Shear Walls

- Types of Shear Walls
- Shear Wall Stiffness
- Maximum Reinforcement Requirements
- Shear Strength
- Example: simple building
- Shear walls with openings

Shear Walls 1

(unreinforced) shear wall (1.17.3.2.2): Unreinforced wall

(with prescriptive reinforcement).

Shear Walls 2

Reinforcement not required at openings smaller than 16 in in either vertical or horizontal direction

Reinforcement of at least 0.2 in²
Shear Walls

___________ reinforced shear wall (1.17.3.2.4): Reinforced wall with prescriptive reinforcement of detailed plain shear wall.

___________ reinforced shear wall (1.17.3.2.5): Reinforced wall with prescriptive reinforcement of detailed plain shear wall. Spacing of vertical reinforcement reduced to 48 inches.

___________ reinforced shear wall (1.17.3.2.6):
1. Maximum spacing of vertical and horizontal reinforcement is min\{1/3 length of wall, 1/3 height of wall, 48 in. [24 in. for masonry in other than running bond]\}.
2. Minimum area of vertical reinforcement is 1/3 area of shear reinforcement
3. Shear reinforcement anchored around vertical reinforcing with standard hook
4. Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
5. Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall [0.0015 for horizontal reinforcement for masonry in other than running bond].

Minimum Reinforcement of Special Shear Walls

<table>
<thead>
<tr>
<th>Reinforcement Ratio</th>
<th>8 in. CMU wall</th>
<th>12 in. CMU wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A_s) (in^2/ft)</td>
<td>(A_s) (in^2/ft)</td>
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<tr>
<td>0.0007</td>
<td>0.064</td>
<td>#4@32</td>
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<tr>
<td></td>
<td></td>
<td>#5@56</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.092</td>
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<tr>
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<td>0.119</td>
<td>#4@16</td>
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<td></td>
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<td>#5@32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#6@40</td>
</tr>
</tbody>
</table>

Use specified dimensions, e.g. 7.625 in. for 8 in. CMU walls.
Special Shear Walls: Design

Minimum strength, strength design (1.17.3.2.6.1.1):
Design shear strength, $\phi V_n$, greater than 1.25 times
nominal flexural strength, $M_n$, except $V_n$ need not be
greater than 2.5$V_u$.

Seismic Design Category

<table>
<thead>
<tr>
<th>Seismic Design Category</th>
<th>Allowed Shear Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>A or B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D and higher</td>
<td></td>
</tr>
</tbody>
</table>

Response modification factor:
Seismic design force divided by response modification factor, which accounts for
ductility and energy absorption.

Knoxville, Tennessee
Category C for Use Group I and II
Category D for Use Group III (essential facilities)
Shear Walls: Stiffness

- **h/d < 0.25**
  - Stiffness predominates

- **0.25 < h/d < 4.0**
  - Both shear and bending stiffness are important

- **h/d > 4.0**
  - Stiffness predominates

Coupled Shear Walls

- _______ Shear Wall
- _________ Shear Wall
Shear Wall Stiffness

3.1.5.1 Deflection calculations shall be based on cracked section properties. Assumed properties shall not exceed half of gross section properties, unless a cracked-section analysis is performed.

Cantilever wall

\[ k_{\text{cant}} = \frac{Et}{h\left(\frac{h}{L}\right)\left(\frac{h}{L}\right)^2 + 3} \]

\( h = \) height of wall
\( A_v = \) shear area; 5/6A for a rectangle
\( G = \) shear modulus; given as 0.4E (1.8.2.2.2)

Fixed wall (fixed against rotation at top)

\[ k_{\text{fixed}} = \frac{Et}{h\left(\frac{h}{L}\right)^2 + 3} \]

\( t = \) thickness of wall
\( L = \) length of wall

Real wall is probably between two cases; diaphragm provides some rotational restraint, but not full fixity.

T- or L- Shaped Shear Walls

Section 1.9.4 Wall intersections designed either to:

a) ____________________:

b) ____________________:

Connection that transfers shear: (must be in running bond)

a) Fifty percent of masonry units interlock

b) Steel connectors at max 4ft.

c) Intersecting bond beams at max 4 ft. Reinforcing of at least 0.1in\(^2\) per foot of wall

Metal lath or wire screen to support grout

1/4in. x 11/2in. x 28in. with 2in. long 90 deg bends at each end to form U or Z shape
Effective Flange Width (1.9.4.2.3)

Effective flange width on either side of web shall be smaller of actual flange width, distance to a movement joint, or:

- Flange in compression: 6t
- Flange in tension:
  - Unreinforced masonry: 6t
  - Reinforced masonry: 0.75 times floor-to-floor wall height

Maximum reinforcing (3.3.3.5)

No limits on maximum reinforcing for following case (3.3.3.5.4):

\[
\frac{M_u}{V_u d_v} \leq 1 \quad \text{and} \quad R \leq 1.5
\]

Squat walls, not designed for ductility

In other cases, can design by either providing boundary elements or limiting reinforcement.

Boundary element design (3.3.6.5):

More difficult with masonry than concrete

Boundary elements not required if:

- Geometrically symmetrical sections:
  \[ P_u \leq 0.1 f_m' A_g \]
- Geometrically unsymmetrical sections:
  \[ P_u \leq 0.05 f_m' A_g \]

AND

\[
\frac{M_u}{V_u I_w} \leq 1 \quad \text{OR} \quad V_u \leq 3 A_n \sqrt{f_m'} \quad \text{AND} \quad \frac{M_u}{V_u I_w} \leq 3
\]
Maximum reinforcing (3.3.3.5)

Reinforcement limits: Calculated using

- Maximum stress in steel of $f_y$
- Axial forces taken from load combination $D + 0.75L + 0.525Q_E$
- Compression reinforcement, with or without lateral ties, permitted to be included for calculation of maximum flexural tensile reinforcement

### Uniformly distributed reinforcement

\[
A_s = \frac{0.64 f'_m b}{d_v} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P}{d_v} \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)
\]

where:
- $\alpha = 1.5$ ordinary walls
- $\alpha = 3$ intermediate walls
- $\alpha = 4$ special walls

### Compression steel with area equal to tension steel

\[
\rho = \frac{A_s}{bd} = \frac{0.64 f'_m b}{f_y - \min \left\{ \frac{d'_v}{d''} \left( \frac{\varepsilon_{mu} + \alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \right\} \frac{\varepsilon_y}{E_s}} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P}{bd} \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)
\]

Maximum reinforcing

Consider a wall with uniformly distributed steel:

\[
C_m + C_s - T_s = P
\]

\[
C_m = 0.8 f'_m \left( 0.8 \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} d_v \right) b
\]

\[
T_s = f_y A_s \left( \frac{\alpha \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) + \frac{1}{2} \frac{\varepsilon_y}{\varepsilon_{mu}}
\]

\[
C_s = f_y A_s \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left[ \frac{\varepsilon_{mu} - \varepsilon_y}{\varepsilon_{mu}} \right] + \frac{1}{2} \frac{\varepsilon_y}{\varepsilon_{mu}}
\]

\[
= f_y A_s \left( \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)
\]

\[
A_s \text{ taken as total steel}
\]

\[
d_i \text{ is actual depth of masonry}
\]
Maximum reinforcing

$$T_s - C_s = C_m - P$$

$$T_s - C_s = f_y A_s \left( \frac{\alpha \varepsilon_y - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - f_y A_s \left( \frac{\varepsilon_{mu} - 0.5 \varepsilon_y}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) = f_y A_s \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)$$

$$T_s - C_s = f_y A_s \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) = 0.8 f_y' \left( 0.8 \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} d_y \right) b - P = C_m - P$$

$$A_s = \frac{0.64 f_y' \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) P}{d_y}$$

Maximum steel on a per foot length of wall basis given for 8 in. wall

Shear Walls 16

Shear Walls 17
Shear Strength (3.3.4.1.2)

\[ V_n = \_ + \_ \quad \phi = 0.8 \]

\[ V_m = \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_n \sqrt{f_m'} + 0.25 P_u \]

- \( M_u/V_u d_v \) need not be taken > 1.0
- \( P_u = \) axial load

\[ V_s = 0.5 \left( \frac{A_v}{s} \right) f_{yv} d_v \]

Vertical reinforcement shall not be less than one-third horizontal reinforcement; reinforcement shall be uniformly distributed, max spacing of 8 ft (3.3.6.2)

Maximum \( V_n \) is:

\[ V_n \leq 6A_n \sqrt{f_m'} \quad (M_u/V_u d_v) \leq 0.25 \]

\[ V_n \leq 4A_n \sqrt{f_m'} \quad (M_u/V_u d_v) \geq 1.0 \]

Interpolate for values of \( M_u/V_u d_v \) between 0.25 and 1.0

\[ V_n = \frac{4}{3} \left( 5 - 2 \frac{M_u}{V_u d_v} \right) A_n \sqrt{f_m'} \]

Example

Given: 10ft. high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, \( f_m = 1500 \text{psi} \); 2-#5 each end; #5 at 48in. (one space will actually be 40 in.); superimposed dead load of 1kip/ft.

**Required:** Maximum wind load based on flexural capacity; shear steel required to achieve this capacity.

**Solution:** Check capacity in flexure using 0.9D+1.0W load combination.

Weight of wall: 38 psf(10ft) = 0.38 k/ft

(Wall weight from lightweight units, grout at 48 in. o.c.)

\[ P_u = 0.9(1 \text{kip/ft} + 0.38 \text{kip/ft}) = 1.24 \text{kip/ft} \]

Total: 1.24 k/ft(16ft) = 19.88 kips
Example

Solve for stresses, forces and moment in terms of \( c \), depth to neutral axis.

- Use trial and error to find \( c \) such that \( P_n = 19.88 \text{ kips} / 0.9 = 22.09 \text{ kips} \)
- \( P_n = C_1 + C_2 - T_1 - T_2 - T_3 - T_4 \)
- \( c = 28.2 \text{ inches} \)
- Sum moments about middle of wall (8 ft from end) to find \( M_n \)
- \( M_n = 1123 \text{ kip-ft}; \quad M_u = 0.9(1123 \text{ kip-ft}) = 1010 \text{ kip-ft} \)
- Wind = \( M_u / h = (1010 \text{ kip-ft}) / (10 \text{ ft}) = 101 \text{ kips} \)
Example

Calculate net area, $A_n$, including grouted cells.

Net area:

$$M_u / (V_u d_v) =$$

Maximum $V_n$

$$V_n \leq 4A_u \sqrt{f_m'} \quad M_u / (V_u d_v) \leq 0.25 \quad V_n \leq 6A_u \sqrt{f_m'} \quad M_u / (V_u d_v) \geq 1.0$$

$$V_n \leq \left(6 - 2 \frac{M_u / V_u d_v - 0.25}{1.0 - 0.25}\right) A_u \sqrt{f_m'} =$$

Maximum $V_u$

$$V_u \leq 0.8(V_n) = 0.8(132.6\text{kips}) = 106.1\text{kips}$$

Example

$$V_m = \left[4.0 - 1.75 \left(\frac{M_u}{V_u d_v}\right)\right] A_u \sqrt{f_m'} + 0.25P_u$$

$$P_u = 0.9(1\text{k/ft}) = 0.9\text{k/ft}$$

Total: $0.9\text{k/ft}(16\text{ft}) = 14.4\text{kips}$

$$V_s = V_n - V_m = 101k / 0.8 - 80.7k = 45.6k$$

Use #5 bars in bond beams. Determine spacing.

$$V_s = 0.5 \left(\frac{A_v}{s}\right) f_y d_v \quad \Rightarrow \quad s = \frac{0.5A_v f_y d_v}{V_s} =$$

Use 4 bond beams, spaced at 32 in. o.c. vertically.

Actual construction has 15 courses: Bond beam in course 4, 8, 12, and 15 from bottom
Example: Maximum Reinforcing

3.3.3.5 No limit to reinforcement since \( M_u/(V_u d_y) < 1 \).

If we needed to check maximum reinforcing, there are at least two ways.

Check on axial load:

a) Set steel strain to limit and find \( c \)
   Steel strain = \( 1.5\varepsilon_y = 0.00310 \)
   \( c = 0.0025/(0.0025+0.00310)*192 = 85.7 \text{ in.} \)

b) Find axial load for this \( c \)
   From spreadsheet, \( P_u = 181 \text{ kips} \) OK

c) If applied axial load is less than axial load, OK

Find steel strain for given axial load:

a) Through trial and error (or solver), find \( c \) for given axial load
   \( P = 22.08 \text{ kips} \) (Dead load)
   \( c = 28.46 \text{ in.} \)

b) Determine steel strain
   \( \varepsilon_s = (192-28.5)/28.5*0.0025 = 0.0143 \)
   \( = 6.93\varepsilon_y \) OK

   Note: did not include compression steel which is allowed when checking max reinforcing

Example: Issues with Strength Design

If this were a special shear wall:

- \( \phi V_n \geq \text{shear corresponding to } 1.25M_n \) (117.3.2.6.1.1)
- \( 1.25M_n = 1.25(1123) = 1404 \text{ k-ft} \)
- \( \phi V_n = M_n/h = 1404 \text{ k-ft/10 ft} = 140.4 \text{ kips} \)
- \( V_n = 140.4 \text{ kips/0.8} = 175.4 \text{ kips} \)
- But, maximum \( V_n \) is 132.6 kips (76% of required).

Cannot build wall. Need to take out some flexural steel or fully grout to get greater shear capacity.
Partially Grouted Shear Walls

Recent research has shown that TMS Code equations overestimate capacity of partially grouted shear walls.

Two possible modifications:
- Only use face shell area, 2.5in(192in) = 480in² (30% reduction)
- Multiply $A_n$ by $A_n/A_g$, an empirical reduction (53% reduction)

Example

Given: Wall system constructed with 8 in. grouted CMU (Type S mortar).
Required: Determine the shear distribution to each wall

Solution: Use gross properties (will not make any difference for shear distribution).
Example

Pier b

\[
k_b = \frac{E_t}{\left(\frac{h}{L}\right)^2 + 3} = \frac{E(7.62\text{in})}{\left(\frac{112\text{in}}{64\text{in}}\right)^2 + 3} = 0.286\text{in}E
\]

Piers a and c  Determine stiffness from basic principles.

\[
A = \ldots
\]

Find centroid from outer flange surface

\[
\bar{y} = \ldots
\]

\[
I = \ldots
\]

\[
A_v \approx A_{\text{web}} = \ldots
\]

Example

\[
k_{a,c} = \frac{P}{\Delta} = \frac{P}{\frac{Ph^3}{3EI} + \frac{Ph}{A_v G}} = \frac{1}{\frac{(112\text{in})^3}{3E(86000in^4)} + \frac{112\text{in}}{305in^2(0.4E)}} = 0.157\text{in}E
\]

Portion of shear load = stiffness of pier over the sum of the stiffnesses.

\[
V_a = \sum k \times V = \frac{0.157\text{in}E}{0.157\text{in}E + 0.286\text{in}E + 0.157\text{in}E} V = 0.26V \quad V_b = 0.48V
\]

Deflection calculations:

\[
k = (0.600\text{in})E = (0.600\text{in})(1350\text{ksi}) = 810\text{kip/in}
\]

Reduce by a factor of 2 to account for cracking:  \[k = 405\text{kip/in}\]
Example

Required: Design the system for a horizontal earthquake load of 35 kips, a dead load of 1200 lb/ft length of building, a live load of 1000 lb/ft length of building, and a vertical earthquake force of 0.2 times the dead load. Use a special reinforced shear wall (R>1.5).

Solution: Use strength design. Check deflection.

\[
\Delta = \frac{P}{k} = \frac{35k}{405k / \text{in}} = 0.086 \text{in}
\]

\[
\frac{\Delta}{h} = \frac{0.086 \text{in}}{112 \text{in}} = 0.000768 \quad \text{or} \quad \Delta = \frac{h}{1302}
\]

Example

Wall b: \( V_{ub} = 0.48V_u = 0.48(35k) = 16.8k \)
\( D = 1.2k/\text{ft}(\text{Tributary width})(1\text{ft/12in}) + 0.075\text{ksf}(112\text{in})(64\text{in})(1\text{ft}^2/144\text{in}^2) = 14.1\text{kips} \)

Use load combination 0.9D+E
\( M_u = 16.8k(112\text{in})(1\text{ft/12in}) = 156.8k-\text{ft} \quad P_u = 0.9(14.1k) - 0.2(14.1k) = 9.87k \)

Minimum prescriptive reinforcement for special shear walls:
Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall

Use prescriptive reinforcement of \((0.002-0.0007)A_g = 0.0013A_g\) in vertical direction
\( A_s = 0.0013(64\text{in})(7.625\text{in}) = 0.63\text{in}^2\)
Example: Flexural Steel

Use load combination 0.9D+E  \( M_u = 156.8\text{k-ft} \)  \( P_u = 9.87\text{k} \)

Try 2-#4 each end; 1-#4 middle: \( A_s=1.00\text{in}^2 \)

Middle bar is at 28 in. (8 cells, cannot place bar right in center)

From spreadsheet: \( \phi M_n=150.5\text{k-ft} \) \text{NG}

Note: used solver to find value

Try 1-#6 each end; 1-#6 middle: \( A_s=1.32\text{in}^2 \)  \( \phi M_n=183.4\text{k-ft} \) \text{OK}

Try 2-#5 each end; 1-#5 middle: \( A_s=1.55\text{in}^2 \)  \( \phi M_n=214.1\text{k-ft} \) \text{OK}

Use 2-#5 each end; 1-#5 middle: \( A_s=1.59\text{in}^2 \)

---

Example: Maximum Reinforcing

Check maximum reinforcing:

Axial force from load combination D+0.75L+0.525QE

\[
L=1\text{k/ft}(104\text{in})(1\text{ft/12in}) = 8.67\text{kip}
\]

\[
D+0.75L+0.525QE = 14.1\text{k}+0.75(8.67\text{k})+0.525(0) = 20.60\text{kip}
\]

For maximum reinforcing calculations, compression steel can be included even if not laterally supported

For #5 bars and P = 20.60kip, c = 7.77in. (from solver seek on spreadsheet)

\[
\varepsilon_s = \frac{d - kd}{kd} e_{nu} = \frac{60\text{in} - 7.77\text{in}}{7.77\text{in}} 0.0025 = 0.0168 = 8.12\varepsilon_y \geq 4\varepsilon_y
\]

\text{OK}
Example: Shear

\[
\frac{M_u}{V_u d_v} = \frac{V_u (112\text{in})}{V_u (64\text{in})} = 1.75 > 1
\]

Use \( M_u/V_u d_v = 1.0 \)

Find \( P_u \) neglecting wall weight
\[
P_u = 0.9(10.4k) - 0.2(10.4k) = 7.28k
\]

\[
V_{nm} = [4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)]A_n\sqrt{f_m'} + 0.25P_u
\]

\[
= [2.25](7.625\text{in})(64\text{in})\sqrt{1500\text{psi} (1k/1000\text{lb})} + 0.25(7.28k) = 44.3\text{kips}
\]

Ignore any shear steel: \( V_n = 44.3\text{kips} \)

\[
\phi V_n = 0.8(44.3\text{kips}) = 35.5\text{kips}
\]

OK

Example: Shear

\[
M_n = \phi M_n/\phi = 214.1\text{k-ft}/0.9 = 237.9\text{k-ft}
\]

\( V \) corresponding to \( M_n \): \([237.9\text{k-ft}](12\text{in/ft}) = 25.5\text{kips}\)

1.25 times \( V \): \( 1.25(25.5k) = 31.9\text{kips} \)< \( \phi V_n = 35.5\text{kips} \) OK

Minimum prescriptive horizontal reinforcement: \( 0.0007A_g \)
\[
A_s = 0.0007(64\text{in})(7.625\text{in}) = 0.34\text{in}^2
\]

Maximum spacing is 48 in.

Use 3 bond beams, 40in. o.c. (top spacing is only 32in.)

Wall b: Vertical bars: 2-#5 each end; 1-#5 middle
Horizontal bars: #4 in bond beam at 40 in. o.c. (Total 3-#4)
Example

Wall a: \( V_{ua} = 0.26V_u = 0.26(35k) = 9.1k \)
\[ D = 1.2k/ft(20in+40in)(1ft/12in) + 0.075ksf(112in)(40in+48in)(1ft^2/144in^2) = 11.1kips \]

Use load combination 0.9D+E  
\[ M_u = 9.1k(112in)(1ft/12in) = 84.9k-ft \]
\[ P_u = 0.9(11.1k) - 0.2(11.1k) = 7.77k \]

Minimum prescriptive reinforcement: \( 0.0013A_g \)
\[ A_s = 0.0013(88in)(7.625in) = 0.87in^2 \]

Design issues:
- Flange in tension: easy to get steel; problems with \( \rho_{max} \)
- Flange in compression: hard to get steel; \( \rho_{max} \) is easy
- Steel in both flange and web affect \( \rho_{max} \)
- Need to also consider loads in perpendicular direction
- Need to check shear at interface of flange and web

Example

Use load combination 0.9D+E  
\[ M_u = 84.9k-ft \]
\[ P_u = 7.77k \]

Axial force for \( \rho_{max} \) from load combination \( D+0.75L+0.525QE \)
\[ L=1k/ft(60in)(1ft/12in) = 5.00kip \]
\[ D+0.75L+0.525QE = 11.1k+0.75(5.00k)+0.525(0) = 14.85kip \]

To be consistent with other wall, try #5 bars
Try 3 in the flange (either distributed as shown, or one in the corner, and two in the jamb
Try 2 in the web jamb

Flange in tension: \( \phi M_n=146.4k-ft \) \( \boxed{OK} \)
Max reinforcing check: \( \epsilon_s=0.0101=4.86\epsilon_y \)
Note: with 2 #5 in flange, \( \phi M_n=104.8k-ft \), but might need 3 bars for perpendicular direction

Flange in compression: \( \phi M_n=124.3k-ft \) \( \boxed{OK} \)
Max reinforcing check: \( \epsilon_s=0.0417=20.2\epsilon_y \)
Example: Shear

\[ \frac{M_u}{V_u d_v} = \frac{V_u (112\text{in})}{V_u (40\text{in})} = 2.8 > 1 \quad \text{Use } M_u/V_u d_v = 1.0 \]

\[ V_m = 2.25 (7.625\text{in})(40\text{in}) \sqrt{1500\text{psi}(1k/1000lb) + 0.25(4.2k)} = 27.6\text{ kips} \]

Ignore any shear steel: \( V_u = 27.6\text{ kips} \quad \phi V_u = 0.8(27.6\text{ kips}) = 22.1\text{kips} \quad \text{OK} \)

\[ M_n = \phi M_u/\phi = 146.4\text{ k-ft}/0.9 = 162.7\text{ k-ft} \]

V corresponding to \( M_n \): \( [162.7\text{k-ft}/(112\text{in})](12\text{in}/\text{ft}) = 17.4\text{kips} \)

1.25 times V: \( 1.25(17.4k) = 21.8\text{kips} < \phi V_n = 22.1\text{kips} \quad \text{OK} \)

Minimum prescriptive horizontal reinforcement: \( 0.0007 A_g \)

Use 3 bond beams, 40in. o.c. (top spacing is only 32in.)

Walls a,c: Vertical bars: 5-#5 as shown in previous slide
Horizontal bars: #4 in bond beam at 40 in. o.c. (Total 3-#4)

Example

Section 1.9.4.2.4 requires shear to be checked at interface. Check intersection of flange and web in walls a and c.

Approximate shear force is tension force in flexural steel.

\[ V_u = A_i f_y = 3(0.3\text{in}^2)(60\text{ksi}) = 55.8\text{kips} \]

Conservatively take \( M_u/V_u d_v = 1.0 \), and \( P_u = 0 \).

\[ V_m = [4.0 - 1.75(M/V d_v)] A_i \sqrt{f_y^2 + 0.25 P_u} \]
\[ = [4.0 - 1.75(1.0)](7.625\text{in})(112\text{in}) \sqrt{1500\text{psi}(1k/1000lb)} = 74.4\text{kips} \]

\[ \phi V_u = 0.8(74.4\text{kips}) = 59.5\text{kips} \quad \text{OK} \]
Example: Drag Struts

Distributed shear force: \( \frac{V_u}{L} = \frac{35k \text{ in}}{224 \text{ in} \times 1 \text{ ft}} = 1.875k/\text{ft} \)

Look at top of wall:

Drag strut

Shear Walls

44

Shear Walls: Building Layout

1. All elements either need to be isolated, or will participate in carrying the load
2. Elements that participate in carrying the load need to be properly detailed for seismic requirements
3. Most shear walls will have openings
4. Can design only a portion to carry shear load, but need to detail rest of structure
Shear Walls with Openings

Shear walls with openings will be composed of solid wall portions, and portions with piers between openings.

Piers:
- Requirements only in strength design
- Length $\geq 3 \times$ thickness; width $\leq 6 \times$ thickness
- Height $< 5 \times$ length
- Factored axial compression $\leq 0.3A_n f_m$ (3.3.4.3.1)
- One bar in each end cell (3.3.4.3.2)
- Minimum reinforcement is 0.0007bd (3.3.4.3.2)

Shear Walls

Shear Walls with Openings

1. Divide wall into piers.
2. Find flexibility of each pier
3. Stiffness is reciprocal of flexibility
4. Distribute load according to stiffness

$$f = \frac{h^3}{6EI} \left( \frac{2k_i + k_b + 2k_i + k_b + 3}{k_i + 2k_i + 2k_b + k_b} \right) + \frac{12h}{5EA}(1 + \nu)$$

$$k_i = \frac{h}{h_i}$$

$$k_b = \frac{h}{h_b}$$

$$M_i = Vh \left( \frac{k_i (1 + k_i)}{k_i + 2k_i + k_b + k_b} \right)$$

$$M_b = Vh \left( \frac{k_b (1 + k_i)}{k_i + 2k_i + k_b + k_b} \right)$$

As the top spandrel decreases in height, the top approaches a fixed condition against rotation.

If $h_b = 0$

$$f = \frac{h^3}{6EI} \left( \frac{2 + k_i}{2k_i + 1} \right) + \frac{12h}{5EA}(1 + \nu)$$

$$M_i = Vh \left( \frac{k_i}{2k_i + 1} \right)$$

$$M_b = Vh \left( \frac{1 + k_i}{2k_i + 1} \right)$$

Example - Shear Walls with Openings

Given:

Required:

A. Stiffness of wall
B. Forces in each pier under 10 kip horizontal load

Sample calculations: Pier B

\[ h = 4\text{ft} \quad h_i = 4\text{ft} \quad h_b = 8\text{ft} \quad k_i = \frac{h_i}{h} = \frac{4\text{ft}}{4\text{ft}} = 0.0 \quad k_b = \frac{h}{h_b} = \frac{4\text{ft}}{8\text{ft}} = 0.5 \]

\[ I = \frac{tL^2}{12} \quad A = tL = (2\text{ft})t \]

\[ f = \frac{h^3}{6EI} \left( \frac{2k_i + k_b}{k_i + 2k_i k_b + k_b} + \frac{12h}{5EA} \right) \left( 1 + \nu \right) \]

\[ k = \frac{1}{f} = \frac{1}{47.6 / Et} = 0.0210 Et \]
**Example - Shear Walls with Openings**

<table>
<thead>
<tr>
<th>Pier</th>
<th>$h$ (ft)</th>
<th>$h_1$ (ft)</th>
<th>$h_b$ (ft)</th>
<th>$k_t$</th>
<th>$k_b$</th>
<th>$l$ (ft$^3$)</th>
<th>$A$ (ft$^2$)</th>
<th>$\Delta_f$</th>
<th>$\Delta_s$</th>
<th>$f$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>4</td>
<td>0</td>
<td>3</td>
<td>inf</td>
<td>0.667$t$</td>
<td>2t</td>
<td>308.4</td>
<td>18</td>
<td>326.4/Et</td>
<td>0.0031Et</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>0.5</td>
<td>0.667$t$</td>
<td>2t</td>
<td>41.6</td>
<td>6</td>
<td>47.6/Et</td>
<td>0.0210Et</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>0.5</td>
<td>0.667$t$</td>
<td>2t</td>
<td>41.6</td>
<td>6</td>
<td>47.6/Et</td>
<td>0.0210Et</td>
</tr>
</tbody>
</table>

$\Delta_f$ is flexural deformation; $\Delta_s$ is shear deformation

Total stiffness is 0.0451Et
Average stiffness from finite element analysis 0.0440Et
Solid wall stiffness 0.1428Et (free at top); 0.25Et (fixed at top)

---

**Example - Shear Walls with Openings**

Sample calculations: Pier B

\[
V_b = \frac{k_b}{\sum k} V = \frac{0.0210Et}{0.0451Et} 10k = 4.66\text{kips}
\]

\[
M_j = Vh \left( \frac{k_t(1+k_b)}{k_t + 2k_t k_b + k_b} \right) =
\]

\[
M_b = Vh \left( \frac{k_t(1+k_b)}{k_t + 2k_t k_b + k_b} \right) =
\]

<table>
<thead>
<tr>
<th>Pier</th>
<th>$V$ (kips)</th>
<th>$M_j$ (k-ft)</th>
<th>$M_b$ (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.68</td>
<td>3.50</td>
<td>4.66</td>
</tr>
<tr>
<td>B</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
<tr>
<td>C</td>
<td>4.66</td>
<td>11.19</td>
<td>7.46</td>
</tr>
</tbody>
</table>
Example - Shear Walls with Openings

To find axial force in piers, look at FBD of top 4ft of wall.

This 65.9k-ft moment is carried by axial forces in the piers. Axial forces are determined from $Mc/I$, where each pier is considered a concentrated area.

Find centroid from left end.

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

Example - Shear Walls with Openings

The axial force in pier A is:

FBD's of piers:
Example - Shear Walls with Openings

FBD's of entire wall to obtain forces in bottom right spandrel:

\[
\sum F_x = 10k - 0.68k - F_x = 0 \\
F_x = 9.32k
\]

\[
\sum F_y = -4.45k + F_y = 0 \\
F_y = 4.45k
\]

\[
\sum M_z = -10k(16\text{ ft}) + (4.66k - ft) \\
-4.45k(1\text{ ft}) + 4.45k(11\text{ ft}) + M_z = 0 \\
M_z = 110.8k - ft
\]