Combined Flexural and Axial Load

- Interaction Diagram
  - Solidly grouted bearing wall
  - Partially grouted bearing wall
- Bearing Walls: Slender Wall Design Procedure
  - Strength
  - Serviceability – Deflections
- Example – Pilaster
- Bearing and Concentrated Loads
- Prestressed Masonry

Concentric Axial Compression (9.3.4.1.1)

\[
P_n = 0.8 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \right] \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \quad \frac{h}{r} \leq 99
\]

\[
P_n = 0.80 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \right] \left( \frac{70r}{h} \right)^2 \quad \frac{h}{r} > 99
\]

\[
P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 EA_r^2}{h^2} = \frac{\pi^2 (900 f_m') A_{st} r^2}{h^2} = A_n f_m' \left( 94.2 \frac{r}{h} \right)^2
\]

- Equation above for CMU; for clay \((E_m = 700f_m')\), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling

Inclusion of wall weight
Wall weight provides uniform axial load over height of wall. Reasonable approximation is to use half the weight of wall acting at top.
Buckling Curve for $A_{st} = 0$

\[ P_n = 0.80 \left( 0.80 A_{n} f'_{m} \right) \left( \frac{70r}{h} \right)^{2} \]

\[ P_n = 0.80 \left( 0.80 A_{n} f'_{m} \right) \left[ 1 - \left( \frac{h}{140r} \right)^{2} \right] \]

Radius of Gyration

4.3.3 Radius of gyration

Radius of gyration shall be computed using average net cross-sectional area of the member considered.

Questions: Is this a strict average or weighted average? What about different types of units (which changes block area)?

NCMA has tabulated values of average radii of gyration. I use $r = \sqrt{I_n/A_n}$ in the examples and spreadsheet.
Interaction Diagram

• Assume strain/stress distribution
• Compute forces in masonry and steel
• Sum forces to get axial force
• Sum moment about centerline to get bending moment
• Key points
  • Pure axial load
  • Pure bending
  • Balanced

Example – 8 in. CMU Bearing Wall

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; solid grout
Required: Interaction diagram in terms of capacity per foot

Pure Moment:

\[ M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8 b f'_m} \right) \]

\[ \phi M_n = 0.9(0.934k - ft / ft) = 0.840k - ft / ft \]

\[ a = \frac{A_s f_y}{0.8 b f'_m} = \]

\[ c = \frac{a}{0.8} = \]
Example – 8 in. CMU Bearing Wall

Pure Axial: \[
r = \frac{1}{\sqrt{12}} \quad t = h = \frac{144\text{in}}{2.201\text{in}} = 65.4
\]

\[
P_n = 0.8 \left[ 0.80 f''_m (A_n - A_{st}) + f_y A_{st} \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \right] \quad \frac{h}{r} \leq 99 \quad \phi P_n = 0.9(68.7 \text{ k/ft}) = 61.8 \text{ k/ft}
\]

For solid sections

\[
\frac{h}{r} = \frac{144\text{in}}{2.201\text{in}} = 65.4
\]

\[
P_n = 0.8 \left[ 0.80(2.0\text{ksi})(7.625\text{in})(12\text{in/ft}) - 0 \right] + 0 \left[ 1 - \left( \frac{65.4}{140} \right)^2 \right] = 117.2 \text{ k/ft}(0.7816) = 91.6 \text{ k/ft}
\]

\[
\phi P_n = 0.9(91.6 \text{ k/ft}) = 82.4 \text{ k/ft}
\]
Example – 8 in. CMU Bearing Wall

Choose strain distribution (alternatively \( c \))

Balanced conditions

\[
\begin{align*}
C_m &= \\
T &= \\
P_n &= C_m \cdot T = \\
M_n &= \\
\phi P_n &= \\
\phi M_n &=
\end{align*}
\]

Example - Interaction Diagram

<table>
<thead>
<tr>
<th>Point</th>
<th>c (in)</th>
<th>( C_m ) (kip/ft)</th>
<th>T (kip/ft)</th>
<th>( \phi P_n ) (kip/ft)</th>
<th>( \phi M_n ) (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = d )</td>
<td>4.76</td>
<td>73.2</td>
<td>0</td>
<td>65.9</td>
<td>10.46</td>
</tr>
<tr>
<td>( c = d )</td>
<td>3.8125</td>
<td>58.6</td>
<td>0</td>
<td>52.7</td>
<td>10.05</td>
</tr>
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<td></td>
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<td>45.3</td>
<td>1.06</td>
<td>39.8</td>
<td>8.95</td>
</tr>
<tr>
<td>Balanced</td>
<td>2.09</td>
<td>32.0</td>
<td>3.0</td>
<td>26.1</td>
<td>7.16</td>
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<td>16.6</td>
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<td>0.9</td>
<td>13.8</td>
<td>3.0</td>
<td>9.7</td>
<td>3.58</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>9.2</td>
<td>3</td>
<td>5.6</td>
<td>2.47</td>
</tr>
<tr>
<td>Pure Moment</td>
<td>0.195</td>
<td>3.0</td>
<td>3.0</td>
<td>0</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Interaction Diagram

Example – Moment at Maximum Axial

Maximum Moment at $\phi P_n = 82.4 \text{ k/ft}$, $P_n = 91.6 \text{ k/ft}$

Assume steel is not in tension, $P_n = C_m$

$$c = \frac{P_n}{0.8(0.8 f_m')}b = \frac{91.6 \text{ k/ft}}{0.8(0.8(2.0 \text{ksi}))(12 \text{ in/ft})} = 5.96 \text{ in}$$

$$a = 0.8c = 4.77 \text{ in}$$

$$M_n = C_m \left( \frac{t}{2} - \frac{a}{2} \right) = 91.6 \text{ k/ft} \left( \frac{7.625 \text{ in}}{2} - \frac{4.77 \text{ in}}{2} \right) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 10.90 \text{ k - ft/ft}$$

$$\phi M_n = 0.9(10.90 \text{ k - ft/ft}) = 9.81 \text{ k - ft/ft}$$
Interaction Diagram – Below Balanced

Below the balanced point:
\[ T = A_s f_y \quad C = 0.8 f'_m b a \]

\[ P_n = 0.8 f'_m b a - A_s f_y \quad \text{or} \quad a = \frac{A_s f_y + P_n}{0.8 f'_m b} \]

\[ M_n = 0.8 f'_m b a \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

\[ = \left( P_n + A_s f_y \left( \frac{t_{sp} - a}{2} \right) \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

If we could only know one point on the interaction diagram, we would want to know the point corresponding to \( \varphi P_n = P_u \)

\[ a = \frac{A_s f_y + P_u / \varphi}{0.8 f'_m b} \]

\[ M_n = \left( P_u / \varphi + A_s f_y \right) \left( \frac{t_{sp} - a}{2} \right) + A_s f_y \left( d - \frac{t_{sp}}{2} \right) \]

These are equations in 9.3.5.2 commentary. They ignore any tension in a possible second layer of steel near the compression face.

For centered bars:
\[ M_n = \left( P_u / \varphi + A_s f_y \right) \left( d - \frac{a}{2} \right) \]
Partially Grouted Bearing Wall

- Small _______ forces
  - Partially grouted walls act as _______ walls
  - Compression area is in ______________
- Strength design
  - Higher axial loads act as __________
  - Very high axial loads act as __________
  - Need to calculate $r$ based on grouted cross-section.

Interaction Diagram: Solid vs. Partial Grout
Walls: Slenderness Effects

Three methods to account for slenderness effects:

1. ______________________________
   a. Axial capacity (and sometimes moment) reduced
   b. Used to be in TMS 402 Code
   c. deleted because it can be unconservative

2. ______________________________
   a. Second-order moment directly added by P-δ
   b. Usually requires iteration
   c. Difficult for hand calculations for other than simple cases
   d. Basis for second-order analysis in computer programs
   e. Historical method used for masonry design

3. ______________________________
   a. Added in 2013 TMS 402 Code
   b. Very general, but a bit conservative.

Walls: Complementary Moment 9.3.5.4.2

Assumes simple support conditions.
Valid only for the following conditions:

\[
\frac{P_u}{A_u} \leq 0.05 f'_m \quad \text{No height limit} \quad \frac{P_u}{A_g} \leq 0.20 f'_m \quad h/t \leq 30
\]

\[
M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + P_u \delta_u
\]

\[
P_u = P_{uw} + P_{uf}
\]

Assumes maximum moment is at midheight
Walls: Deflections

Deflection Calculation

\[
\delta = \frac{5Mh^2}{48E_mI_n} \quad M < M_{cr}
\]

\[
\delta = \frac{5M_{cr}h^2}{48E_mI_n} + \frac{5(M - M_{cr})h^2}{48E_mI_{cr}} \quad M > M_{cr}
\]

Deflection Limit \( \delta_s \leq 0.007h \) Calculated under service loads

\[
I_{cr} = n \left( A_s + \frac{P_u}{f_y} \frac{t_{sp}}{2d} \right) (d - c)^2 + \frac{bc^3}{3} \quad c = \frac{A_s f_y + P_u}{0.64 f_y b}
\]

For centered bars: \( I_{cr} = n \left( A_s + \frac{P_u}{f_y} \right) (d - c)^2 + \frac{bc^3}{3} \)

Walls: Design

Design Procedure
1. Solve for \( M_u \). Compare to \( \phi M_n \).
2. Solve for \( M_{seg} \). Compute deflection, and compare to allowable.

Solving for \( M \)

\[
M \left( 1 - \frac{5Ph^2}{48E_mI_{cr}} \right) = \frac{wh^2}{8} + P_f \frac{e}{2} + \frac{5M_{cr}Ph^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right) \quad M > M_{cr}
\]

\[
M \left( 1 - \frac{5Ph^2}{48E_mI_n} \right) = \frac{wh^2}{8} + P_f \frac{e}{2} \quad M < M_{cr}
\]

Equations can be used for either service or ultimate loads.
Walls - Design Procedure

Preliminary estimate of steel; assume steel yields
1. Determine \( a \), depth of compressive stress block

\[
a = d - \sqrt{d^2 - \frac{2[P_u(d - t/2) + M_u]}{\phi(0.8f'_m'b)}}
\]

2. Solve for \( A_s \)

\[
A_s = \frac{0.8f'_m'ba - P_u / \phi}{f_y}
\]

This neglects increase in moment due to second order effects. Can estimate increase in moment, such as 10% for a preliminary estimate of amount of reinforcement.

Example – Out-of-Plane Wall

Given: 18 ft high CMU bearing wall, with 2.5 ft parapet (total height is 20.5 ft); Type S masonry cement mortar; Grade 60 steel in center of wall; Dead load from roof of 500 lb/ft; Roof live load of 400 lb/ft; Lateral wind load of 32 psf, 48 psf on parapet; Wind uplift of 360 lb/ft; Roof forces act on 3 in. wide bearing plate at edge of wall.

Required: Reinforcing steel.

Solution:

Determine eccentricity
\( e = 7.625\text{in}/2 - 1.0\text{ in.} = 2.81\text{ in.} \)
Example - OOP: Estimate Steel

Use $1.2D + 1.0W + 0.5L_r$ without second-order effects, parapet, and wall weight

$$P_{uf} = 1.2(500\text{ lb/ft}) + 1.0(-360\text{ lb/ft}) + 0.5(400\text{ lb/ft}) = 440\text{ lb/ft}$$

$$M_{u,app} = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} = \frac{1.0(32 \text{ psf})(18 \text{ ft})^2}{8} \left(\frac{12\text{ in}}{\text{ft}}\right) + 440\text{ lb/ft} \frac{2.81\text{ in}}{2} = 16.2k\text{ in/ft}$$

Estimate steel: try an 8 in. CMU wall

$$a = d - \sqrt{d^2 - \frac{2(P_u(d-t/2)+M_u)}{\phi(0.8f_m''b)}}$$

$$= 3.81\text{ in} - \sqrt{(3.81\text{ in})^2 - \frac{2[0.44\text{ kips/ft}(3.81-7.62/2)\text{ in}+16.2\text{ in}/\text{ft}]}{0.9(0.8(2.0\text{ ksi})(12\text{ in/ft})}} = 0.254\text{ in}$$

$$A_s = \frac{0.8f_m''ba - P_u}{f_y} = \frac{0.8(2.0\text{ ksi})(12\text{ in/ft})(0.254\text{ in})-(0.44\text{ kips/ft})/0.9}{60\text{ksi}} = 0.0733\text{ in}^2/\text{ft}$$

Try #5@48 in., $A_s = 0.0775\text{ in}^2/\text{ft}$

---

Example - OOP

Summary of Strength Design Load Combination Axial Forces

(wall weight is 38 psf for 48 in. grout spacing)

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>$P_{uf}$ (kip/ft)</th>
<th>$P_{uw}$ (kip/ft)</th>
<th>$P_u$ (kip/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D+1.0W</td>
<td>0.090</td>
<td>0.393</td>
<td>0.483</td>
</tr>
<tr>
<td>1.2D+1.6L_r+0.5W</td>
<td>1.240</td>
<td>0.524</td>
<td>1.764</td>
</tr>
<tr>
<td>1.2D+1.0W+0.5L_r</td>
<td>0.440</td>
<td>0.524</td>
<td>0.964</td>
</tr>
</tbody>
</table>

$P_{uf} =$ Factored floor load (just eccentrically applied load)

$P_{uw} =$ Factored wall load (includes wall and parapet weight; found at midheight of wall between supports (9 ft from bottom)
Example - OOP: $M_{cr}$

Find modulus of rupture; use linear interpolation between no grout and full grout

Ungroused (Type S masonry cement): _____ psi

Fully grouted (Type S masonry cement): 153 psi

$$f_r = ____ psi + (153 psi - ____ psi) \left( \frac{1 \text{ cell grouted}}{6 \text{ cells}} \right) = ____ psi$$

Find $M_{cr}$, cracking moment:

Commentary allows one to include axial load

Use minimum axial load (once wall has cracked, it has cracked)

$$M_{cr} = \left( \frac{P_u/A_n + f_r I_z}{t/2} \right)$$

Example - OOP

Check strength: $1.2D + 1.0W + 0.5L_r$

$(P_{ult}/2) = \text{mid-height moment from moment at top of wall.}$

- There will be a moment at the top of the wall from eccentric load and lateral forces on parapet
- Method assumes wind is providing suction on wall
  - Moments from lateral wind and eccentric load add together
- Lateral wind load on parapet will cause moment at top to decrease
  - Decrease in moment from parapet wind is ignored in calculations (that is, wind load is considered to be zero on parapet)
- Earthquake lateral forces on parapet would be included; first mode has motion in opposite directions.
Out-of-Plane Loads

Wind Load

Seismic Load

Example - OOP: Moment of Inertia

Find $I_{cr}$, cracked moment of inertia.

\[ c = \frac{A_s f_y + P_u}{0.64 f'_m b} \]

\[ n = \frac{E_s}{E_m} = \frac{29000ksi}{1800ksi} = 16.1 \]

\[ I_{cr} = n \left( A_s + \frac{P_u l_{sp}}{f_y 2d} \right) (d - c)^2 + \frac{bc^3}{3} \]
**Example - OOP**

Check strength; \(1.2D + 1.0W + 0.5L_r\).

Solving for \(M\):

\[ M_u \left(1 - \frac{5P_u h^2}{48E_m I_{cr}}\right) = \frac{w_u h^2}{8} + \frac{P_{uf} e}{2} + \frac{5M_{cr} P_u h^2}{48E_m} \left(\frac{1}{I_g} - \frac{1}{I_{cr}}\right) \]

\[ M_u > M_{cr} \]

\[
\frac{w_u h^2}{8} + \frac{P_{uf} e}{2} + \frac{5M_{cr} P_u h^2}{48E_m} \left(\frac{1}{I_g} - \frac{1}{I_{cr}}\right) = \]

---

**Example - OOP**

Check strength; \(1.2D + 1.0W + 0.5L_r\).

Compare to capacity:

\[
a = \frac{A_s f_y + P_u / \phi}{0.80 f_m' b} = \]

\[
\phi M_u = \phi P_u / \phi + A_s f_y \left(d - \frac{a}{2}\right) \]
Example – OOP Strength

Summary of Strength Design Load Combinations

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>$M_u$ (kip-in/ft)</th>
<th>$\phi M_n$ (kip-in/ft)</th>
<th>$M_u/\phi M_n$</th>
<th>Second Order $M / \text{First Order } M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D+1.0W</td>
<td>16.4</td>
<td>17.1</td>
<td>0.96</td>
<td>1.05</td>
</tr>
<tr>
<td>1.2D+1.6$L_r$+0.5W</td>
<td>10.0</td>
<td>21.0</td>
<td>0.48</td>
<td>1.08</td>
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<tr>
<td>1.2D+1.0W+0.5$L_r$</td>
<td>17.8</td>
<td>18.9</td>
<td>0.94</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Example – OOP Deflections

Check deflections:

\[
\delta = \frac{5M_{cr}h^2}{48E_mI_n} + \frac{5(M - M_{cr})h^2}{48E_mI_{cr}}
\]

Deflection Limit \( \delta_s \leq 0.007h = 0.007(216\text{in}) = 1.51\text{in} \)

\[
M > M_{cr}
\]
Example: Moment Magnification

Resolve the problem using moment magnification method.

\[ M_u = \psi M_{u,0} \]
\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} \]
\[ P_e = \frac{\pi^2 E_m I_{\text{eff}}}{h^2} \]

A good approximation for most simply supported walls is that maximum moment occurs at midheight. The maximum moment can be calculated as:

\[ M_{u,0} = \frac{w_u h^2}{8} + \frac{P_u e}{2} + \frac{(P_u e)^2}{2w_u h^2} \]

\[ x = \frac{h}{2} - \frac{P_u e}{w_u h} \]

Example – Moment Magnification

Check strength; \(1.2D + 1.0W + 0.5L_r\) (\(P_{uf} = 0.440\text{k/ft}; P_u = 0.964\text{k/ft}\))

Solving for \(\psi\)

\[ P_e = \frac{\pi^2 E_m I_{\text{eff}}}{h^2} \]
\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} = \]

\[ M_{u,0} = \frac{w_u h^2}{8} + \frac{P_u e}{2} + \frac{(P_u e)^2}{2w_u h^2} \]

\[ = \frac{0.032\text{ksf}(216\text{in})^2(1\text{ft/12in})}{8} + \frac{(0.440\text{k/ft})(2.81\text{in})}{2} + \frac{(0.440\text{k/ft})(2.81\text{in})^2}{2(0.032\text{ksf})(216\text{in})^2(1\text{ft/12in})} \]
\[ = (15.55 + 0.62 + 0.01)k\text{-in/ft} = 16.18k\text{-in/ft} \]

\[ M_u = \psi M_{u,0} = 1.162(16.18k\text{-in/ft}) = 18.81\text{kip-in/ft} \]
Example – Moment Magnification

Check strength: \(1.2D+1.0W+0.5L_r\).

Compare to capacity of 18.86 kip-in/ft:

\[
M_u = 18.81 \text{kip} - \text{in} / \text{ft} < 18.86 \text{kip} - \text{in} / \text{ft} = \phi M_n \quad \text{OK}
\]

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>(M_{u,0}) (kip-in/ft)</th>
<th>(\psi)</th>
<th>(M_u) (kip-in/ft)</th>
<th>(\phi M_n) (kip-in/ft)</th>
<th>(M_u/\phi M_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9D+1.0W</td>
<td>15.7</td>
<td>1.086</td>
<td>17.0</td>
<td>17.2</td>
<td>0.99</td>
</tr>
<tr>
<td>1.2D+1.6L_r+0.5W</td>
<td>9.3</td>
<td>1.268</td>
<td>11.7</td>
<td>21.0</td>
<td>0.56</td>
</tr>
<tr>
<td>1.2D+1.0W+0.5L_r</td>
<td>16.2</td>
<td>1.162</td>
<td>18.8</td>
<td>18.9</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Moment Magnification - Deflections

First-order deflection (simply supported wall):

\[
\delta_0 = \frac{5wh^4}{384EI_e} + \frac{P_jeh^2}{16EI_e}
\]

The million dollar question. What do you use for \(I_e\)?

Rewriting TMS 402 OOP equations:

Same as proposed by Bischoff, P. (2005).


\[
I_e = \frac{I_{cr}}{1 - \frac{M_{cr}}{M} \left(1 - \frac{I_{cr}}{I_n}\right)}
\]

TMS 402 beam deflections:

\[
I_e = I_n \left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr} \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]
\]

SEAOSC Task group report on slender concrete walls:

\[
\frac{1}{I_e} = \frac{1}{I_n} + \left(\frac{M_a - M_{cr}}{M_a - M_{cr}}\right) \left(\frac{1}{I_{cr}} - \frac{1}{I_n}\right)
\]
Example – Moment Magnification

Check deflections: Load Combination \( D+0.6W \)

\[
P_f = 500\text{lb/ft} - 0.6(360\text{lb/ft}) = 284\text{lb/ft}
\]

\[
P = 284\text{lb/ft} + 38\text{psf}(9\text{ft}+2.5\text{ft}) = 721\text{lb/ft}
\]

\[
w = 0.6(32\text{psf}) = 19.2\text{psf}
\]

\[
c = \frac{A_x f_y + P}{0.64 f_m' b} = \frac{0.0775in^2 / ft(60ksi) + 0.721k / ft}{0.64(2.0ksi)12in / ft} = 0.350in
\]

\[
I_{cr} = n \left( A_x + \frac{P u t_{sp}}{f_y 2d} \right) (d - c)^2 + \frac{bc^3}{3}
\]

\[
= 16.1 \left( 0.0775in^2 / ft + \frac{0.721k / ft}{60ksi} \right) (3.812in - 0.350in)^2 + \frac{12in / ft (0.350in)^3}{3} = 17.5in^4 / ft
\]

I_{cr} – Chase a rabbit

Some engineers think you should find the neutral axis based on Allowable Stress Design assumptions \((kd)\). TMS 402 code uses \(c\):

- simplicity (same as for strength)
- \(c\) was used in the original development of the provisions.

\[
(n\rho)_{eff} = n\rho + \frac{nP}{F_y bd} = 16.1 \left( \frac{0.0775in^2 / ft}{12in / ft (3.8125in.)} \right) + \frac{16.1(0.721kip / ft)}{32ksi(12in / ft)(3.815in.)} = 0.0352
\]

\[
k = \sqrt{(n\rho)^2_{eff} + 2(n\rho)_{eff} - (n\rho)^2_{eff}} = \sqrt{(0.0352)^2 + 2(0.0352) - (0.0352)} = 0.232
\]

\[
I_{cr} = n \left( A_x + \frac{P u t_{sp}}{F_y 2d} \right) (d - kd)^2 + \frac{b(kd)^3}{3} = 16.6in^4 / ft
\]
Example – Moment Magnification

Solving for \( \psi \)

\[
P_e = \frac{\pi^2 E_m I_{eff}}{h^2} = \frac{\pi^2 (1800ksi)(17.5in^4 / ft)}{(216in)^2} = 6.65k / ft
\]

\[
\psi = \frac{1}{1 - \frac{P_u}{P_e}} = \frac{1}{1 - \frac{0.721k / ft}{6.65k / ft}} = 1.122
\]

\[
M_o = \frac{wh^2}{8} + \frac{P_f e}{2} + \left( \frac{P_f e}{2wh^2} \right)^2
\]

\[
= \frac{0.0192ksf(216in)^2(1ft / 12in)}{8} + \frac{(0.284k / ft)(2.81in)}{2} + \frac{(0.284k / ft)(2.81in)^2}{2(0.0192ksf)(216in)^2(1ft / 12in)}
\]

\[
= (9.33 + 0.40 + 0.01)k-in / ft = 9.73k-in / ft
\]

\[
M = \psi M_o = 1.122(9.73k-in / ft) = 10.92kip-in / ft
\]

Example – Moment Magnification

\[
I_e = \frac{I_{cr}}{1 - \frac{M_{cr}}{M} \left( \frac{1 - I_{cr}}{I_n} \right)} = \frac{17.5in^4 / ft}{1 - \frac{6.96k-in / ft}{10.92k-in / ft} \left( \frac{1 - 17.5in^4 / ft}{332.0in^4 / ft} \right)} = 44.2in^4 / ft
\]

\[
\delta_0 = \frac{5wh^4}{384E_m I_e} + \frac{P_f e h^2}{16E_m I_e}
\]

\[
= \frac{5(0.0192ksf)(216in)^4(1ft / 12in)}{384(1800ksi)(44.2in^4 / ft)} + \frac{0.284k / ft(2.81in)(216in)^2}{16(1800ksi)(44.2in^4 / ft)} = 0.599in.
\]

\[
\delta = \psi \delta_0 = 1.122(0.599in.) = 0.672in.
\]

Deflection Limit \( \delta_s \leq 0.007h = 0.007(216in) = 1.51in \)
Example – Moment Magnification

Check deflections:

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>$\delta_0$ (in)</th>
<th>$\psi$</th>
<th>$\delta$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D+0.6W</td>
<td>0.599</td>
<td>1.112</td>
<td>0.672</td>
</tr>
<tr>
<td>D+0.75(0.6W)+0.75$L_r$</td>
<td>0.351</td>
<td>1.181</td>
<td>0.415</td>
</tr>
<tr>
<td>0.6D+0.6W</td>
<td>0.528</td>
<td>1.058</td>
<td>0.559</td>
</tr>
</tbody>
</table>

Deflection Limit $\delta_s \leq 0.007h = 0.007(216\text{in}) = 1.51\text{in}$

Example – Comparison of $I_e$

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>TMS 402 OOP</th>
<th>TMS 402 Beam</th>
<th>SEAOSC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_e$ (in^4/ft)</td>
<td>$\delta$ (in)</td>
<td>$I_e$ (in^4/ft)</td>
</tr>
<tr>
<td>D+0.6W</td>
<td>44.0</td>
<td>0.675</td>
<td>98.4</td>
</tr>
<tr>
<td>D+0.75(0.6W)+0.75$L_r$</td>
<td>62.1</td>
<td>0.415</td>
<td>147.8</td>
</tr>
<tr>
<td>0.6D+0.6W</td>
<td>48.4</td>
<td>0.559</td>
<td>122.0</td>
</tr>
</tbody>
</table>
Example - Pilaster Design

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; f'\text{m}=2000\ \text{psi}; Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Wind load of 26 psf (pressure and suction) and uplift of 8.1 kips (e=5.8 in.); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Reinforcement

Solution:

Vertical Spanning

\[ d = 11.8\ \text{in} \]

Lateral Load

\[ w = 26\text{psf}(16\text{ft}) = 416\text{lb/ft} \]

Combined Flexural and Axial Loads

Combined Flexural and Axial Loads

Critical location is top of pilaster. \( P_u = 26.9 \text{ kips} \) \( M_u = 156.0 \text{ kip-in} \)

\[ a = d - \sqrt{d^2 - \frac{2\left(P_u(d - h/2) + M_u\right)}{\phi(0.8f'_mb)}} \]

\[ A_s = \frac{0.8f'_mba - P_u}{f_y} / \phi \]
Example - Pilaster Design

Why the negative area of steel?
Sufficient area from just masonry to resist applied forces.
Determine \(a\) from just compression.

\[
a = \frac{P_u}{0.8 f'_m b} = \frac{26.9 \text{kip}}{0.8(2.0 \text{ksi}) \cdot 5.6 \text{in}} = 1.08 \text{in}
\]

Find the moment

\[
M = P_u \left( \frac{t}{2} - \frac{a}{2} \right) = 26.9 \text{kip} \left( \frac{15.6 \text{in}}{2} - \frac{1.08 \text{in}}{2} \right) = 195 \text{kip-in}
\]

\(M_u = 156 \text{ kip-in}\)

Sufficient capacity from just masonry. No steel needed.

---

0.9D + 1.0W  
Check wind suction

At top of pilaster. \(P_u = 0.9(9.6) - 1.0(8.1) = 0.54 \text{ kips}\)
\(M_u = 0.54 \text{ kips}(5.8 \text{ in}) = 3.1 \text{ kip-in}\)

\[
M_{\text{max}} = \frac{M}{2} + \frac{wL^2}{8} + \frac{M^2}{2wL^2}
\quad
x = \frac{L}{2} - \frac{M}{wL}
\quad
\text{If } x < 0 \text{ or } x > L, M_{\text{max}} = M
\]

Find axial force at this point. Include weight of pilaster.
Example - Pilaster Design

0.9D + 1.0W
At top: \( P_u = 0.5 \text{ k} \) \( M_u = 3 \text{ k-in} \)
x = 144 in \( P_u = 2.7 \text{ k} \) \( M_u = 361 \text{ k-in} \)
\[ a = 1.49 \text{ in} \hspace{1cm} A_s = 0.57 \text{ in}^2 \]

1.2D + 1.0W + 0.5S
At top: \( P_u = 8.2 \text{ k} \) \( M_u = 48 \text{ k-in} \)
x = 139 in \( P_u = 11.0 \text{ k} \) \( M_u = 384 \text{ k-in} \)
\[ a = 1.74 \text{ in} \hspace{1cm} A_s = 0.52 \text{ in}^2 \]

1.2D + 1.6S + 0.5W
At top: \( P_u = 22.8 \text{ k} \) \( M_u = 132 \text{ k-in} \)
x = 117 in \( P_u = 25.2 \text{ k} \) \( M_u = 252 \text{ k-in} \)
\[ a = 1.41 \text{ in} \hspace{1cm} A_s = 0.12 \text{ in}^2 \]

Required steel = 0.57 \text{ in}^2
Use 2-#5 each face, \( A_s = 0.62 \text{ in}^2 \)
Total bars, 4-#5, one in each cell

Combined Flexural and Axial Loads
Columns

- Structural member, not built integrally into a wall, designed primarily to resist compressive loads parallel to its longitudinal axis.
- Minimum side dimension is 8 in. (5.3.1.1(b))
- Distance between lateral supports ≤ 99r {h/r ≤ 99} (5.3.1.1(a))
- Minimum reinforcement is 0.0025A_n (5.3.1.3)
- Maximum reinforcement is 0.04A_n (5.3.1.3)
  - Additional maximum reinforcement requirements in strength design
- Minimum of 4 bars (5.3.1.3)
- Fully grouted (5.3.1.2)

Ties: (5.3.1.4)
- ≥ 1/4 in. diameter; located in mortar joint or grout
- spacing ≤ 16 longitudinal bar diameter, 48 tie diameter, or least cross-sectional dimension

Bearing Walls

Location of Reaction:
Wall section

- Members that rotate will cause reaction to shift towards edge
- Members that experience little rotation (deep truss)

Bearing area (4.3.4):

\[ A_1 \sqrt{\frac{A_2}{A_1}} \leq 2A_1 \]

Wall section

- A_2 ends at edge of member or head joint in stack bond

Plan view

Strength Design:
\[ \varphi = 0.6 \ (9.1.4.2) \]
\[ B_n = 0.8f_{m}'A_{br} \ (9.1.8) \]
Bearing Walls

**Distribution of Concentrated Loads Along Wall:** (5.1.3)
Load is dispersed along a 2 vertical: 1 horizontal line.

(a) Distribution of concentrated load through bond beam

(b) Distribution of concentrated load in wall
Combined Flexural and Axial Loads

Prestressed Masonry

Load indicating washer (LIW)