FE Review – Mechanics of Materials

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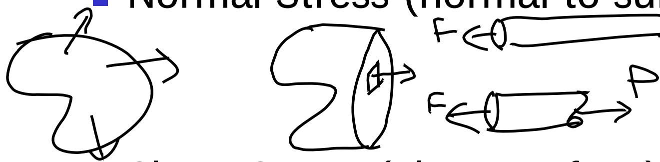
Resources

- You can get the sample reference book:
  www.ncees.org – main site
  http://www.ncees.org/exams/study_materials/fe_handbook
- Multimedia learning material web site: http://web.umr.edu/~mecmovie/index.html

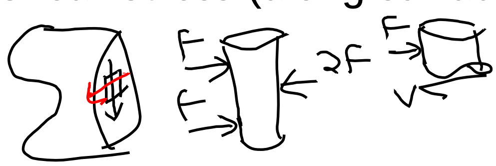
First Concept – Stress

- Normal Stress (normal to surface)



$\sigma = \frac{P}{A}$

- Shear Stress (along surface)



$\tau_{ave} = \frac{V}{A}$

Second Concept – Strain

- Normal Strain – length change
  - Mechanical $\varepsilon = \frac{\delta}{L}$
  - Thermal $\varepsilon = \alpha \Delta T$ ← change in Temp (α: coeff. of thermal expansion)
- Shear Strain – angle change

$\gamma = \frac{\pi}{2} - \theta'$ ← in rad's

Material Properties

- Hooke's Law
  - Normal (1D) $\sigma_x = E\varepsilon_x$
  - Normal (3D) $\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$
    $\varepsilon_y = \cdots -$
    $\varepsilon_z = \cdots$
  - Shear $\tau = G\gamma$

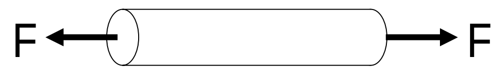
Material Properties

- Poisson's ratio

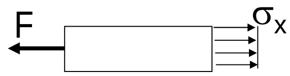
$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

$$G = \frac{E}{2(1+\nu)}$$

Axial Loading

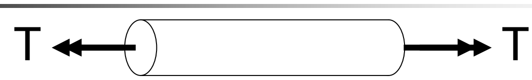


- Stress $\sigma_x = \frac{P}{A}$

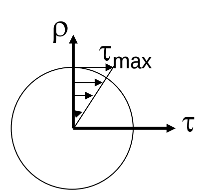


- Deformation $\delta = \sum \frac{PL}{AE}$

Torsional Loading

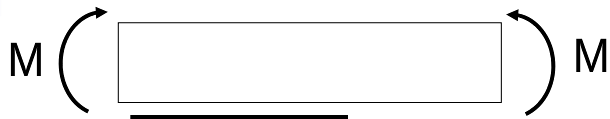


- Stress $\tau = \frac{T\rho}{J}$ $\tau_{\max} = \frac{Tc}{J}$

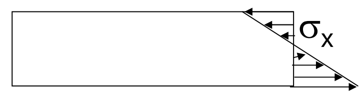


- Deformation $\theta = \sum \frac{TL}{JG}$

Bending Stress

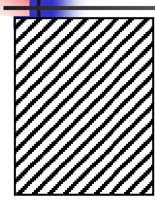


- Stress $\boxed{\sigma_x = -\frac{M_r y}{I}}$ $\sigma_{max} = \frac{M_r c}{I}$



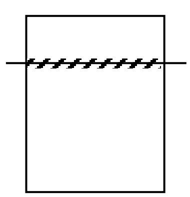
- Find centroid of cross-section
- Calculate I about the Neutral Axis

Transverse Shear Equation



$$\tau_{ave} = \frac{V}{A}$$

Average over entire cross-section



$$\tau_{ave} = \frac{VQ}{Ib}$$

Average over line

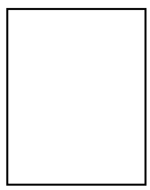
V = internal shear force

b = thickness

I = 2^{nd} moment of area

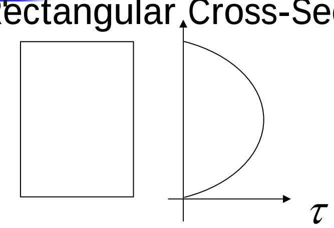
Q = 1^{st} moment of area of partial section

Partial 1st Moment of Area (Q)



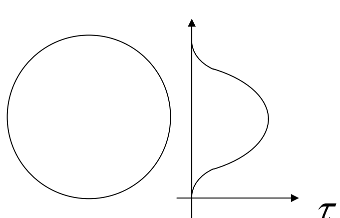
Max. Shear Stresses on Specific Cross-Sectional Shapes

Rectangular Cross-Section



$$\tau_{max} = \frac{3V}{2A}$$

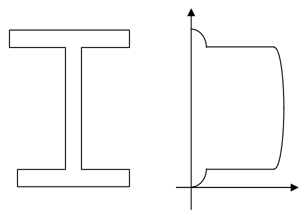
Circular Cross-Section



$$\tau_{max} = \frac{4V}{3A}$$

Max. Shear Stresses on Specific Cross-Sectional Shapes

Wide-Flange Beam



$$\tau_{max} \approx \frac{V}{A_{web}}$$

*10.1 A structural member with the same material properties in all directions at any particular point is

a) homogeneous
b) isotropic
c) isentropic
d) holomorphic

*10.2 The amount of lateral strain in a tension member can be calculated using

a) the bulk modulus
b) Poisson's ratio
c) the yield stress
d) Hooke's law

10.3 Wood has grain resulting in material properties quite different normal to the grain compared with properties parallel to the grain. Such a material is

a) nonhomogeneous
b) nonholomorphic
c) nonorthotropic
d) nonisotropic

*10.4 Find the allowable load, in kN, on a 2-cm-dia, 1-m-long, steel rod if its maximum elongation cannot exceed 0.1 cm.

a) 35 b) 45 c) 55 d) 66

10.5 An elevator is suspended by a 2-cm-dia, 30-m-long steel cable. Twenty people, with a total weight of 14 000 N, enter. How far, in millimeters, does the elevator drop?

a) 3.5 b) 4.5 c) 5.5 d) 6.4

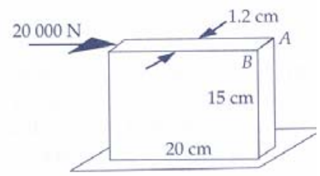
10.6 A hole, one meter from the end of a structural steel member fixed at one end, is 0.8 mm shy of matching another hole for possible connection. What force, in kN, is necessary to stretch it for connection? The cross section is 25 mm × 3 mm.

a) 12.6 b) 13.6 c) 14.7 d) 15.8

10.7 As the load is applied, edge *A B* moves 0.03 mm to the right. Determine the shear modulus, in MPa.

a) 50 300 c) 38 600

b) 41 700 d) 32 500



10.8 A 5-cm-dia steel shaft is subjected to an axial tensile force of 600 kN. What is the diameter, in cm, after the force is applied? Use $\nu = 0.28$.

a) 4.998 b) 4.996 c) 4.994 d) 4.992

*10.9 An aluminum cylinder carries an axial compressive load of 1500 kN. Its diameter measures exactly 12.015 cm and its height 19.311 cm. What was its original diameter, in cm?

a) 12.010 b) 12.008 c) 12.006 d) 12.004

10.10 A tensile stress of 100 MPa exists in a 2-cm-dia steel rod that is fastened securely between two rigid walls. If the temperature increases by 30° C, determine the final stress, in MPa in the rod.

a) 46.7 b) 41.2 c) 36.9 d) 26.2

*10.11 A steel bridge span is normally 300 m long. What is the difference in length, in cm, between January (–35° C) and August (40° C)?

a) 26 b) 28 c) 30 d) 32

10.12 An aluminum bar at 30° C is inserted between two rigid stationary walls by inducing a compressive stress of 70MPa. At what temperature, in ° C, will the bar drop out?

a) 10 b) 0 c) –8 d) –14

*10.13 Brass could not be used to reinforce concrete because
a) its density is too large.
b) its density is too low.
c) it is too expensive.
d) its coefficient of thermal expansion is not right.

*10.14 The maximum shearing stress, in MPa, that exists in a 6-cm-dia shaft subjected to a 200 N·m torque is
a) 4.72 b) 5.83 c) 7.29 d) 8.91

10.15 The shaft of Prob. 10.14 is replaced with a 6-cm-outside diameter, 5-cm inside diameter hollow shaft. What is the maximum shearing stress, in MPa?
a) 5.5 b) 6.4 c) 7.3 d) 9.1

10.16 The maximum allowable shear stress in a 10-cm-dia shaft is 140 MPa. What maximum torque, in N·m, can be applied?
a) 27 500 b) 21 400 c) 19 300 d) 17 100

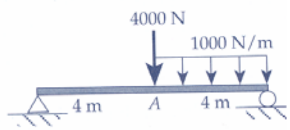
10.17 A builder uses a 50-cm-long, 1-cm-dia steel drill. If two opposite forces of 200 N are applied normal to the shaft, each with a moment arm of 15 cm, what angle of twist, in degrees, occurs in the drill?
a) 29.3 b) 24.6 c) 22.8 d) 21.1

10.18 A solid circular shaft, 8 cm in diameter, transmits a torque of 1200 N·m. Calculate the maximum normal stress in the shaft.
a) 24 MPa b) 18 MPa c) 14 MPa d) 12 MPa

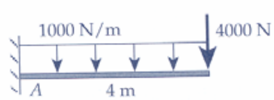
10.19 The maximum bending stress at a given cross section of an I-beam occurs
a) where the shearing stress is maximum.
b) at the outermost fiber.
c) at the joint of the web and the flange.
d) at the neutral axis.

10.20 The moment diagram for a simply-supported beam with a load at the mid-point is a
a) triangle b) parabola c) trapezoid d) rectangle

10.21 Find the bending moment, in N·m, at *A*.
a) 12 000 c) 16 000
b) 14 000 d) 18 000

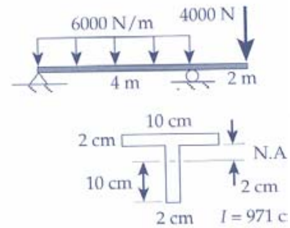


*10.22 What is the bending moment, in N·m, at *A*?
a) 26 000 c) 22 000
b) 24 000 d) 20 000



*10.23 Find the maximum tensile stress, in MPa.

a) 94
b) 86
c) 82
d) 76



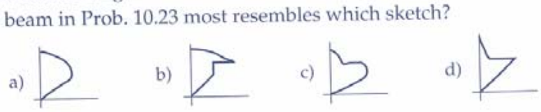
*10.24 What is the maximum compressive stress, in MPa, in the beam of Prob. 10.2

a) 96 b) 90 c) 82 d) 76

*10.25 What is the maximum shearing stress, in MPa, in the beam of Prob. 10.23?

a) 7.2 b) 8.2 c) 9.6 d) 11.3

*10.26 The shearing stress distribution $\tau = VQ/Ib$ on the cross section of the T-beam in Prob. 10.23 most resembles which sketch?

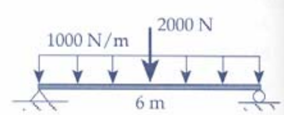


10.27 If the allowable bending stress is 140 MPa in the beam of Prob. 10.22, calculate the *section modulus* defined by I/y, in cm^3.

a) 196 b) 184 c) 171 d) 162

10.28 Find the maximum bending stress, in MPa, if the 10-cm-wide beam is 5 cm deep.

a) 200 c) 160
b) 180 d) 140



10.29 If the beam of Prob. 10.28 were 5 cm wide and 10 cm deep, find the maximum bending stress, in MPa.

a) 80 b) 90 c) 110 d) 120

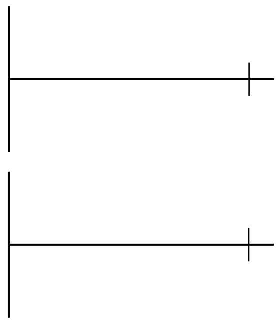
10.30 Find the maximum shearing stress, in MPa, of a simply supported, 6-m-long beam with a 5 cm × 5 cm cross section if it has a 2000 N load at the mid point.

a) 0.6 b) 0.9 c) 1.2 d) 1.6

V & M Diagrams

V

M



$$w = \frac{dV}{dx}$$

$$V = \frac{dM}{dx}$$

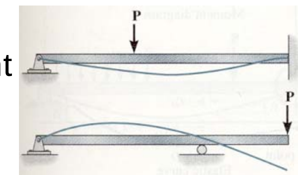
Six Rules for Drawing V & M Diagrams

1. w = dV/dx
   The value of the distributed load at any point in the beam is equal to the slope of the shear force curve.
2. V = dM/dx
   The value of the shear force at any point in the beam is equal to the slope of the bending moment curve.
3. The shear force curve is continuous unless there is a point force on the beam. The curve then "jumps" by the magnitude of the point force (+ for upward force).
4. The bending moment curve is continuous unless there is a point moment on the beam. The curve then "jumps" by the magnitude of the point moment (+ for CW moment).
5. The shear force will be zero at each end of the beam unless a point force is applied at the end.
6. The bending moment will be zero at each end of the beam unless a point moment is applied at the end.

Deflection Equation

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

y = deflection of midplane
M = internal bending moment
E = elastic modulus
I = 2[nd] moment of area with respect to neutral axis

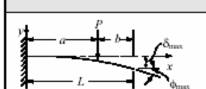
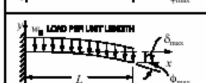
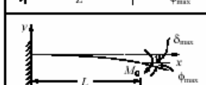
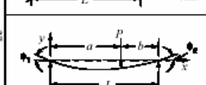
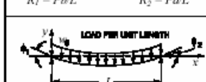
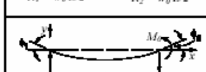


To solve bending deflection problems (find y):

1. Write the moment equation(s) *M(x)*
2. Integrate it twice
3. Apply boundary conditions
4. Apply matching conditions (if applicable)

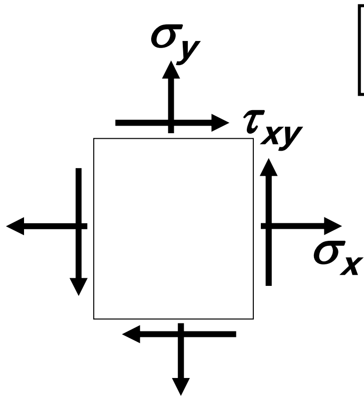
Method of Superposition

Beam Deflection Formulas – Special Cases
(δ is positive downward)

	$\delta = \frac{Pa^2}{6EI}(3x-a)$, for $x > a$ $\delta = \frac{Px^2}{6EI}(-x+3a)$, for $x \le a$	$\delta_{max} = \frac{Pa^2}{6EI}(3L-a)$	$\phi_{max} = \frac{Pa^2}{2EI}$
	$\delta = \frac{w_o x^2}{24EI}(x^2+6L^2-4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
	$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
 $R_1 = Pb/L$ $R_2 = Pa/L$	$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2-b^2)x\right]$, for $x > a$ $\delta = \frac{Pb}{6LEI}\left[-x^3 + (L^2-b^2)x\right]$, for $x \le a$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$ $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
 $R_1 = w_0 L/2$ $R_2 = w_0 L/2$	$\delta = \frac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
 $R_1 = M_0/L$ $R_2 = M_0/L$	$\delta = \frac{M_o Lx}{6EI}\left(1 - \frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$ $\phi_2 = \frac{M_o L}{3EI}$

Stress Transformation

Plane Stress Transformation Equations:



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\frac{\left(\sigma_x - \sigma_y\right)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Stress Transformation

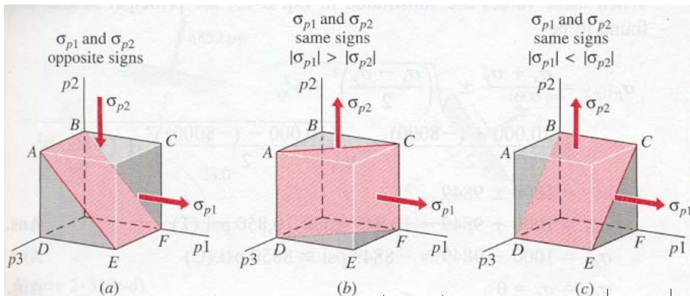
Principal Stresses:

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan\left(2\theta_p\right) = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

Stress Transformation

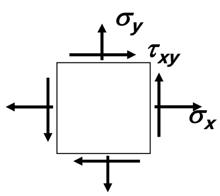
Max Shear Stress:

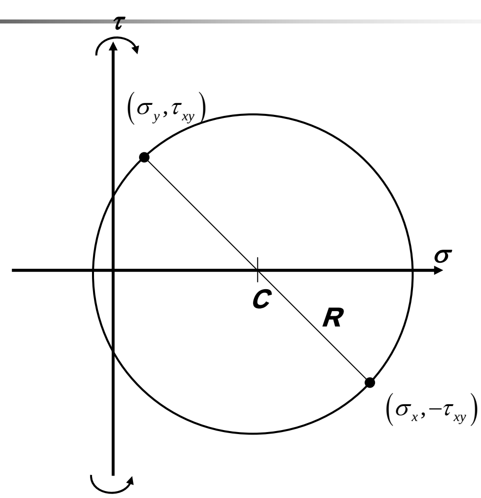


$$\tau_{\max} = \left|\frac{\sigma_{p1} - \sigma_{p2}}{2}\right| \qquad \tau_{\max} = \left|\frac{\sigma_{p1}}{2}\right| \qquad \tau_{\max} = \left|\frac{\sigma_{p2}}{2}\right|$$

Stress Transformation

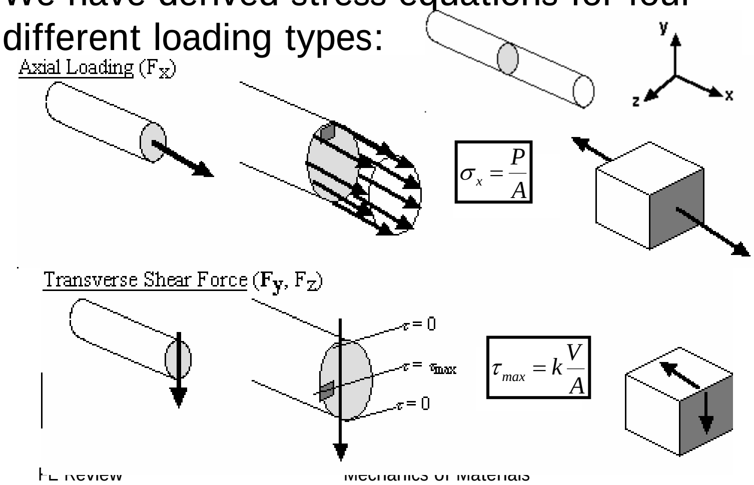
Mohr's Circle

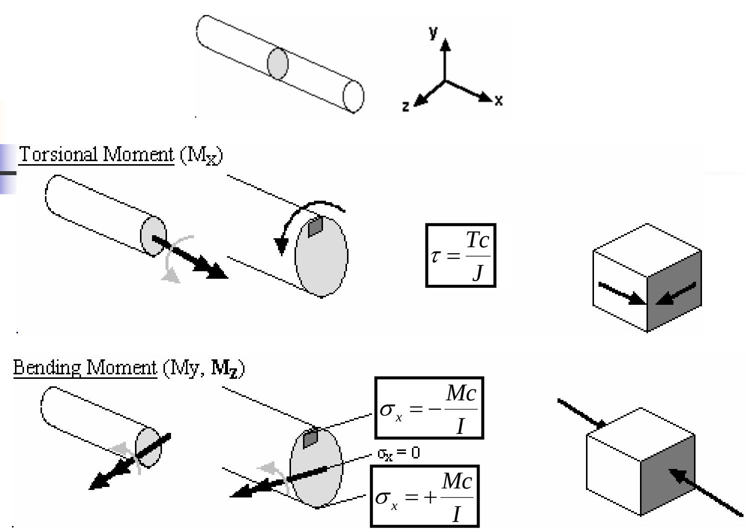




Combined Loading

We have derived stress equations for four different loading types:





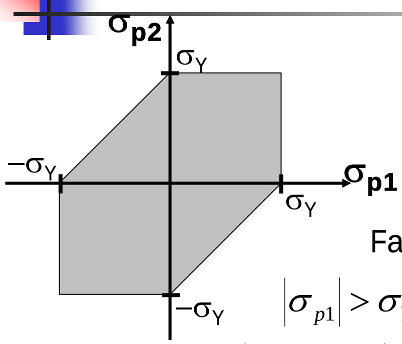
Method for Solving Combined Loading Problems

1. Find internal forces and moments at cross-section of concern.
2. Find stress caused by each individual force and moment at the point in question.
3. Add them up.

Thin-Walled Pressure Vessels

Column Buckling

Maximum Shear Stress Theory



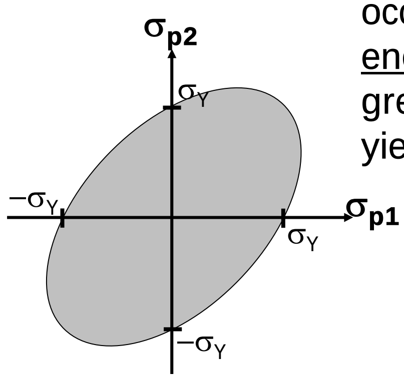
Failure occurs when:

$$|\sigma_{p1}| > \sigma_Y$$ if σ_{p1} and σ_{p2} have the same sign

$$|\sigma_{p1} - \sigma_{p2}| > \sigma_Y$$ if σ_{p1} and σ_{p2} have different signs

where σ_{p1} is the largest principal stress.

Maximum Distortion Energy Theory



This theory assumes that failure occurs when the distortion energy of the material is greater than that which causes yielding in a tension test.

Failure occurs when:

$$\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2 > \sigma_Y^2$$

*10.31 What is the maximum deflection, in cm, of a simply supported, 6-m-long steel beam with a 5 cm × 5 cm cross-section if it has a 2000 N load at the midpoint?

a) 6.35 b) 7.02 c) 7.63 d) 8.23

10.32 Find the maximum deflection, in cm, for the steel beam of Prob. 10.28.

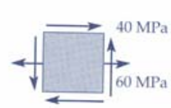
a) 39.7 b) 32.4 c) 28.3 d) 11.8

10.33 If the deflection of the right end of the 5-cm-dia steel beam is 10 cm, what is the load P, in N?

a) 403
b) 523
c) 768
d) 872

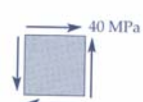
*10.34 Find the maximum shearing stress, in MPa.

a) 80
b) 70
c) 60
d) 50



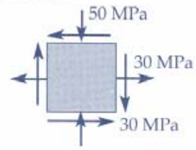
10.35 What is the maximum tensile stress, in MPa?

a) 40
b) 30
c) 20
d) 10



10.36 Determine the maximum shearing stress, in MPa.

a) 80 c) 50
b) 60 d) 40



*10.37 Find the maximum shearing stress, in MPa, in the shaft.

a) 29.5 c) 27.5
b) 28.5 d) 26.5

10.38 The maximum normal stress, in MPa, in the shaft of Prob. 10.37 is

a) 52.8 b) 41.7 c) 36.7 d) 30.1

10.39 The normal stress, in MPa, at pt. *A* is

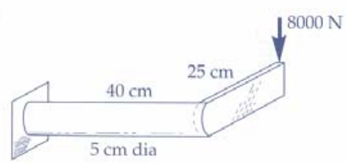
a) 263 c) 228
b) 241 d) 213

10.40 The maximum shearing stress, in MPa, at pt. *A* in Prob. 10.39 is

a) 140 b) 130 c) 120 d) 110

10.41 The maximum shearing stress, in MPa, in the circular shaft is

a) 171 c) 154
b) 167 d) 142



10.42 The maximum tensile stress, in MPa, in the circular shaft of Prob. 10.41 is

a) 284 b) 248 c) 223 d) 212

10.43 The allowable tensile stress for a pressurized cylinder is 180 MPa. What maximum pressure, in kPa, is allowed if the 80-cm-dia cylinder is made of 0.5 cm thick material?

a) 2400 b) 2250 c) 2150 d) 2050

10.44 The maximum normal stress that can occur in a 120-cm-dia steel sphere is 200 MPa. If it is to contain a pressure of 8000 kPa, what must be the minimum thickness, in cm?

a) 1.6 b) 1.4 c) 1.2 d) 1.0

10.45 What is the maximum shearing stress, in MPa, in the sphere of Prob. 10.44?

a) 0 b) 50 c) 100 d) 150

10.49 What is the minimum length, in meters, for which a 10 cm × 10 cm wooden post can be considered a long column? Assume a maximum slenderness ratio of 60.

a) 4.03 b) 3.12 c) 2.24 d) 1.73

*10.50 A free-standing platform, holding 2000 N, is to be supported by a 10-cm-dia vertical aluminum strut. How long, in meters, can it be if a safety factor of 2 is used?

a) 18.3 b) 16.6 c) 14.6 d) 12.2

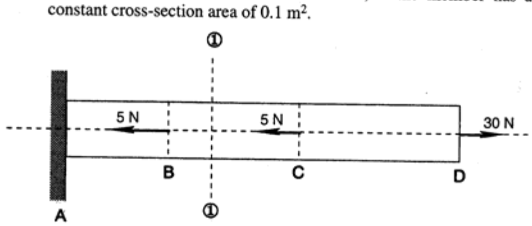
10.51 What increase in temperature, in °C, is necessary to cause a 2-cm-dia, 4-m-long, steel rod with fixed ends to buckle? There is no initial stress.

a) 4.63 b) 5.27 c) 6.34 d) 7.12

10.52 A column with both ends fixed buckles when subjected to a force of 30 000 N. One end is then allowed to be free. At what force, in newtons, will it buckle?

a) 2025 b) 1875 c) 1725 d) 1650

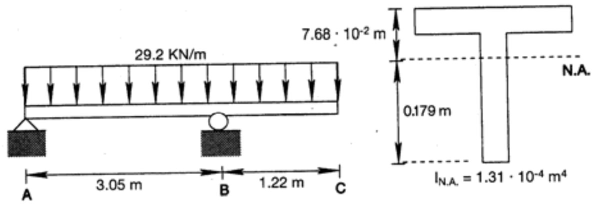
97. Find the normal stress on section ① – ①, if the member has a constant cross-section area of 0.1 m^2.



(A) 250 N/m^2 (tension) (C) 50 N/m^2 (tension)

(B) 250 N/m^2 (compression) (D) 50 N/m^2 (compression)

98. Find the maximum tensile stress of the given beam.

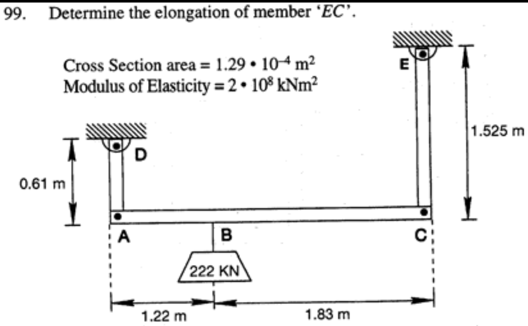


(A) $1.38 \cdot 10^4$ kN/m^2 (C) $5.51 \cdot 10^4$ kN/m^2

(B) $3.45 \cdot 10^4$ kN/m^2 (D) $8.27 \cdot 10^4$ kN/m^2

99. Determine the elongation of member '*EC*'.

Cross Section area = $1.29 \cdot 10^{-4}$ m^2
Modulus of Elasticity = $2 \cdot 10^8$ kNm^2



(A) $5.1 \cdot 10^{-4}$ m (C) $5.2 \cdot 10^{-3}$ m

(B) $1.02 \cdot 10^{-3}$ m (D) $7.87 \cdot 10^{-3}$ m

100. '*AB*' and '*BC*' are circular shafts made from the same material. A twisting moment, or torque, of 271 kNm is applied at the connection '*B*'. Determine the reaction at '*A*'.

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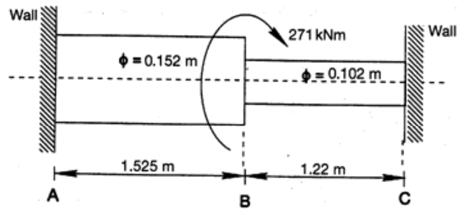
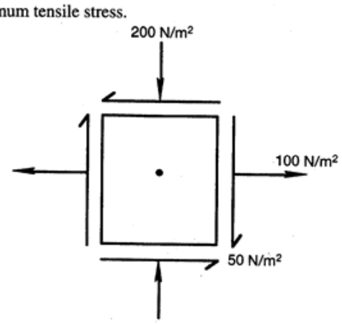


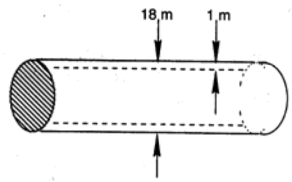
(A) 122 kNm
(B) 136 kNm
(C) 217 kNm
(D) 272 kNm

101. Find the maximum tensile stress.

(A) 100 N/m²
(B) 108 N/m²
(C) 112 N/m²
(D) 206 N/m²



102. Find the maximum pressure that the cylindrical vessel can withstand.



Given: The longitudinal stress cannot exceed 20 N/m^2.

The circumferential (tangential) stress cannot exceed 8 N/m^2.

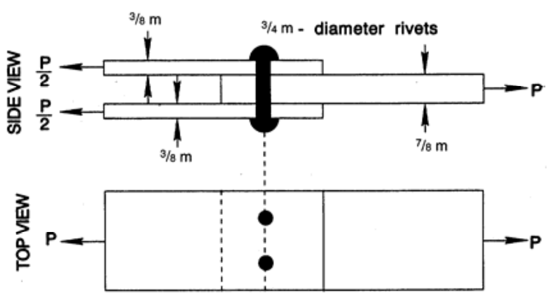
The wall thickness is 1 m.

The diameter of the vessel is 18 m.

(A) 445 N/m^2 (C) 1,780 N/m^2

(B) 890 N/m^2 (D) 2,220 N/m^2

103. What is the maximum load, P, that can be applied to the connection shown if the shear stress in the rivets is limited to 14 N/m^2?



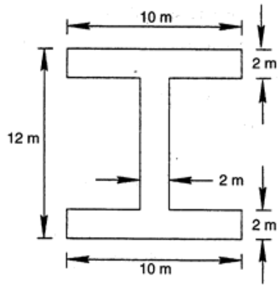
(A) 6 N (C) 25 N

(B) 12 N (D) 50 N

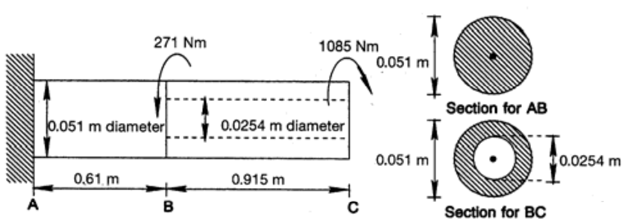
104. Find the maximum shear stress on the given section due to the vertical shear force of 50 N. (See following figure.)

(A) 1,250 N/m^2 (C) 3,210 N/m^2

(B) 2,639 N/m^2 (D) 4,860 N/m^2



105. Find the maximum shear stress.



(A) 1.34 • 10^7 N/m^2 (C) 4.42 • 10^7 N/m^2

(B) 3.16 • 10^7 N/m^2 (D) 7.65 • 10^7 N/m^2

106. Find the maximum compressive stress in concrete. Given the moment at the section is 1,000,000 Nm.

Modulus of Elasticity

Concrete	$Ec = 3 \cdot 10^6$ N/m^2
Steel	$Es = 30 \cdot 10^6$ N/m^2

(A) 750 N/m^2

(B) 1,500 N/m^2

(C) 1,660 N/m^2

(D) 3,220 N/m^2