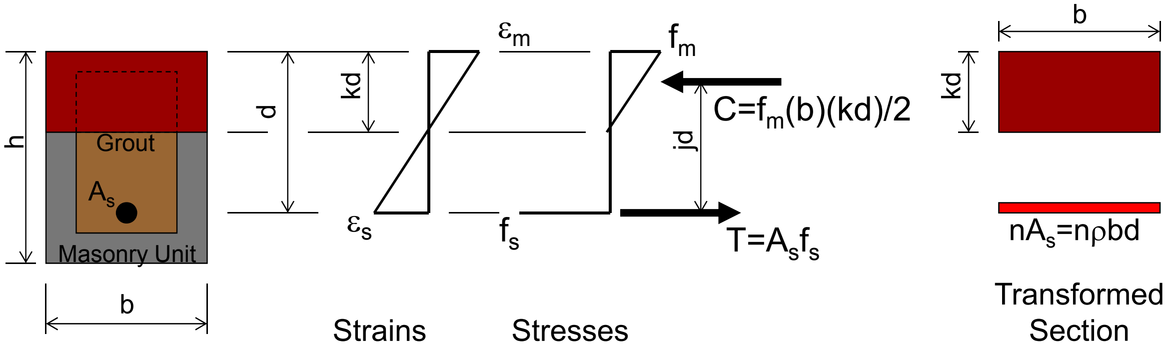
f_r: Table 9.1.9.2

Deflections do not need to be checked when L ≤ 8d (5.2.1.4.3).
End restraint of masonry beams reduces deflections from 20 to 45% of those of simply supported beams.

Doubly reinforced beam

$$I_{cr} = \frac{bk^3 d^3}{3} + nA_s (d - kd)^2 + (n-1)A_s' (kd - d')^2$$

Flexural Members – Determination of k



$$\rho = \frac{A_s}{bd} \qquad n = \frac{E_s}{E_m} \qquad \boxed{k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho}$$

Doubly reinforced beam

$$k = \sqrt{(n\rho)^2(1+r)^2 + 2n\rho\left(\frac{1+rd'}{d}\right)} - n\rho(1+r) \qquad r = \frac{(n-1)A_s'}{nA_s}$$

Example

Given: 10 ft. opening; dead load (including beam) of 2.5 kip/ft; live load of 1.0 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; f'_m = 2000 psi

Required: Design beam

Solution:

5.2.1.3: Length of bearing of beams shall be a minimum of 4 in.; typically assumed to be 8 in.

5.2.1.1.1: Span length of members not built integrally with supports shall be taken as the clear span plus depth of member, but need not exceed distance between center of supports.

- Span = 10 ft +2(4 in.) =10.67 ft

5.2.1.2: Compression face of beams shall be laterally supported at a maximum spacing of

- 32 multiplied by the beam thickness. 32(7.625 in.) = 244 in. = 20.3 ft
- 120b^2/d. 120(7.625 in.)^2 / (20 in.) = 349 in. = 29.1 ft

Example – Flexure

Flexural Design:

$$w_u = 1.2D + 1.6L = 1.2(2.5k/ft) + 1.6(1.0k/ft) = 4.6k/ft$$

$$M_u = \frac{w_u L^2}{8} = \frac{(4.6k/ft)(10.67ft)^2}{8} = 65.42k - ft$$

$$a = d - \sqrt{d^2 - \frac{2M_n}{0.8 f'_m b}} = 20in - \sqrt{(20in)^2 - \frac{2\left(\frac{65.42k - ft}{0.9}\right)\left(\frac{12in}{ft}\right)}{0.8(2.0ksi)(7.625in)}} = 3.97in$$

$$A_s = \frac{0.8 f'_m ba}{f_y} = \frac{0.8(2.0ksi)(7.625in)(3.97in)}{60ksi} = 0.807in^2$$

Use 2 - #6 (A_s = 0.88in^2)

M_n = 78.48 k-ft
φM_n = 70.63 k-ft

Example – Flexure

Minimum Reinforcement Check:

Maximum Reinforcement Check:

Example - Development

$$l_{de} = \frac{0.13 d_b^2 f_y \gamma}{K\sqrt{f'_m}}$$

Cover = 1.25 (face shell thickness)
+ 0.5 (coarse grout)
+ 0.375 (assumed #3 stirrup)
= 2.125 in.

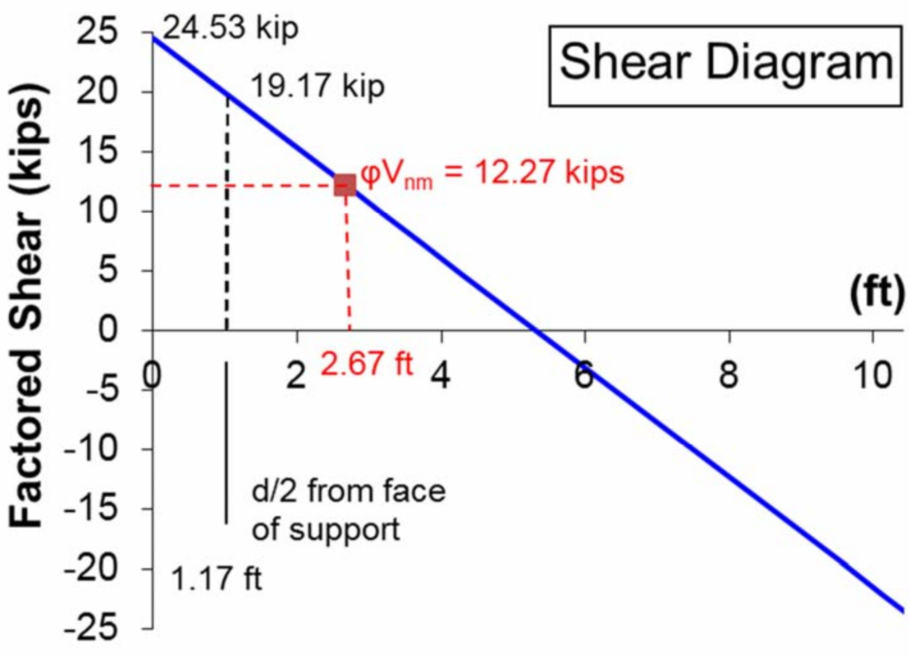
γ=1.3 (#6, #7)

K = min{masonry cover, clear spacing between adjacent bars, $9d_b$}

$$K = \min\{2.125, 9(0.75)\} = 2.125in$$

$$l_{de} = \frac{0.13(0.75in)^2(60000\,psi)(1.3)}{2.125in\sqrt{2000\,psi}} = 60.0in = 5.00\,ft$$

Example - Shear



Example - Shear

Conservative approximation, $M_u/(V_u d_v)$ = 1.0

Bennett design: Use d instead d_v for beams

$$A_{nv} = bd_v = (7.625in)(20in) = 152.5in^2$$

$$V_{nm} =$$

$$\phi V_{nm} =$$

$$(V_n)_{\max} \le 4A_{nv}\sqrt{f'_m} = 4(152.5in^2)\sqrt{2000\,psi} = 27.28kips$$

$$(\phi V_n)_{\max} \le 0.8(27.28kips) = 21.82kips$$

V_u at $d/2$ from face of supports is 19.17 kips; solve for A_v

Example - Shear

$$\phi V_{ns} = V_u - \phi V_{nm} = 19.17 - 12.27 = 6.90kips$$

$$V_{ns} = \frac{6.90kips}{0.8} = 8.62kips$$

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$

Space bars at every 8 in. (every cell)

$$A_v =$$

Example - Details

9.3.4.2.3 *Transverse reinforcement*

When transverse reinforcement is required, the following provisions shall apply:

(a)Transverse reinforcement shall be a single bar with a 180° hook at each end.

(b) Transverse reinforcement shall be hooked around longitudinal reinforcement.

(c) The minimum area of transverse reinforcement shall be $0.0007bd_v$.

(d) The first transverse bar shall not be located more than one-fourth of the beam depth, d_v , from the end of the beam.

(e) The maximum spacing shall not exceed 1/2 the depth of the beam nor 48 in.

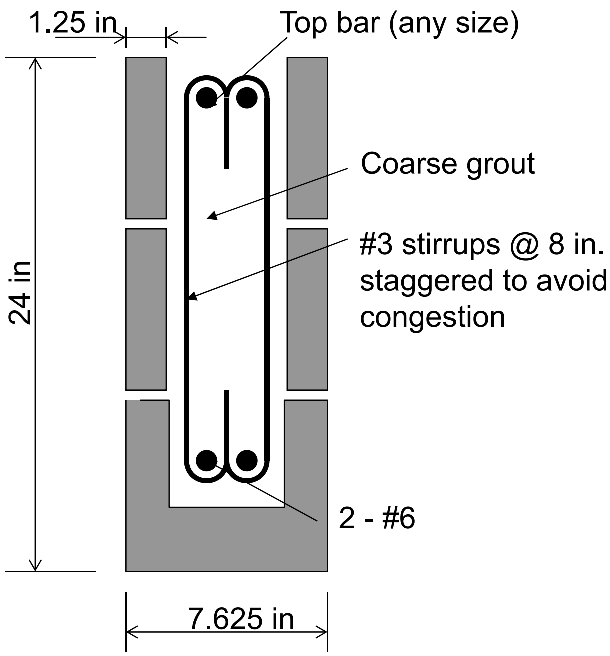
Intent of provision (c) is area over a length of d_v.

$$0.0007bd_v = 0.0007(7.625in)(24in) = 0.128in^2$$

$$l_{de} = \frac{0.13d_b^2 f_y \gamma}{K\sqrt{f'_m}} = \frac{0.13(0.375)^2(60000)(1.0)}{\min\{9(0.375), 1.25 + 0.5\}\sqrt{2000}} = 14.0in$$

With hook: $l_{de} = 14.0 - 13(0.375) = 9.1in$

Example - Details



<u>Check width:</u>

2(1.25) Face shells

2(0.75) #6 bars

2(0.5) coarse grout space

2(0.375) #3 stirrups

1.0 space between bars

Total 6.75 in. OK

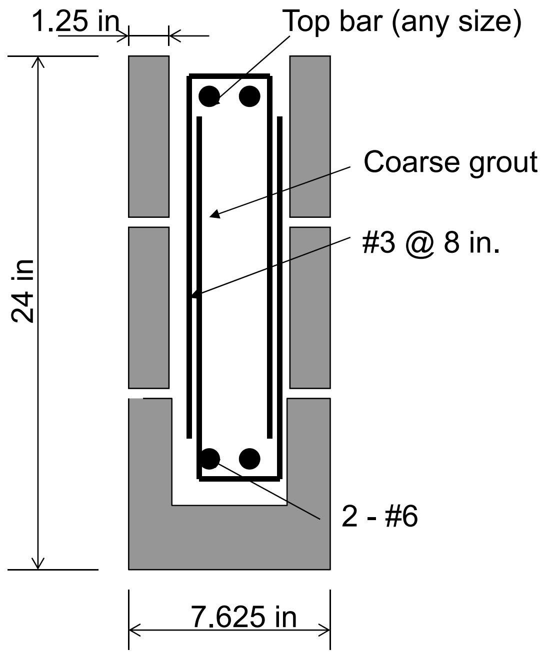
Need stirrups over first 2.67 ft = 32 in.

First stirrup is at 4 in.

Use stirrups over next 32 in.

Use 2-#3 stirrups at 8 in.

Example – Alternate Details



8.1.6.6.1.5 Pairs of U-stirrups or ties placed to form a closed unit shall be considered properly spliced when length of laps are $1.7l_d$. In grout at least 18 in. deep, such splices with $A_v f_y$ not more than 9,000 lb per leg shall be permitted to be considered adequate if legs extend the full available depth of grout.

$$A_v f_y = \left(0.11 in^2\right)\left(60000\, psi\right) = 6600\, lb$$

Example - Deflections

$$L/d = 10.67\, ft / 20in\left(12in/\ ft\right) = 6.4$$

Although deflections are not required to be checked, we will check to illustrate process.

$$w = D + L = 2.5k/\ ft + 1.0k/\ ft = 3.5k/\ ft$$

$$M_a = \frac{wL^2}{8} = \frac{\left(3.5k/\ ft\right)\left(10.67\, ft\right)^2}{8} = 49.78k - ft$$

$$\rho = 0.00577$$

$$n\rho = 0.09296$$

$$n = \frac{E_s}{E_m} = \frac{29000ksi}{900\, f'_m} = \frac{29000ksi}{900\left(2.0ksi\right)} = \frac{29000ksi}{1800ksi} = 16.11$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho = \sqrt{(0.0930)^2 + 2\left(0.0930\right)} - 0.0930 = 0.348$$

$$kd = 6.96in$$

$$I_n = \frac{bh^3}{12} = \frac{7.625in(24in)^3}{12} = 8784in^4$$

$$I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 = \frac{7.625in\left(6.96in\right)^3}{3} + \left(16.11\right)\left(0.88in^2\right)\left(20in - 6.96in\right)^2 = 3268in^4$$

Example - Deflections

$$I_{eff} = I_n\left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] = 8784in^4\left(\frac{9.76k-ft}{49.78k-ft}\right)^3 + 4020in^4\left[1-\left(\frac{9.76k-ft}{49.78k-ft}\right)^3\right] = 3309in^4$$

$$\delta = \frac{5wL^4}{384EI} = \frac{5(3500lb/ft)(10.67\,ft)^4}{384(1800000\,psi)(3309in^4)}\frac{1728in^3}{1\,ft^3} = 0.171in$$

$$\frac{L}{600} = \frac{(10.67\,ft)}{600}\frac{12in}{1\,ft} = 0.213in$$

Deflection OK

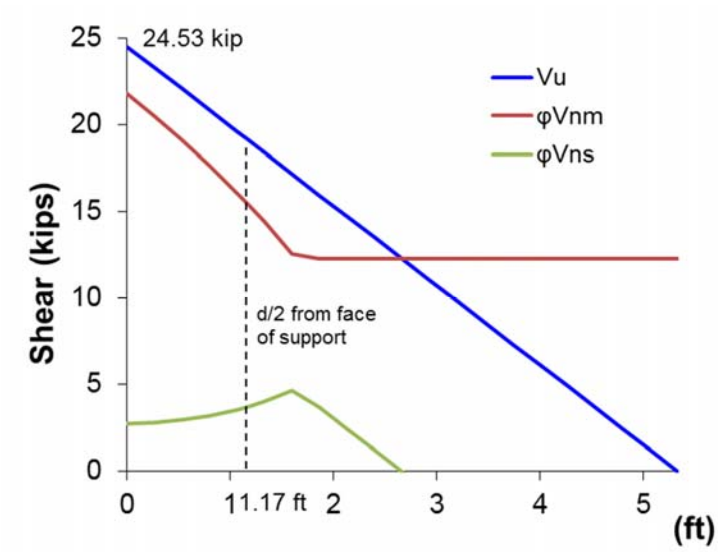
Example – Shear Revisited

$$V_{nm} = \left[4-1.75\left(\frac{M_u}{V_u d_v}\right)\right]A_{nv}\sqrt{f'_m}$$

Simply supported beam; uniform load

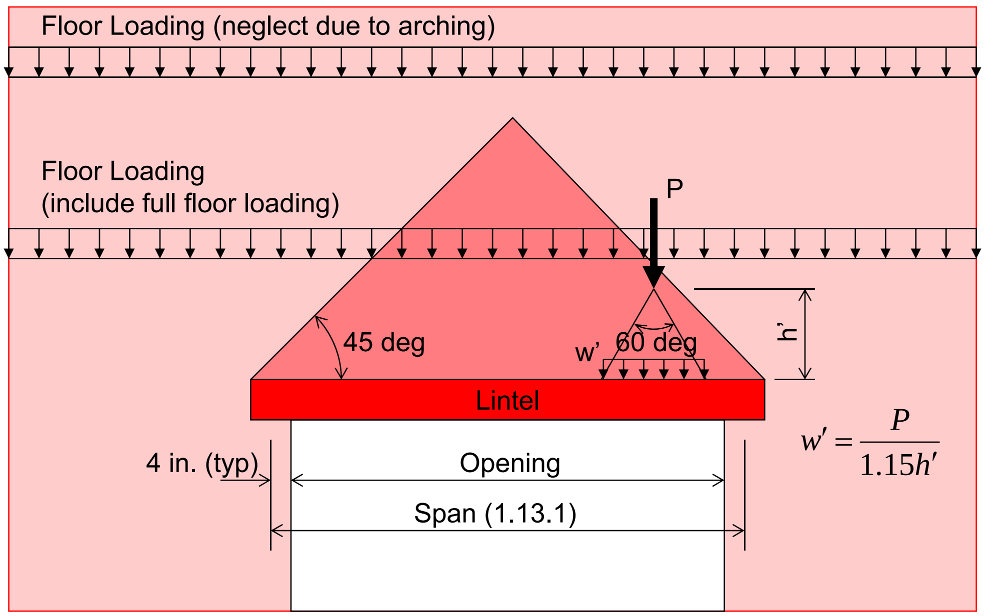
$$V_u = w_u\left[\frac{L}{2}-x\right]$$

$$M_u = w_u\left[\frac{L}{2}x-\frac{x^2}{2}\right]$$



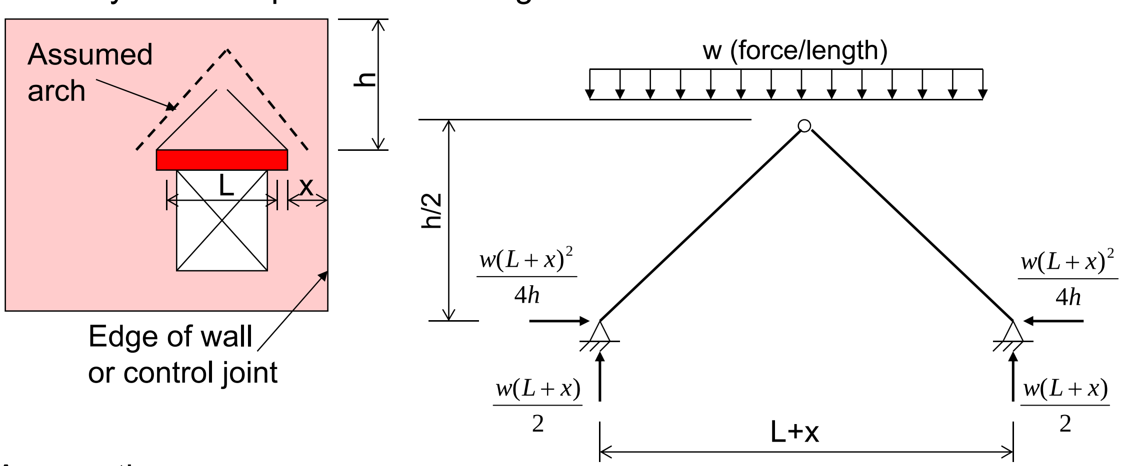
$(\varphi V_{ns})_{max}$ = 4.73 kips at x = 1.64 ft
Previous was 6.90 kips

Lintels



Lintels - Arching

Arching creates thrust force. Shoring must remain in place long enough for masonry to develop sufficient strength to resist thrust.



Assumptions:

- Height of arch is half the height above the lintel
- *w* is conservatively taken as entire wall weight above lintel plus any superimposed load

Lintels - Arching, Shear strength

Equate shear force (horizontal reaction) and shear strength. Solve for x.

Allowable shear stress: $1.5\sqrt{f'_m}$ $\qquad V = 1.5\sqrt{f'_m}\,bx\left(\tfrac{2}{3}\right) = \sqrt{f'_m}\,bx$

${}^{2}/_{3}$ from parabolic shear stress distribution; *b* is 2x face shell thickness for ungrouted, wall thickness for grouted.

$$\frac{w(L+x)^2}{4h} = \sqrt{f'_m}\,bx \qquad \frac{w}{4h}x^2 + \left(\frac{wL}{2h} - b\sqrt{f'_m}\right)x + \frac{wL^2}{4h} = 0 \qquad \text{(A)}$$

Allowable shear stress: $v + 0.45 N_v / A_n$ $\qquad V = \frac{2}{3}\left[vbx + 0.45\frac{w(L+2x)}{2bx}bx\right]$

$$\frac{w}{4h}x^2 + \left(\frac{wL}{2h} - \frac{2}{3}vb - \frac{2}{3}0.45w\right)x + \left[\frac{wL^2}{4h} - \frac{2}{3}(0.45)\frac{wL}{2}\right] = 0 \qquad \text{(B)}$$

Solve for x using quadratic formula; use maximum value of x.

Lintels - Arching, Example

Given: 6 ft. opening in 8 in. CMU wall; f'_m=1350 psi; face shell bedding; normal weight units.
Required: Distance x vs. height above opening
Solution: L=6ft.+4in.+4in.=6.67ft. $\qquad$ b=2.5in. $\qquad$ ν=37psi

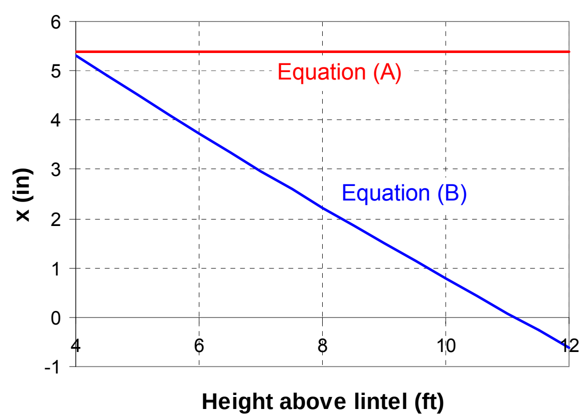
If load is just from wall, then w/h = wall weight (psf)

Equation (A) is independent of wall height. x = 5.4 in.

There is also a maximum value of x, which is 99.2 ft. This is a real flat arch with a large thrust.

For practical heights, Equation (B) does not control.

Negative values are from friction theory that friction force ($0.45N_v$) is independent of area. This is not true - some area is needed.



Not much masonry is needed to take thrust from an arch.

Deep Beams, 5.2.2

- Lintels in which there is a large height of masonry above the opening
- Walls not continuously supported (supported on pier foundations)

- Plane sections do not remain plane
- Internal level arm smaller than computed from linear strain distribution

Effective span length, l_{eff}, smaller of:
- center-to-center distance between supports
- 1.15 multiplied by the clear span

$$\frac{l_{eff}}{d_v} \le \begin{cases} 3 & \text{continuous spans} \\ 2 & \text{simple spans} \end{cases}$$

z – internal lever arm	
Simple spans	Continuous spans
$1 \le \frac{l_{eff}}{d_v} < 2$ $\quad z = 0.2(l_{eff} + 2d_v)$	$1 \le \frac{l_{eff}}{d_v} < 3$ $\quad z = 0.2(l_{eff} + 1.5d_v)$
$\frac{l_{eff}}{d_v} < 1$ $\quad z = 0.6 l_{eff}$	$\frac{l_{eff}}{d_v} < 1$ $\quad z = 0.5 l_{eff}$

Deep Beams

- Flexural reinforcement
  - distributed flexural reinforcement for half beam depth
  - maximum spacing of one-fifth d_v or 16 in.
  - joint reinforcement can be used as flexural reinforcement
  - horizontal reinforcement anchored to develop yield strength at face of supports
- Shear reinforcement (when required)
  - minimum area of vertical reinforcement is $0.0007bd_v$
  - horizontal shear reinforcement shall have area ≥ half vertical shear reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth d_v or 16 in.
- Total reinforcement: sum of horizontal and vertical reinforcement shall be at least $0.001bd_v$.

Example 1: Deep Beams

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 3.0 kip/ft; live load of 2.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; f'_m = 2000 psi

Required: Design beam

Solution:

Center-to-center between supports = 10 ft + 2(4 in.) = 10.67 ft
1.15(clear span) = 1.15(10 ft) = 11.5 ft
Effective span length, l_{eff} = min(10.67, 11.5) = 10.67 ft

$$M_u = \frac{[1.2(3.0 kip/ft) + 1.6(2.0 kip/ft)](10.67 ft)^2}{8} = 96.7 kip - ft$$

$$\frac{l_{eff}}{d_v} = \frac{10.67 ft}{6 ft} = 1.78$$

$$z = 0.2(l_{eff} + 2d_v) = 0.2(10.67 + 2(6 ft)) = 4.53 ft = 54.4 in$$

Example 1: Deep Beams, cont.

$$A_s = \frac{(M_u/\phi)}{zf_y} = \frac{(96.7 kip - ft/0.9)}{4.53 ft(60 ksi)} = 0.395 in^2$$

Use 2-#4 bars

Using standard beam theory, $A_{s,req}$ = 0.320 in^2 (19% less)

Flexural reinforcement requirements:

- distributed flexural reinforcement for half beam depth
  - need flexural reinforcement over bottom 36 inches.
- maximum spacing of one-fifth d_v or 16 in.
  - (1/5)(72) = 14.4 in.
- Use W1.7 (9 gage) joint reinforcement every 8 in. in bottom 5 bed joints (as a practical matter, use in every bed joint in beam)

Example 1: Deep Beams, cont.

$$V_u = \frac{[1.2(3.0kip/ft)+1.6(2.0kip/ft)](10.67ft)}{2} = 36.3kip$$

$$V_{nm} = \left[4-1.75\left(\frac{M_u}{V_u d_v}\right)\right]A_{nv}\sqrt{f'_m} = 2.25[(7.625in)(68in)]\sqrt{2000psi}\frac{1kip}{1000lb} = 52.2kips$$

$$\phi V_{nm} = 0.8(52.2kip) = 41.7kips$$

No shear reinforcement required

Total Reinforcement: $0.001bd_v$ = 0.001(7.625in)(72in) = 0.55 in^2

2-#4 (0.40in^2) + 5(2)(0.017in^2) = 0.57in^2 **OK**

joint reinforcement

Example 1: Deep Beams, cont.

Development:

Clear cover = 1.25 (face shell) + 0.5 (coarse grout) = 1.75 in.

$$l_{de} = \frac{0.13 d_b^2 f_y \gamma}{\min\{9d_b, \text{clear cover}\}\sqrt{f'_m}}$$

$$= \frac{0.13(0.5in.)^2(60000psi)1.0}{\min\{9(0.5in), 1.75in\}\sqrt{2000psi}} = 24.9in.$$

Extend bars 24 in. beyond face of support

(close enough; cover of 1.82 in., 1/16 in. more than 1.75 in., results in l_{de} = 24 in.)

Example 2: Deep Beams

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 6.0 kip/ft; live load of 3.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; f'_m = 2000 psi

Required: Design beam

Solution:

$$M_u = \frac{[1.2(6.0kip/ft) + 1.6(3.0kip/ft)](10.67ft)^2}{8} = 170.7kip-ft$$

$$z = 4.53ft \qquad A_s = \frac{(M_u/\phi)}{zf_y} = \frac{(170.7kip-ft/0.9)}{4.53ft(60ksi)} = 0.697in^2$$

Use 2-#4 bars in each of bottom 2 courses (4 total bars)

Using standard beam theory, $A_{s,req}$ = 0.650 in^2 (7% less)

Use joint reinforcement in every course for distributed reinforcement

Example 2: Deep Beams, cont.

$$V_u = \frac{[1.2(6.0kip/ft) + 1.6(3.0kip/ft)](10.67ft)}{2} = 64.0kip$$

$$\phi V_{n,\max} = 0.8\left[4A_{nv}\sqrt{f'_m}\right] = 0.8\left[4(518in^2)\sqrt{2000psi}\,\frac{1kip}{1000lb}\right] = 74.1kips$$

$$\phi V_{nm} = 41.7kips \qquad \phi V_{ns.requ} = 64.0 - 41.7 = 22.3kips$$

$$A_v = \frac{V_{ns}s}{0.5f_yd_v} = \frac{(22.3k/0.8)(8in)}{0.5(60ksi)(72in)} = 0.103in^2$$

Use 1-#3 at 8 in. vertical shear reinforcement

Example 2: Deep Beams, cont.

- Shear reinforcement details
  - minimum area of vertical reinforcement is $0.0007bd_v$
    - $0.0007bd_v = 0.0007(7.625)(72) = 0.384 \text{ in}^2$
    - Total vertical is $0.11\text{in}^2(15) = 1.65 \text{ in}^2$
  - horizontal shear reinforcement shall have area ≥ half vertical shear reinforcement
    - Use 1-#4 each course
    - Do not need joint reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth d_v or 16 in.
    - Maximum spacing is 14.4 in.